

Use A Double-angle Identity To Find The Exact Value Of Sin 120 .

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In trigonometry, understanding how to manipulate and evaluate sine values at various angles is fundamental. One effective method to determine the exact value of sine for certain angles, such as 120 degrees, is through the application of double-angle identities. These identities allow us to express trigonometric functions of multiple angles in terms of functions of single angles, simplifying the calculation process significantly. In this comprehensive guide, we will explore how to use the double-angle identity to find the exact value of $\sin 120^\circ$, walk through the step-by-step process, and highlight related concepts to deepen your understanding of trigonometric identities.

Understanding Double-Angle Identities in Trigonometry

What Are Double-Angle Identities?

Double-angle identities are formulas that relate the sine, cosine, and tangent of an angle to the same functions of twice that angle. These identities are derived from the sum formulas for sine and cosine and are essential tools for simplifying trigonometric expressions.

The primary double-angle identities are:

- Sine Double-Angle Identity:

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

- Cosine Double-Angle Identity:

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

or equivalently,

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

- Tangent Double-Angle Identity:

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

These identities are incredibly useful for transforming complex trigonometric expressions into more manageable forms.

Relevance of Double-Angle Identities in Finding $\sin 120^\circ$

The double-angle identities enable us to relate the sine of 120° to known sine and cosine values of specific angles, like 60° , which are standard and well-understood. Since 120° is related to 60° through angle addition or subtraction, the double-angle identity can help us express $\sin 120^\circ$ in terms of simpler, known functions.

Step-by-Step Guide to Using Double-Angle Identity to Find $\sin 120^\circ$

Step 1: Recognize the Angle and Its Relation to Double Angles

Notice that:

$$\begin{aligned} & \left[\right. \\ & 120^\circ = 2 \times 60^\circ \\ & \left. \right] \end{aligned}$$

This suggests that the double-angle identity for sine could be directly applied:

$$\begin{aligned} & \left[\right. \\ & \sin 2\theta = 2 \sin \theta \cos \theta \\ & \left. \right] \end{aligned}$$

with $\theta = 60^\circ$.

Step 2: Recall Known Values of $\sin 60^\circ$ and $\cos 60^\circ$

From standard trigonometric values:

- $\sin 60^\circ = \frac{\sqrt{3}}{2}$
- $\cos 60^\circ = \frac{1}{2}$

These are fundamental values that are often memorized or easily derived from the unit circle.

Step 3: Apply the Double-Angle Identity

Using $\sin 2\theta = 2 \sin \theta \cos \theta$, with $\theta = 60^\circ$:

$$\sin 120^\circ = 2 \sin 60^\circ \cos 60^\circ$$

Substituting the known values:

$$\sin 120^\circ = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

Simplify step-by-step:

$$\sin 120^\circ = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= (2 \times \frac{\sqrt{3}}{2}) \times \frac{1}{2}$$

$$= \sqrt{3} \times \frac{1}{2}$$

$$\begin{aligned} & \sqrt{\frac{\sqrt{3}}{2}} \\ & \end{aligned}$$

Result:

$$\boxed{\sin 120^\circ = \frac{\sqrt{3}}{2}}$$

This confirms the exact value of $\sin 120^\circ$.

Alternative Approach Using Angle Addition Formula

While the primary method involves the double-angle identity, it's beneficial to understand how the angle addition formula can also be used to verify the result.

Using the Angle Addition Formula for Sine

Recall:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

Express $\sin(120^\circ)$ as $\sin(60^\circ + 60^\circ)$:

$$\sin 120^\circ = \sin (60^\circ + 60^\circ)$$

Applying the formula:

$$\sin 120^\circ = \sin 60^\circ \cos 60^\circ + \cos 60^\circ \sin 60^\circ$$

Simplify:

$$\sin 120^\circ = 2 \times \sin 60^\circ \times \cos 60^\circ$$

This is the same as the double-angle identity, confirming the consistency of the approach.

Understanding the Sign of $\sin 120^\circ$

Since 120° lies in the second quadrant (between 90° and 180°), the sine function is positive in this quadrant. This aligns with the positive value of $\frac{\sqrt{3}}{2}$.

Key Point:

- Quadrant II: $\sin$ is positive.
- Quadrant III: $\sin$ is negative.
- Quadrant I: $\sin$ is positive.

- Quadrant IV: $\sin$ is negative.

Therefore, the positive value aligns with the known behavior of sine in the second quadrant.

Related Trigonometric Identities and Concepts

Using Cosine Double-Angle Identity to Find $\cos 120^\circ$

Similarly, the cosine double-angle identity:

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

Applying for $\theta = 60^\circ$:

$$\cos 120^\circ = 2 \times \left(\frac{1}{2}\right)^2 - 1 = 2 \times \frac{1}{4} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$$

This helps in understanding the cosine value at 120° , which is useful for other trigonometric calculations.

Summary:

Angle	$\sin$ value	$\cos$ value
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|-----|-----|-----|
| 60° | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ |
| 120° | $\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ |

Other Useful Identities Related to 120°

- Complementary angles: $(180^\circ - \theta)$ often helps find related sine and cosine values.
- Symmetry in the unit circle: Sine and cosine functions exhibit symmetry which can be exploited for quick calculations.

Practical Applications of Finding $\sin 120^\circ$

Understanding how to evaluate $\sin 120^\circ$ precisely is crucial in several practical scenarios:

- Engineering: Signal processing and wave analysis often require precise trigonometric values.
- Physics: Calculations involving projectile motion or wave phenomena.
- Mathematics education: Building foundational knowledge in trigonometry and preparing for advanced topics.
- Computer graphics: Rotation and transformation calculations often involve trigonometric functions.

Summary and Key Takeaways

- The double-angle identity is a powerful tool for evaluating sine at angles like 120° , which is double 60° .
- Recognizing the relationship between angles and using known values simplifies the calculation.
- The exact value of $\sin 120^\circ$ is $\frac{\sqrt{3}}{2}$.
- The sine function is positive in the second quadrant, confirming the sign of the value.
- Other identities, such as the cosine double-angle identity, complement this approach and provide additional insights.

Conclusion

Applying the double-angle identity to find $\sin 120^\circ$ demonstrates the elegance and utility of trigonometric identities. By recognizing that $120^\circ = 2 \times 60^\circ$, we can directly use the identity $\sin 2\theta = 2 \sin \theta \cos \theta$, substituting the known sine and cosine values of 60° , to arrive at the exact value efficiently. Mastery of these identities enhances problem-solving skills in mathematics and physics.

Frequently Asked Questions

How can I use the double-angle identity to find the exact value of $\sin 120^\circ$?

You can express 120° as $2 \times 60^\circ$ and then use the double-angle identity for sine: $\sin 2\theta = 2 \sin \theta \cos \theta$. By finding $\sin 60^\circ$ and $\cos 60^\circ$, and applying the identity, you can compute $\sin 120^\circ$.

What are the steps to find $\sin 120^\circ$ using the double-angle formula?

First, recognize that $120^\circ = 2 \times 60^\circ$. Then, apply $\sin 2\theta = 2 \sin \theta \cos \theta$ with $\theta = 60^\circ$. Substitute $\sin 60^\circ = \frac{\sqrt{3}}{2}$ and $\cos 60^\circ = 1/2$, resulting in $\sin 120^\circ = 2 \times (\frac{\sqrt{3}}{2}) \times (1/2) = \frac{\sqrt{3}}{2}$.

Why is the double-angle identity useful for calculating $\sin 120^\circ$?

Because 120° is twice 60° , the double-angle identity allows us to find $\sin 120^\circ$ by using known values of $\sin 60^\circ$ and $\cos 60^\circ$, simplifying the calculation without a calculator.

Can the double-angle identity be applied to angles other than 120° ?

Yes, the double-angle identity for sine can be applied to any angle θ to find $\sin 2\theta$, making it a versatile tool for calculating sines of doubled angles.

What is the exact value of $\sin 120^\circ$, and how does the double-angle identity confirm it?

The exact value of $\sin 120^\circ$ is $\frac{\sqrt{3}}{2}$. Using the double-angle identity with $\theta = 60^\circ$, $\sin 120^\circ = 2 \sin 60^\circ \cos 60^\circ = 2 \times (\frac{\sqrt{3}}{2}) \times (1/2) = \frac{\sqrt{3}}{2}$, confirming the exact value.

How does understanding the double-angle identity enhance trigonometry problem-solving skills?

It allows for easier computation of sine values of doubled angles, simplifies complex expressions, and helps in deriving other identities, thereby improving overall problem-solving efficiency in trigonometry.

Additional Resources

[Use A Double-angle Identity To Find The Exact Value Of Sin 120°](#)

In the realm of trigonometry, understanding the relationships between angles and their sine, cosine,

and tangent values is fundamental. Among these techniques, the use of double-angle identities stands out for its power and elegance, especially when calculating exact values of trigonometric functions for angles that are not immediately familiar. One such application involves finding the exact value of $\sin 120^\circ$, a question often posed to students learning about identities and transformations. This article explores the process thoroughly, examining the theoretical foundations, step-by-step derivations, and broader implications of using double-angle identities to evaluate $\sin 120^\circ$.

Understanding the Foundation: Trigonometric Identities and Angle Relationships

Before delving into the specific problem, it is essential to establish a solid understanding of the basic trigonometric identities and the angle relationships that serve as the backbone of such calculations.

Basic Trigonometric Identities

Trigonometry relies heavily on identities—equations that relate different trigonometric functions or manipulate angles to simplify calculations. Some fundamental identities include:

- Pythagorean identities:

$$\sqrt{\sin^2 \theta + \cos^2 \theta} = 1$$

- Angle sum and difference identities:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

\\

\\

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

\\

- Double-angle identities:

\\

$$\sin 2\theta = 2\sin \theta \cos \theta$$

\\

\\

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta$$

\\

These identities allow us to express trigonometric functions of multiple angles in terms of functions of a single angle, which is invaluable when exact values are needed.

Relationship Between 120° and Known Angles

The angle 120° is a prominent member of the second quadrant, where the sine value is positive, and cosine is negative. Recognizing that 120° can be expressed in terms of familiar angles—specifically as 180° - 60°—is key to simplifying calculations:

\\

$$120^\circ = 180^\circ - 60^\circ$$

\\

This relationship enables the use of supplementary angle identities, which state:

\\

$$\sin(180^\circ - \theta) = \sin \theta$$

\]

Applying this to 120° , we get:

\[

$$\sin 120^\circ = \sin 60^\circ$$

\]

While this direct approach provides an immediate answer, it bypasses the opportunity to demonstrate how double-angle identities can be used to arrive at the same result, reinforcing the interconnectedness of trigonometric formulas.

Applying the Double-Angle Identity to Find $\sin 120^\circ$

The core of this investigation lies in applying the double-angle identity strategically to evaluate $\sin 120^\circ$ precisely, rather than relying solely on known values or geometric interpretations.

Step 1: Recognize the Angle as a Sum or Difference

To employ the double-angle identity, consider expressing 120° as a multiple of 60° , or relate it to angles where sine and cosine values are known—such as 60° and 30° . Since 120° is twice 60° , it suggests using the double-angle formula for sine:

\[

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

\]

If we let:

$$\begin{aligned} &2\theta = 120^\circ \\ &\end{aligned}$$

then:

$$\begin{aligned} &\theta = 60^\circ \\ &\end{aligned}$$

Our goal becomes finding $\sin 120^\circ$ via the double-angle identity, knowing $\theta = 60^\circ$.

Step 2: Find $\sin 60^\circ$ and $\cos 60^\circ$

The exact values of $\sin 60^\circ$ and $\cos 60^\circ$ are well-known:

$$\begin{aligned} \sin 60^\circ &= \frac{\sqrt{3}}{2} \\ \cos 60^\circ &= \frac{1}{2} \end{aligned}$$

These serve as the foundation for applying the double-angle formula.

Step 3: Use the Double-Angle Identity

Applying the double-angle formula:

$$\begin{aligned} & \backslash[\\ & \sin 2\theta = 2 \sin \theta \cos \theta \\ & \backslash] \end{aligned}$$

Substituting $(\theta = 60^\circ)$:

$$\begin{aligned} & \backslash[\\ & \sin 120^\circ = 2 \times \sin 60^\circ \times \cos 60^\circ \\ & \backslash] \end{aligned}$$

Plugging in the known values:

$$\begin{aligned} & \backslash[\\ & \sin 120^\circ = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} \\ & \backslash] \end{aligned}$$

Simplify:

$$\begin{aligned} & \backslash[\\ & \sin 120^\circ = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} \\ & \times \frac{1}{2} = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2} \\ & \backslash] \end{aligned}$$

Thus, the exact value of $(\sin 120^\circ)$ derived via the double-angle identity is:

$$\begin{aligned} & \backslash[\\ & \boxed{\sin 120^\circ = \frac{\sqrt{3}}{2}} \end{aligned}$$

\]

Alternative Approaches and Confirmations

While the above method directly applies the double-angle identity for $(\sin 2\theta)$, alternative methods can confirm this result, such as the sine addition formula or geometric constructions.

Using the Sine Addition Formula

Express 120° as a sum:

\[

$$120^\circ = 60^\circ + 60^\circ$$

\]

Applying the sine addition formula:

\[

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

\]

Set $(A = B = 60^\circ)$:

\[

$$\sin 120^\circ = \sin 60^\circ \cos 60^\circ + \cos 60^\circ \sin 60^\circ$$

\]

Simplify:

$$\begin{aligned} & \sin 120^\circ = 2 \sin 60^\circ \cos 60^\circ = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \\ & \frac{\sqrt{3}}{2} \end{aligned}$$

This corroborates the earlier result, illustrating the consistency of trigonometric identities.

Broader Implications and Applications

The process of applying double-angle identities extends beyond simple calculations, serving as a gateway to understanding more complex trigonometric relationships, solving equations, and modeling real-world phenomena.

Educational Significance

Understanding how to derive $\sin 120^\circ$ using identities reinforces conceptual comprehension and enhances problem-solving skills. It illustrates how multiple identities interconnect, empowering students to approach unfamiliar problems with confidence.

Practical Applications

- Engineering and Physics: Precise trigonometric calculations underpin wave analysis, signal processing, and mechanical design.

- Computer Graphics: Rotation, shading, and transformations often utilize sine and cosine values for arbitrary angles.
- Navigation and Geospatial Analysis: Accurate angle measurements rely on exact trigonometric evaluations.

Mathematical Generalization

The methodology demonstrated—using double-angle identities—can be generalized to find the sine of other angles expressed as multiples of known angles, aiding in the development of series and Fourier analysis.

Conclusion

The investigation into using a double-angle identity to find the exact value of $\sin 120^\circ$ reveals a fundamental aspect of trigonometry: the interconnectedness of identities and the elegance of mathematical symmetry. By recognizing that 120° is twice 60° , applying the double-angle formula, and leveraging known sine and cosine values of 60° , we find that:

$$\boxed{\sin 120^\circ = \frac{\sqrt{3}}{2}}$$

This result not only demonstrates the power of double-angle identities in simplifying calculations but also underscores their importance as foundational tools in mathematics, engineering, and science. The approach exemplifies how theoretical understanding and practical computation intersect, allowing us to

evaluate exact values of trigonometric functions confidently and accurately.

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