

The First Term Of An A. P Is 5 And The 10th Term Is 95 To Find S10

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When working with arithmetic progressions (A.P.), one common problem involves finding the sum of the first several terms when given specific details about the sequence. In this article, we will explore how to determine the sum of the first 10 terms, denoted as S_{10} , given that the first term (a_1) of an A.P. is 5 and the 10th term (a_{10}) is 95. Understanding this process not only enhances your grasp of sequences but also sharpens your problem-solving skills in algebra.

Understanding Arithmetic Progression (A.P.)

Before diving into calculations, it's essential to understand the basic components of an arithmetic progression.

What is an Arithmetic Progression?

An arithmetic progression is a sequence of numbers in which the difference between any two consecutive terms is constant. This constant difference is called the common difference (d). The general form of an A.P. is:

$$a_1, a_1 + d, a_1 + 2d, \dots, a_1 + (n-1)d$$

where each term after the first is obtained by adding the common difference to the previous term.

Key Components of an A.P.

- First term (a_1): The initial term of the sequence.
- Common difference (d): The fixed amount added to each term to get the next.
- nth term (a_n): The term at position n , calculated as $a_n = a_1 + (n - 1)d$.
- Sum of first n terms (S_n): The total sum of the first n terms.

Given Data and What We Need to Find

In our problem, the given data are:

- First term, $a_1 = 5$
- 10th term, $a_{10} = 95$

Our goal is to find:

- The sum of the first 10 terms, S_{10}

To achieve this, we need to determine the common difference (d) first, then use the sum formula for an A.P.

Step-by-Step Solution to Find S₁₀

Let's outline the steps systematically.

Step 1: Find the Common Difference (d)

Using the formula for the nth term of an A.P.:

$$a_n = a_1 + (n - 1)d$$

Plugging in the known values for the 10th term:

$$95 = 5 + (10 - 1)d$$

Simplify:

$$95 = 5 + 9d$$

Subtract 5 from both sides:

$$95 - 5 = 9d$$

$$90 = 9d$$

Divide both sides by 9:

$$d = 10$$

Result: The common difference, d, is 10.

Step 2: List the First 10 Terms (Optional for Understanding)

Although not necessary for calculating the sum, listing terms can help understand the sequence:

$$a_1 = 5$$

$$a_2 = 5 + 10 = 15$$

$$a_3 = 15 + 10 = 25$$

$$a_4 = 25 + 10 = 35$$

$$a_5 = 35 + 10 = 45$$

$$a_6 = 45 + 10 = 55$$

$$a_7 = 55 + 10 = 65$$

$$a_8 = 65 + 10 = 75$$

$$a_9 = 75 + 10 = 85$$

$$a_{10} = 95 \text{ (matches the given data)}$$

Step 3: Use the Sum Formula for the First n Terms

The sum of the first n terms of an A.P. is given by:

$$S_n = (n/2) [2a_1 + (n - 1)d]$$

For $n = 10$, $a_1 = 5$, and $d = 10$:

$$S_{10} = (10/2) [25 + (10 - 1)10]$$

Calculate step by step:

$$- (10/2) = 5$$

$$- 25 = 10$$

$$- (10 - 1) = 9$$

$$- 910 = 90$$

Now, sum inside brackets:

$$10 + 90 = 100$$

Finally, multiply:

$$S_{10} = 5 \cdot 100 = 500$$

Result: The sum of the first 10 terms, S_{10} , is 500.

Additional Insights and Applications

Understanding how to find S_{10} in this context opens doors to solving similar problems involving arithmetic progressions.

Application in Real-Life Scenarios

- Financial Planning: Calculating total savings over time with regular increments.
- Engineering: Summing incremental measurements or steps.
- Education: Analyzing patterns in sequences and series for academic purposes.

Variations and Extensions

- Finding the sum of a different number of terms: For example, S_{20} , using the same method.
- Determining the n th term for other positions: Using $a_n = a_1 + (n - 1)d$.
- Analyzing non-uniform progressions: Extending concepts to geometric progressions or other series.

Summary

In summary, given the first term and the 10th term of an arithmetic progression, we can efficiently find the common difference, and subsequently, the sum of the first 10 terms. The key steps involve:

- Applying the n th term formula to find the common difference (d).
- Using the sum formula for an arithmetic series to find S_{10} .

In our specific problem, the first term was 5, the 10th term was 95, and through calculation, we found the common difference to be 10, leading to a sum of 500 for the first 10 terms.

Mastering these steps helps you confidently solve similar problems involving arithmetic progressions, enhancing your algebraic understanding and problem-solving skills.

Frequently Asked Questions

What is the first term of the arithmetic progression if $a_1 = 5$ and the 10th term is 95?

The first term a_1 is given as 5.

How do you find the common difference of an arithmetic progression with given terms?

Use the formula for the n th term: $a_n = a_1 + (n - 1)d$. Plug in the known values to solve for d .

Given $a_1 = 5$ and $a_{10} = 95$, what is the common difference d ?

Calculate d as $(a_{10} - a_1) / (10 - 1) = (95 - 5) / 9 = 90 / 9 = 10$.

What is the formula for the sum of the first n terms of an arithmetic progression?

Sum $S_n = n/2 (2a_1 + (n - 1)d)$.

How do you find the sum of the first 10 terms, S_{10} , of the AP?

Use the formula $S_n = n/2 (a_1 + a_n)$. Here, $S_{10} = 10/2 (a_1 + a_{10}) = 5 (5 + 95) = 5 \cdot 100 = 500$.

What is the value of S_{10} for the given AP?

The sum of the first 10 terms, S_{10} , is 500.

Can you verify the 10th term using the common difference and first

term?

Yes, $a_{10} = a_1 + (10 - 1)d = 5 + 9 \cdot 10 = 5 + 90 = 95$, which matches the given 10th term.

What are the key steps to solve for the sum of an AP when given specific terms?

Identify the first term, find the common difference using known terms, then apply the sum formula for n terms.

Why is understanding the relationship between terms and sum important in AP problems?

It allows you to efficiently calculate total sums and understand the progression pattern, which is useful in many applications.

Additional Resources

The First Term Of An A. P Is 5 And The 10th Term Is 95 To Find S_{10}

Understanding the fundamentals of arithmetic progressions (A.P.) is essential for students and enthusiasts of mathematics. An arithmetic progression is a sequence of numbers in which the difference between consecutive terms remains constant. This constant difference is known as the common difference, denoted as d . When analyzing or solving problems involving A.P., knowing specific terms allows us to determine other properties such as the sum of a certain number of terms, which is often a key objective. In this article, we explore a classic problem: given that the first term of an A.P. is 5 and its tenth term is 95, how do we find the sum of the first ten terms, denoted as S_{10} ? We will dissect this problem step-by-step, clarifying the underlying principles and providing a comprehensive understanding.

Understanding the Basics of Arithmetic Progression

Before tackling the specific problem, it is important to establish a clear understanding of the fundamental concepts related to arithmetic progressions.

What Is an Arithmetic Progression?

An arithmetic progression (A.P.) is a sequence of numbers where each term after the first is obtained by adding a constant difference to the previous term. Mathematically, a sequence $\{a_1, a_2, a_3, \dots\}$ is an A.P. if:

$$a_2 = a_1 + d$$

$$a_3 = a_2 + d = a_1 + 2d$$

$$a_4 = a_3 + d = a_1 + 3d$$

... and so forth.

General Term of an A.P.

The n th term, denoted as a_n , of an arithmetic progression can be expressed as:

$$a_n = a_1 + (n - 1)d$$

where:

- a_1 is the first term,
- d is the common difference,
- n is the term number.

Sum of the First n Terms (S_n)

The sum of the first n terms of an A.P. is given by:

$$S_n = (n/2) [2a_1 + (n - 1)d]$$

This formula is fundamental in solving many problems related to arithmetic progressions.

Analyzing the Given Data

In the problem at hand, we are told:

- The first term, $a_1 = 5$
- The 10th term, $a_{10} = 95$

Our goal is to find the sum of the first 10 terms, S_{10} .

This problem involves two key steps:

1. Determining the common difference d .
2. Calculating the sum S_{10} using the known terms.

Let's explore each step in detail.

Step 1: Finding the Common Difference (d)

Given the first term and the 10th term, we can use the general term formula to find the common difference.

Applying the nth Term Formula

Recall:

$$a_n = a_1 + (n - 1)d$$

Plugging in the known values:

$$a_{10} = a_1 + (10 - 1)d$$

$$95 = 5 + 9d$$

Solving for d

Subtract 5 from both sides:

$$95 - 5 = 9d$$

$$90 = 9d$$

Divide both sides by 9:

$$d = 90 / 9 = 10$$

Therefore, the common difference d is 10.

Step 2: Calculating the Sum of the First 10 Terms (S₁₀)

With the common difference known, we can now compute S₁₀.

Using the Sum Formula

Recall:

$$S_n = (n/2) [2a_1 + (n - 1)d]$$

Substitute $n = 10$, $a = 5$, and $d = 10$:

$$S_{10} = (10/2) [25 + (10 - 1)10]$$

Simplify step-by-step:

$$- (10/2) = 5$$

$$- 25 = 10$$

$$- (10 - 1) = 9$$

$$- 90 = 90$$

Now, compute inside the brackets:

$$10 + 90 = 100$$

Finally, multiply:

$$S_{10} = 5 \cdot 100 = 500$$

Thus, the sum of the first ten terms, S_{10} , is 500.

Interpreting the Results and Practical Implications

The calculation reveals that the sum of the first ten terms of this arithmetic progression is 500. This result has several interpretations and applications:

Real-World Contexts

- Financial Planning: If each term represents an amount received or invested annually, knowing the

total over a decade is crucial.

- Progressive Growth: In scenarios like population growth or resource accumulation, understanding the sum helps in forecasting total growth over a period.
- Educational Settings: This problem exemplifies how to utilize basic formulas to solve for unknowns in sequences, reinforcing foundational algebra skills.

Mathematical Significance

The problem demonstrates the power of relationships within an A.P. — knowing just two terms allows us to uncover the entire sequence's behavior and total sum. It emphasizes the importance of the formula:

$$S_n = (n/2) [2a_1 + (n - 1)d]$$

which is a cornerstone in the study of sequences and series.

Broader Applications and Related Problems

Understanding and calculating sums of arithmetic progressions are fundamental across various disciplines, including:

- Economics: Calculating total revenue or expenses over time with fixed increments.
- Engineering: Summing series of measurements or signals with steady increases.
- Computer Science: Analyzing algorithms with linear complexities where sequence sums are relevant.

Other related problems involve:

- Finding a specific term given the sum and initial term.
- Calculating the sum for different values of n .

- Extending concepts to geometric progressions.

Conclusion

The problem of finding the sum of the first ten terms of an arithmetic progression when given the first and tenth terms illustrates the elegance and utility of mathematical formulas. Starting with the known values, we determined the common difference, then applied the sum formula to find that S_{10} equals 500. This example underscores the importance of understanding fundamental concepts like the n th term and the sum of an A.P., which serve as building blocks for more complex mathematical and real-world problems.

Mathematics becomes more approachable and less intimidating when broken down into clear steps, emphasizing the value of logical reasoning and algebraic manipulation. Whether in academic pursuits or practical applications, mastering these techniques empowers problem-solvers to analyze sequences efficiently and accurately.

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