Solve The Matrix Equation Ax=B For X. A=[[1,6],[7,5]],B=[[35],[60]]

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When faced with a matrix equation of the form Ax = B, solving for the unknown matrix X is a fundamental problem in linear algebra with numerous applications across engineering, computer science, physics, and data analysis. This article provides a comprehensive guide to solving the specific matrix equation where matrix A is [[1, 6], [7, 5]] and matrix B is [[35], [60]]. We will explore the step-by-step process, mathematical concepts involved, methods to find the solution, and practical tips to ensure accuracy and efficiency.

Understanding the Matrix Equation Ax = B

What is the Matrix Equation?

The matrix equation Ax = B involves three matrices:

- A: The coefficient matrix, which in this case is [[1, 6], [7, 5]].
- x: The unknown matrix or vector we want to solve for.
- B: The constant matrix, given as [[35], [60]].

The goal is to find the matrix x such that when it multiplies A, the result is B.

Why is Solving Ax = B Important?

Solving such equations is essential because:

- They model systems of linear equations.
- They are used in computer graphics to transform coordinates.
- They underpin algorithms in machine learning and data science.
- They help in solving engineering problems related to systems control and signal processing.

Mathematical Foundations for Solving Ax = B

Key Concepts in Linear Algebra

Before solving the specific problem, it’s important to understand some fundamental concepts:

- Matrix Inversion: The process of finding a matrix A^{-1} such that $A A^{-1} = I$, where I is the identity matrix.
- Determinant: A scalar value that indicates whether a matrix is invertible (non-zero determinant) or singular (zero determinant).
- Matrix Multiplication: The method of multiplying matrices to produce a new matrix.
- Row Operations: Techniques used in methods like Gaussian elimination to solve linear systems.

Conditions for the Existence of a Solution

A solution for x exists if and only if matrix A is invertible, i.e., its determinant is non-zero. If A is invertible, then:

$x = A^{-1} B$

This formula provides a straightforward way to solve the matrix equation.

Step-by-Step Solution for A=[[1,6],[7,5]] and B=[[35],[60]]

Step 1: Verify that the Matrix A is Invertible

First, compute the determinant of matrix A:

$$
\det(A) = (1)(5) - (6)(7) = 5 - 42 = -37
$$

Since $\det(A) = -37 \neq 0$, matrix A is invertible, and the solution exists.

Step 2: Calculate the Inverse of Matrix A

For a 2x2 matrix:

\[

A = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\]

the inverse is:

\[
A^{-1} = \frac{1}{ad - bc} \begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}
\]

Applying this to matrix A:

\[
A^{-1} = \frac{1}{-37} \begin{bmatrix}
5 & -6 \\
-7 & 1
\end{bmatrix}
\]

which simplifies to:

\[
A^{-1} = \begin{bmatrix}
\frac{5}{-37} & \frac{-6}{-37} \\
\frac{-7}{-37} & \frac{1}{-37}
\end{bmatrix} = \begin{bmatrix}
-\frac{5}{37} & \frac{6}{37} \\
\frac{7}{37} & -\frac{1}{37}
\end{bmatrix}
\]

Step 3: Multiply A^{-1} by B to Find X

Recall:

\[
x = A^{-1} \times B
\]

Given:

\[
A^{-1} = \begin{bmatrix}
-\frac{5}{37} & \frac{6}{37} \\

\frac{7}{37} & -\frac{1}{37}
\end{bmatrix}
\]

and

\[
B = \begin{bmatrix}
35 \\
60
\end{bmatrix}
\]

Compute each element of x:

First element (row 1):

\[
x_1 = \left(-\frac{5}{37}\right) \times 35 + \left(\frac{6}{37}\right) \times 60
\]

Calculate:

\[
x_1 = -\frac{5 \times 35}{37} + \frac{6 \times 60}{37} = -\frac{175}{37} +
\frac{360}{37} = \frac{-175 + 360}{37} = \frac{185}{37} \approx 5
\]

Second element (row 2):

\[
x_2 = \left(\frac{7}{37}\right) \times 35 + \left(-\frac{1}{37}\right) \times 60
\]

Calculate:

\[
x_2 = \frac{7 \times 35}{37} - \frac{60}{37} = \frac{245}{37} - \frac{60}{37} =
\frac{185}{37} \approx 5
\]

Final solution:

\[
X = \begin{bmatrix}
5 \\
5
\end{bmatrix}
\]

or in vector form:

$$
x = \begin{bmatrix} 5 \\ 5 \end{bmatrix}
$$

Interpreting the Solution

The solution to the matrix equation Ax = B is the vector x = [5, 5]. This means:

- When the vector [5, 5] is multiplied by matrix A, the result is matrix B.
- The solution confirms that the system of equations:

$$
\begin{cases}
1 \times x_1 + 6 \times x_2 = 35 \\
7 \times x_1 + 5 \times x_2 = 60
\end{cases}
$$

holds true when $x_1 = 5$ and $x_2 = 5$.

Additional Methods to Solve Ax = B

While matrix inversion is the most straightforward method for small matrices, other techniques are valuable for larger or more complex systems.

1. Gaussian Elimination

- A systematic method involving row operations to reduce the matrix to row-echelon form.
- Suitable for solving larger systems where inversion becomes computationally expensive.

2. LU Decomposition

- Factorizes matrix A into lower and upper triangular matrices.
- Efficient for multiple systems with the same A but different B.

3. Using Computational Tools

- Software like MATLAB, NumPy (Python), or online calculators can quickly compute solutions.
- Ideal for real-world applications involving large matrices.

Practical Tips for Solving Matrix Equations

- Always verify that the coefficient matrix is invertible before attempting to invert.
- Use accurate calculations or computational tools to avoid errors, especially with large or complex matrices.
- Understand the properties of matrices, such as rank and determinant, to determine the feasibility of solutions.
- When dealing with high-dimensional matrices, consider numerical stability and computational efficiency.

Summary

Solving the matrix equation Ax = B for the specific matrices A=[[1,6],[7,5]] and B=[[35],[60]] involves calculating the inverse of A and multiplying it by B. The process confirms that the solution vector x = [5, 5] satisfies the system. This step-by-step approach highlights the importance of matrix invertibility, determinant calculation, and matrix multiplication in solving linear algebra problems. Whether you're a student, engineer, or data scientist, mastering these techniques enhances your ability to solve complex systems efficiently.

Conclusion

Understanding how to solve the matrix equation Ax = B, especially with concrete examples like A=[[1,6],[7,5]] and B=[[35],[60]], is essential for anyone working with linear systems. By following the process of calculating determinants, inverses, and performing matrix multiplication, you can confidently find solutions to similar problems. Leveraging computational tools can further streamline the process, making it accessible for large-scale applications. Developing proficiency in these methods provides a solid foundation for advanced studies and practical problem-solving in various scientific and engineering domains.

This comprehensive guide aims to equip you with the knowledge and techniques necessary to solve matrix equations effectively, ensuring accuracy and confidence in your linear algebra applications.

Frequently Asked Questions

How do I solve the matrix equation Ax = B for X when A = [[1,6],[7,5]] and B = [[35],[60]]?

To solve for X, find the inverse of A (if it exists) and multiply it by B: $X = A^{-1} B$. First, compute the determinant of A to ensure it's invertible, then find its inverse, and multiply by B.

Is the matrix A = [[1,6],[7,5]] invertible?

Yes, to check invertibility, compute the determinant: det(A) = (1)(5) - (6)(7) = 5 - 42 = -37. Since the determinant is non-zero, A is invertible.

How do I compute the inverse of matrix A = [[1,6],[7,5]]?

The inverse of a 2x2 matrix [[a,b],[c,d]] is (1/det) [[d, -b], [-c, a]]. For A, det = -37, so A^{-1} = (1/-37) [[5, -6], [-7, 1]].

What is the inverse of matrix A = [[1,6],[7,5]]?

The inverse is A^{-1} = (1/ -37) [[5, -6], [-7, 1]], which simplifies to [[-5/37, 6/37], [7/37, -1/37]].

How do I multiply the inverse of A by B to find X?

Multiply each element of A^{-1} by the corresponding elements in B: $X = A^{-1} B$ = [[-5/37, 6/37], [7/37, -1/37]] [[35], [60]]. Perform standard matrix multiplication to get the solution.

What is the solution X for the matrix equation Ax = B given A and B?

Calculating, $X = A^{-1} B$ = [[(-5/37)35 + (6/37)60], [(7/37)35 + (-1/37)60]] = [[(-175/37) + (360/37)], [(245/37) - (60/37)]]. Simplify to get X = [[185/37], [185/37]].

Can I verify the solution X = [[185/37], [185/37]] by

plugging back into the original equation?

Yes, multiply A by X: A X = [[1,6],[7,5]] [[185/37], [185/37]] = [[(1)(185/37) + 6(185/37)], [7(185/37) + 5(185/37)]]. Simplify to confirm it equals B = [[35],[60]].

What are common mistakes to avoid when solving the matrix equation Ax = B?

Common mistakes include forgetting to check if A is invertible (determinant zero), incorrectly calculating the inverse, or making errors in matrix multiplication. Double-check calculations to ensure accuracy.

Are there alternative methods to solve Ax = B besides finding the inverse of A?

Yes, you can also use methods like Gaussian elimination or LU decomposition to solve the system without explicitly calculating A's inverse, which can be more efficient and numerically stable.

Additional Resources

Solve The Matrix Equation Ax=B For X is a fundamental problem in linear algebra, with applications spanning engineering, physics, computer science, and many other scientific disciplines. At its core, it involves finding an unknown matrix X that satisfies the matrix equation $AX = B$, where A and B are known matrices. This problem is not only theoretical but also highly practical, as many real-world systems can be modeled using matrix equations. In this article, we will explore how to solve the specific matrix equation:

$$
A = \begin{bmatrix} 1 & 6 \\ 7 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 35 \\ 60 \end{bmatrix}
$$

to find the matrix X. We will delve into the theoretical background, step-by-step solution process, and discuss various methods, their advantages, disadvantages, and applications.

Understanding the Matrix Equation $AX = B$

What is the Equation About?

The equation $AX = B$ involves:
- A, a known square matrix (in this case 2×2)

- B, a known matrix (here a 2×1 column vector)
- X, an unknown matrix (which can be a vector in this context)

The goal is to determine the matrix X such that when A multiplies X, the result is B.

Why Is It Important?

Solving such equations allows us to:
- Find unknown parameters in systems of linear equations
- Model and analyze real-world systems
- Perform transformations in graphics and data science
- Solve differential equations numerically

In our specific case, the goal is to find X such that:

$$
\begin{bmatrix} 1 & 6 \\ 7 & 5 \end{bmatrix} \times X = \begin{bmatrix} 35 \\ 60 \end{bmatrix}
$$

Mathematical Approach to Solving $AX = B$

Method 1: Direct Matrix Inversion

The most straightforward method for solving $AX = B$ when A is square and invertible is to compute:

$$
X = A^{-1} B
$$

This approach involves:
- Checking if A is invertible
- Calculating the inverse A^{-1}
- Multiplying A^{-1} by B

Method 2: Cramer's Rule

For 2×2 systems, Cramer's rule provides an explicit formula:

$$
X = \frac{\text{adj}(A) B}{\det(A)}
$$

where:

- \(\det(A) \) is the determinant of \(A \)
- \(\text{adj}(A) \) is the adjugate of \(A \)

This method is efficient for small matrices but becomes impractical for larger systems.

Method 3: Gaussian Elimination

Transform the augmented matrix \([A | B]\) into row echelon form and solve for \(X \) via back-substitution.

Step-by-Step Solution for \(A \) and \(B \)

Step 1: Verify if \(A \) is invertible

Calculate the determinant of \(A \):

\[
\det(A) = (1)(5) - (6)(7) = 5 - 42 = -37
\]

Since \(\det(A) \neq 0 \), \(A \) is invertible, and we can proceed with the inverse-based solution.

Step 2: Find \(A^{-1} \)

For a \(2 \times 2 \) matrix \(A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \),
\[
A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\]

Applying this to our matrix:

\[
A^{-1} = \frac{1}{-37} \begin{bmatrix} 5 & -6 \\ -7 & 1 \end{bmatrix}
\]

which simplifies to:

\[
A^{-1} = \begin{bmatrix} -\frac{5}{37} & \frac{6}{37} \\ \frac{7}{37} & -\frac{1}{37}
\end{bmatrix}
\]

Step 3: Compute \(X = A^{-1} B \)

Multiply \(A^{-1} \) by \(B \):

\[
X = \begin{bmatrix} -\frac{5}{37} & \frac{6}{37} \\ \frac{7}{37} & -\frac{1}{37} \end{bmatrix} \times \begin{bmatrix} 35 \\ 60 \end{bmatrix}
\]

Calculate each component:

- First component:

\[
X_1 = \left(-\frac{5}{37}\right) \times 35 + \left(\frac{6}{37}\right) \times 60
\]

\[
X_1 = -\frac{175}{37} + \frac{360}{37} = \frac{185}{37}
\]

- Second component:

\[
X_2 = \left(\frac{7}{37}\right) \times 35 + \left(-\frac{1}{37}\right) \times 60
\]

\[
X_2 = \frac{245}{37} - \frac{60}{37} = \frac{185}{37}
\]

Thus,

\[
X = \begin{bmatrix} \frac{185}{37} \\ \frac{185}{37} \end{bmatrix}
\]

which simplifies to:

\[
X = \begin{bmatrix} 5 \\ 5 \end{bmatrix}
\]

Interpreting the Solution

The solution \(X = \begin{bmatrix} 5 \\ 5 \end{bmatrix} \) indicates that when \(X \) is this vector, multiplying \(A \) by \(X \) yields \(B \). To verify:

$$
A X = \begin{bmatrix} 1 & 6 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \times 5 + 6 \times 5 \\ 7 \times 5 + 5 \times 5 \end{bmatrix} = \begin{bmatrix} 5 + 30 \\ 35 + 25 \end{bmatrix} = \begin{bmatrix} 35 \\ 60 \end{bmatrix}
$$

which matches the matrix B, confirming the correctness of the solution.

Features, Pros, and Cons of the Solution Method

Features:
- Applicable to square matrices where the determinant is non-zero
- Uses algebraic formulas for quick computation in small systems
- Provides explicit solutions that can be verified easily

Pros:
- Straightforward for small matrices (like 2x2)
- Computationally efficient for small systems
- Easily verifiable

Cons:
- Not suitable for large matrices (computationally expensive)
- Requires the matrix to be invertible
- Numerical instability can occur if the matrix is close to singular

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Alternative Methods and Their Applicability

LU Decomposition:
- Useful for larger matrices
- More efficient and numerically stable than direct inversion

QR Decomposition:
- Suitable for overdetermined systems
- Provides least-squares solutions

Iterative Methods (e.g., Jacobi, Gauss-Seidel):
- Useful for very large, sparse matrices
- Approximate solutions, especially when exact inversion is impractical

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Conclusion

Solving the matrix equation \(AX = B \) involves understanding the properties of matrices, especially invertibility. For the specific matrices provided, the direct inverse approach is straightforward and effective. The detailed calculation confirms that the solution \(X = \begin{bmatrix} 5 \\ 5 \end{bmatrix} \) satisfies the original equation, demonstrating the power and elegance of linear algebra techniques.

In practice, choosing the right method depends on the size and properties of the matrices involved. For small, well-conditioned matrices like in this example, direct inversion and Cramer's rule are quick and reliable. For larger systems, more sophisticated methods like LU or QR decomposition provide better numerical stability and efficiency. Understanding these methods and their contexts is essential for effectively solving matrix equations in diverse scientific and engineering applications.

Solve The Matrix Equation Ax B For X A 1 6 7 5 B 35 60

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