Use (x-3)^2 -6to Solve X^2-6x+3=0 Leave Your Answer In Surd Form

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Solving quadratic equations is a fundamental aspect of algebra, often requiring different methods such as factoring, completing the square, or applying the quadratic formula. In this article, we will focus on solving the quadratic equation $X^2 - 6x + 3 = 0$ by completing the square, specifically using the expression $(x - 3)^2 - 6$, and we will leave the answers in surd (radical) form for precision. Understanding how to manipulate quadratic equations into perfect square form not only simplifies solving but also deepens comprehension of quadratic functions.

Understanding the Quadratic Equation X^2 - 6x + 3 = 0

Before diving into the solution, it's essential to understand the structure of the quadratic equation in question.

Standard Form of a Quadratic Equation

A quadratic equation typically takes the form:

$$
ax^2 + bx + c = 0
$$

where a, b, and c are constants, and $a \neq 0$.

In our case:

$$
X^2 - 6x + 3 = 0
$$

Here, $a = 1$, $b = -6$, and $c = 3$.

Why Complete the Square?

Completing the square is a method that transforms a quadratic into a perfect square trinomial plus or minus a constant. This approach makes solving for x straightforward and provides insight into the graph of the quadratic function.

In particular, the expression $(x - 3)^2 - 6$ is directly related to the quadratic, allowing us to rewrite the original equation in a form that reveals the roots more clearly.

Expressing the Quadratic in Completed Square Form

To solve the quadratic equation using the expression $(x - 3)^2 - 6$, we need to manipulate the original quadratic into this form.

Step 1: Recall the Completing the Square Formula

For a quadratic:

$$
ax^2 + bx + c
$$

when $a = 1$ (which is our case), the quadratic can be written as:

$$
(x + \frac{b}{2})^2 - \left(\frac{b}{2}\right)^2 + c
$$

This is derived from the identity:

$$
x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c
$$

Applying this to our quadratic:

$$
x^2 - 6x + 3
$$

we proceed as follows.

Step 2: Complete the Square

- Take half of the coefficient of x, which is -6:

$$
\frac{-6}{2} = -3
$$

- Square this value:

$$
(-3)^2 = 9
$$

- Rewrite the quadratic as:

$$
x^2 - 6x + 9 - 9 + 3
$$

or:

$$
(x - 3)^2 - 6
$$

Note: We added and subtracted 9 to complete the square, which doesn't change the value of the quadratic expression.

Step 3: Write the Quadratic in Completed Square Form

Thus, the original quadratic can be expressed as:

$$
x^2 - 6x + 3 = (x - 3)^2 - 6
$$

This form is crucial because it directly relates to the expression we wanted to explore and simplifies solving the equation.

Solving the Equation Using the Completed Square Form

Now that we've expressed the quadratic as $(x - 3)^2 - 6$, we proceed to solve the original equation:

$$
X^2 - 6x + 3 = 0
$$

which is equivalent to:

$$
(x - 3)^2 - 6 = 0
$$

Step 1: Set the Expression Equal to Zero

$$
(x - 3)^2 - 6 = 0
$$

Add 6 to both sides:

$$
(x - 3)^2 = 6
$$

Step 2: Solve for x

Take the square root of both sides:

$$
x - 3 = \pm \sqrt{6}
$$

Note that $\sqrt{6}$ is already in surd form, which preserves the exactness of the solution.

Finally, solve for x:

$$
x = 3 \pm \sqrt{6}
$$

Expressing the Solutions in Surd Form

The solutions are:

$$
x = 3 + \sqrt{6} \quad \text{and} \quad x = 3 - \sqrt{6}
$$

These are the exact solutions in surd form, which avoids decimal approximations and maintains mathematical precision.

Why Leave the Roots in Surd Form?

Leaving roots in surd form offers several advantages:

- Exactness: Surd form provides precise solutions without rounding errors.
- Mathematical Clarity: It clearly shows the nature of the roots, especially whether they are rational or irrational.
- Further Calculations: Surd form is essential when these roots are used in subsequent algebraic or calculus problems for exact results.
- Standard Practice: In higher mathematics, expressing roots in surd form is often required for clarity and correctness.

Additional Insights into the Roots and the Quadratic Function

Understanding the roots in surd form also gives insight into the quadratic's graph and properties.

Vertex of the Parabola

The vertex form of a quadratic:

$$
y = (x - h)^2 + k
$$

where (h, k) is the vertex.

Our quadratic:

$$
y = (x - 3)^2 - 6
$$

has a vertex at (3, -6). This confirms that the parabola opens upwards (since the coefficient of x^2 is positive), with its lowest point at (3, -6).

Axis of Symmetry

The axis of symmetry is the vertical line passing through the vertex:

$$
x = 3
$$

which aligns with the solutions found, $x = 3 \pm \sqrt{6}$, symmetric about $x = 3$.

Discriminant and Nature of Roots

The discriminant Δ of a quadratic:

$$
ax^2 + bx + c
$$

is:

$$
\Delta = b^2 - 4ac
$$

Calculating for our quadratic:

$$
\Delta = (-6)^2 - 4 \times 1 \times 3 = 36 - 12 = 24
$$

Since $\Delta > 0$, the quadratic has two distinct real roots, which we confirmed are in surd form.

Summary of the Solution Process

To recap, solving the quadratic equation $X^2 - 6x + 3 = 0$ using the method of completing the square involves these steps:

1. Rewrite the quadratic in the form $(x - 3)^2 - 6$.
2. Set the expression equal to zero: $(x - 3)^2 - 6 = 0$.
3. Isolate the squared term: $(x - 3)^2 = 6$.
4. Take the square root of both sides: $x - 3 = \pm \sqrt{6}$.
5. Solve for x: $x = 3 \pm \sqrt{6}$.

This approach emphasizes the utility of completing the square and illustrates how to find exact roots in surd form without resorting to decimal approximations.

Applications and Importance of Solving Quadratics in Surd Form

Quadratic equations are ubiquitous across various fields such as physics, engineering, economics, and computer science. Mastering solutions in surd form is essential because:

- Precision in Calculations: Exact roots prevent the accumulation of rounding errors.
- Analytical Geometry: Surd roots help in understanding the parabola's properties, intersections, and vertex accurately.
- Advanced Mathematics: Many higher-level problems require expressing solutions in surd form for algebraic manipulation and proof.

Final Tips for Solving Quadratic Equations Using Completing the Square

- Always check if the quadratic is easily factorable before proceeding.
- When completing the square, remember to add and subtract the same number to maintain equality.

- Keep roots in surd form unless a decimal approximation is explicitly required.
- Use the discriminant to determine the nature of roots before solving.

Conclusion

Using the expression \((x - 3)^2 - 6\) provides an elegant and precise method to solve the quadratic equation \(X^2 - 6x + 3 = 0\). By completing the square, we find that the roots are \(x = 3 + \sqrt{6}\) and \(x = 3 - \sqrt{6}\), both in surd form, which

Frequently Asked Questions

How can I use the expression (x - 3)^2 - 6 to solve the quadratic equation x^2 - 6x + 3 = 0?

Start by recognizing that (x - 3)^2 expands to x^2 - 6x + 9. Rewrite the equation using this: (x - 3)^2 - 6 = 0. Then, add 6 to both sides: (x - 3)^2 = 6. Take the square root of both sides: x - 3 = ±√6. Finally, solve for x: x = 3 ± √6.

Why is expressing the solution in surd form important for quadratic equations like x^2 - 6x + 3 = 0?

Expressing solutions in surd form preserves the exact value of roots, avoiding rounding errors that occur with decimal approximations. It provides precise solutions, especially when roots are irrational.

What steps are involved in solving x^2 - 6x + 3 = 0 using the method with (x - 3)^2?

First, rewrite the quadratic as (x - 3)^2 - 6 = 0. Next, isolate the perfect square: (x - 3)^2 = 6. Then, take the square root of both sides, remembering to include both positive and negative roots: x - 3 = ±√6. Finally, solve for x: x = 3 ± √6.

Can you explain how completing the square relates to using (x - 3)^2 in solving quadratic equations?

Completing the square involves rewriting a quadratic in the form (x - h)^2 = k. In this case, x^2 - 6x + 3 is rewritten as (x - 3)^2 - 6, which simplifies solving by taking square roots, leading directly to the solutions in surd form.

What is the significance of leaving the solution in the form x = 3 ± √6 for the quadratic equation?

Leaving the solution as x = 3 ± √6 provides an exact expression for the roots, showing clearly that the solutions are irrational. It maintains mathematical precision and is preferred in higher-level mathematics.

Additional Resources

Use of $(x - 3)^2 - 6$ to Solve $x^2 - 6x + 3 = 0$

Mathematics, renowned for its elegance and precision, often offers multiple pathways to reach the same destination—solving quadratic equations being a prime example. Among these methods, completing the square stands out as a particularly insightful approach, illuminating the structure of quadratic functions and their roots. In this article, we delve into the application of a specific algebraic identity—using $(x - 3)^2 - 6$—to solve the quadratic equation $x^2 - 6x + 3 = 0$, leaving the answers in surd form. Our exploration combines theoretical understanding with practical steps, ensuring clarity and depth for students, educators, and math enthusiasts alike.

Understanding the Quadratic Equation $x^2 - 6x + 3 = 0$

Before jumping into solution techniques, it’s vital to comprehend the structure of the given quadratic:

$$
x^2 - 6x + 3 = 0
$$

- Standard form: The equation is quadratic in x, with coefficients 1 (for x^2), -6 (for x), and 3 (constant term).
- Nature of roots: Since the quadratic’s discriminant, $\Delta = b^2 - 4ac$, will guide us on the roots’ nature, let's compute it:

$$
\Delta = (-6)^2 - 4 \times 1 \times 3 = 36 - 12 = 24
$$

- Discriminant analysis: Because $\Delta = 24$, which is positive but not a perfect square, the roots are real and irrational, hence best expressed in surd form.

Motivation for Completing the Square

The method of completing the square transforms quadratic equations into a perfect square binomial, making roots more transparent, especially when roots are irrational. It is not only a computational tool but also offers insight into the geometry of quadratics.

Why use the expression $(x - 3)^2 - 6$?

- The quadratic $x^2 - 6x + 3$ can be rewritten in a way that isolates a perfect square $(x - 3)^2$.
- Recognizing that the coefficient of x is -6, which relates to the expansion of $(x - 3)^2 = x^2 - 6x + 9$, provides a natural pathway.

Key observation:

$$
x^2 - 6x + 3 = (x - 3)^2 - 6
$$

This identity simplifies solving the quadratic and reveals the roots directly in surd form.

Step-by-Step Solution Using $(x - 3)^2 - 6$

1. Express the quadratic in completed square form

The core idea is to recognize the perfect square:

$$
x^2 - 6x + 3 = (x - 3)^2 - 6
$$

- Verification:

$$
(x - 3)^2 = x^2 - 6x + 9
$$

Subtract 6:

$$
(x - 3)^2 - 6 = x^2 - 6x + 9 - 6 = x^2 - 6x + 3
$$

which confirms the identity.

2. Rewrite the original equation

Substitute:

\[

(x - 3)^2 - 6 = 0
\]

to obtain:

\[
(x - 3)^2 = 6
\]

This step transforms the quadratic into a simple square root problem.

3. Solve for \(x\)

Taking the square root of both sides:

\[
x - 3 = \pm \sqrt{6}
\]

which yields:

\[
x = 3 \pm \sqrt{6}
\]

Final answer:

\[
\boxed{
x = 3 + \sqrt{6} \quad \text{or} \quad x = 3 - \sqrt{6}
}
\]

This solution leaves the roots in surd form, as requested.

Analysis of the Roots in Surd Form

Expressing roots in surd form offers several advantages:

- Exactness: Surds represent irrational roots precisely without approximation.
- Insight: The roots’ structure reveals their relation to the quadratic's coefficients.
- Further calculations: Maintaining surds allows for algebraic manipulations without loss of precision.

In this case, the roots \(3 \pm \sqrt{6} \) are irrational, confirming the earlier discriminant analysis.

Comparison with the Quadratic Formula

The quadratic formula for roots of $ax^2 + bx + c = 0$:

$$
x = \frac{-b \pm \sqrt{\Delta}}{2a}
$$

Applying to our quadratic:

$$
x = \frac{-(-6) \pm \sqrt{24}}{2 \times 1} = \frac{6 \pm \sqrt{24}}{2}
$$

Simplify $\sqrt{24}$:

$$
\sqrt{24} = \sqrt{4 \times 6} = 2 \sqrt{6}
$$

Thus:

$$
x = \frac{6 \pm 2 \sqrt{6}}{2} = 3 \pm \sqrt{6}
$$

This matches our earlier solution, confirming the consistency and elegance of completing the square.

Advantages of Using $(x - 3)^2 - 6$ for Solving

Intuitive understanding: Recognizing the quadratic as a perfect square minus a constant simplifies root extraction.

Simplification: Instead of dealing with the quadratic formula’s fractions and radicals, the completed square approach reduces the problem to taking a square root, which is often more straightforward.

Educational value: Demonstrates the power of algebraic identities and their application in solving equations efficiently.

Summary of the Solution Process

- Recognize the quadratic as a perfect square minus a constant.
- Rewrite $x^2 - 6x + 3$ as $(x - 3)^2 - 6$.
- Set equal to zero: $(x - 3)^2 = 6$.
- Take square roots: $x - 3 = \pm \sqrt{6}$.
- Solve for x: $x = 3 \pm \sqrt{6}$.

This method is not only elegant but also reinforces understanding of quadratic identities and their utility in solving equations precisely.

Conclusion: The Power of Completing the Square and Surd Solutions

The exploration of solving $x^2 - 6x + 3 = 0$ via the identity $(x - 3)^2 - 6$ underscores the importance of algebraic manipulation in problem-solving. It highlights how recognizing familiar patterns—such as perfect squares—can lead to cleaner, more insightful solutions, especially when roots are irrational.

By expressing the roots as $3 \pm \sqrt{6}$, we preserve the exactness of the solutions, avoiding approximations inherent in decimal representations. This approach exemplifies the elegance of algebra and its capacity to turn complex problems into straightforward solutions, reinforcing the foundational skills necessary for advanced mathematical pursuits.

In sum, whether for educational purposes or mathematical elegance, leveraging identities like $(x - 3)^2 - 6$ to solve quadratics exemplifies the beauty of algebraic reasoning and the power of surd notation in capturing the essence of irrational roots.

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