

What Are The X- And Y-intercepts Of The Line? $2x + 3y = 12$ Show Work

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Understanding the x- and y-intercepts of a line is fundamental in coordinate geometry, as they provide valuable insights into the graph's behavior and the line's position within the coordinate plane. When analyzing the linear equation $2x + 3y = 12$, determining these intercepts helps visualize where the line crosses the axes, making it easier to sketch the graph accurately and interpret the relationship between x and y. In this comprehensive guide, we will explore what x- and y-intercepts are, how to find them step-by-step using the equation $2x + 3y = 12$, and why they are important in various mathematical and real-world contexts.

Understanding the Basics: What Are X- and Y-intercepts?

Before delving into calculations, it's crucial to understand what x- and y-intercepts represent.

Definition of X-intercept

- The x-intercept is the point where a line crosses the x-axis.
- At this point, the y-coordinate is always zero.
- To find the x-intercept, set $y = 0$ in the equation and solve for x.

Definition of Y-intercept

- The y-intercept is the point where a line crosses the y-axis.
- At this point, the x-coordinate is always zero.
- To find the y-intercept, set $x = 0$ in the equation and solve for y.

Given Equation: $2x + 3y = 12$

Our goal is to find both the x-intercept and y-intercept for the line described by the equation $2x + 3y = 12$.

Step-by-Step Process to Find the Intercepts

This section provides detailed instructions on how to compute each intercept.

Finding the X-intercept

1. Set $y = 0$ in the equation:

$$\begin{aligned} & \backslash[\\ & 2x + 3(0) = 12 \end{aligned}$$

$\backslash]$

2. Simplify:

$$\begin{aligned} & \backslash[\\ & 2x = 12 \end{aligned}$$

$\backslash]$

3. Solve for x :

$$\begin{aligned} & \backslash[\\ & x = \frac{12}{2} = 6 \end{aligned}$$

$\backslash]$

4. Interpretation:

- The x-intercept is at the point $(6, 0)$.

Finding the Y-intercept

1. Set $x = 0$ in the equation:

$$\begin{aligned} & \backslash[\\ & 2(0) + 3y = 12 \end{aligned}$$

$\backslash]$

2. Simplify:

$$\begin{aligned} & \backslash[\\ & 3y = 12 \end{aligned}$$

$\backslash]$

3. Solve for y :

$$\begin{aligned} & \backslash[\\ & y = \frac{12}{3} = 4 \end{aligned}$$

$\backslash]$

4. Interpretation:

- The y-intercept is at the point $(0, 4)$.

Graphing the Line Using Intercepts

Once the intercepts are known, plotting the line becomes straightforward.

Plotting the Intercepts

- Mark the points (6, 0) and (0, 4) on the coordinate plane.
- Draw a straight line through these two points.
- This line represents all solutions to the equation $2x + 3y = 12$.

Additional Points for Accuracy

- To improve the accuracy of your graph, you can find additional points by choosing arbitrary x-values or y-values and solving for the other variable.
- For example, choose $x = 3$:

$$\begin{aligned} & \backslash [\\ 2(3) + 3y &= 12 \rightarrow 6 + 3y = 12 \rightarrow 3y = 6 \rightarrow y = 2 \\ & \backslash] \end{aligned}$$

- Corresponding point: (3, 2).

Understanding the Graph of the Equation

The line passes through the points:

- (6, 0) — x-intercept
- (0, 4) — y-intercept
- (3, 2) — additional point for accuracy

This line has a slope that can be calculated as follows:

Calculating the Slope

- The slope (m) of the line is the change in y over the change in x between two points:

$$\begin{aligned} & \backslash [\\ m &= \frac{y_2 - y_1}{x_2 - x_1} \\ & \backslash] \end{aligned}$$

Using points (6, 0) and (0, 4):

$$m = \frac{4 - 0}{0 - 6} = \frac{4}{-6} = -\frac{2}{3}$$

The negative slope indicates the line decreases from left to right.

Applications of Intercepts in Real-World Contexts

Understanding intercepts isn't just a mathematical exercise; it has practical implications across various fields:

1. **Business and Economics:** To determine break-even points, where revenue or profit lines cross axes.
2. **Physics:** Analyzing motion graphs where intercepts indicate initial conditions or thresholds.
3. **Engineering:** Designing systems where certain parameters reach zero at specific points.
4. **Statistics:** Interpreting regression lines and their intercepts to understand data trends.

Common Mistakes to Avoid

While calculating intercepts is straightforward, several pitfalls can lead to errors:

- **Forgetting to set the correct variable to zero:** Always set $y=0$ for the x-intercept and $x=0$ for the y-intercept.
- **Incorrect algebraic manipulation:** Double-check calculations when solving for x or y.
- **Ignoring the domain and range:** Intercepts are points on the axes, but the line may extend beyond them; always consider the context.

- **Mislabeling points:** Clearly label intercept points when plotting to avoid confusion.

Conclusion

Determining the x- and y-intercepts of a line like $2x + 3y = 12$ is an essential skill in algebra and geometry. By setting the other variable to zero and solving, you can find the points where the line crosses the axes, facilitating graphing and interpretation of the line's behavior. The x-intercept at (6, 0) and y-intercept at (0, 4) provide a clear picture of the line's position in the coordinate plane. Whether for academic purposes, solving real-world problems, or understanding the fundamentals of linear equations, mastering intercepts is a vital step in developing a strong mathematical foundation.

Additional Practice Problems

To reinforce your understanding, try solving these problems:

1. Find the intercepts of the line $3x - 4y = 12$.
2. Given the equation $y = 2x + 5$, determine the x- and y-intercepts.
3. Plot the line passing through points (2, 3) and (4, 7) and identify its intercepts.
4. Explain the significance of intercepts in the context of a linear function representing profit over units sold.

Final Thoughts

Understanding how to find and interpret the x- and y-intercepts of a line like $2x + 3y = 12$ not only

enhances your grasp of algebra but also equips you with practical skills for analyzing linear relationships in various disciplines. With consistent practice and attention to detail, you'll be able to quickly identify intercepts, graph lines accurately, and apply these concepts to complex problems with confidence.

Frequently Asked Questions

What is the general method to find the X-intercept of the line given by $2x + 3y = 12$?

To find the X-intercept, set $y = 0$ in the equation and solve for x . So, substitute $y=0$ into $2x + 3(0) = 12$, which simplifies to $2x = 12$, giving $x = 6$.

How do you determine the Y-intercept of the line $2x + 3y = 12$?

To find the Y-intercept, set $x = 0$ in the equation and solve for y . Substituting $x=0$ gives $2(0) + 3y = 12$, which simplifies to $3y = 12$, so $y = 4$.

What are the X- and Y-intercepts of the line $2x + 3y = 12$?

The X-intercept is at $(6, 0)$, and the Y-intercept is at $(0, 4)$.

Can the intercepts of the line $2x + 3y = 12$ be negative? Why or why not?

Yes, the intercepts can be negative if the line crosses the axes in the negative direction. For example, if we solve for x when y is negative or vice versa, we may get negative intercepts. In this case, the intercepts are positive, but depending on the line's position, they can be negative.

What is the significance of the X- and Y-intercepts in graphing the line $2x + 3y = 12$?

The intercepts provide key points that help in graphing the line quickly. Plotting the intercepts $(6,0)$ and $(0,4)$ allows you to draw the line accurately through these points.

If the equation of a line is $2x + 3y = 12$, what is the slope of the line?

First, rewrite the equation in slope-intercept form: $y = - (2/3)x + 4$. The slope is $-2/3$.

Why is it helpful to find both intercepts when analyzing the line $2x + 3y = 12$?

Finding both intercepts helps visualize the line's position on the coordinate plane, determine its slope, and understand how it crosses the axes, making graphing and analysis easier.

Additional Resources

What Are The X- And Y-intercepts Of The Line? $2x + 3y = 12$ Show Work

Understanding the intercepts of a line is fundamental in algebra and coordinate geometry because they offer a quick way to graph the line and understand its position relative to the axes. The x-intercept and y-intercept are points where the line crosses the x-axis and y-axis respectively. For the linear equation $2x + 3y = 12$, finding these intercepts involves a straightforward process of substitution and solving. This article provides a detailed explanation of these concepts, demonstrates step-by-step solutions, and discusses the importance of intercepts in graphing and analyzing linear equations.

What Are Intercepts in a Line?

Before diving into the specific equation, it's essential to understand what intercepts are in the context of a line.

Definition of X-Intercept and Y-Intercept

- X-intercept: The point where a line crosses the x-axis. At this point, the y-coordinate is zero.
- Y-intercept: The point where a line crosses the y-axis. At this point, the x-coordinate is zero.

These intercepts are represented as ordered pairs:

- X-intercept: $(x, 0)$
- Y-intercept: $(0, y)$

Understanding these points helps in sketching the graph of the line without plotting numerous points.

Why Are Intercepts Important?

- They provide quick insights into the line's position relative to the axes.
- They are easy to compute and can serve as starting points for graphing.
- They help in understanding the behavior of the line in the coordinate plane.

Finding the X- and Y-intercepts of $2x + 3y = 12$

Let's explore how to find these intercepts step by step for the specific linear equation:

Equation: $2x + 3y = 12$

Finding the X-Intercept

The x-intercept occurs where $y = 0$. To find it:

1. Substitute $y = 0$ into the equation:

$$2x + 3(0) = 12$$

2. Simplify the equation:

$$2x + 0 = 12$$

3. Solve for x :

$$2x = 12$$

$$x = 12 / 2$$

$$x = 6$$

Result: The x-intercept is at the point $(6, 0)$.

Finding the Y-Intercept

The y-intercept occurs where $x = 0$. To find it:

1. Substitute $x = 0$ into the equation:

$$2(0) + 3y = 12$$

2. Simplify:

$$0 + 3y = 12$$

3. Solve for y:

$$3y = 12$$

$$y = 12 / 3$$

$$y = 4$$

Result: The y-intercept is at the point (0, 4).

Graphing the Line Using Intercepts

Knowing the intercepts makes graphing much easier. The line passes through the points (6, 0) and (0, 4). Plotting these two points on the coordinate plane and drawing a straight line through them provides a visual representation of the line.

Advantages of Using Intercepts for Graphing:

- Quick and efficient.
- Requires only two points to sketch the line accurately.
- Helps verify the slope and behavior of the line.

Additional Methods for Finding Intercepts

While substitution is the most straightforward method for finding intercepts, there are alternative approaches:

Rearranging the Equation into Slope-Intercept Form

The slope-intercept form is:

$$y = mx + b$$

where:

- m is the slope,
- b is the y-intercept.

Let's convert $2x + 3y = 12$ into this form:

1. Solve for y:

$$3y = -2x + 12$$

2. Divide both sides by 3:

$$y = (-2/3)x + 4$$

In this form:

- The y-intercept is clearly 4 (when $x=0$).
- The slope is $-2/3$.

To find the x-intercept using this form, set $y=0$:

$$0 = (-2/3)x + 4$$

Solve for x:

$$(-2/3)x = -4$$

$$x = (-4) (-3/2) = 6$$

Same as earlier, confirming the intercepts.

Advantages:

- Provides both slope and intercepts at once.
- Useful for quick graphing and understanding line behavior.

Pros and Cons of Using Intercepts to Graph Lines

Pros:

- Simplicity: Only two points needed for accurate graphing.
- Efficiency: Faster than plotting many points.
- Clarity: Immediate visualization of where the line crosses axes.
- Educational Value: Helps students grasp the relationship between algebraic equations and geometric graphs.

Cons:

- Limited to Linear Equations: Not applicable for nonlinear graphs.
- Requires Basic Algebra Skills: Needs substitution and solving equations.
- Potential for Errors: Mistakes in substitution or arithmetic can lead to incorrect intercepts.

Features and Applications of Intercepts

Features:

- They are always on the axes, simplifying identification.
- The intercepts are essential in the graphical analysis of equations.
- They help in understanding the slope and position of the line.

Applications:

- Graphing Linear Equations: Quick plotting points.
- Word Problems: Interpreting real-world data where intercepts represent initial conditions or thresholds.
- Calculus and Advanced Math: Intercepts serve as boundary points in various analyses.

Summary and Final Thoughts

The process of finding x- and y-intercepts for the line given by $2x + 3y = 12$ is straightforward and provides valuable insights into the line's behavior. By setting $y=0$, we determined the x-intercept at $(6, 0)$, and by setting $x=0$, we found the y-intercept at $(0, 4)$. These points serve as the foundation for graphing and understanding the linear relationship.

Understanding intercepts not only simplifies the visualization process but also deepens comprehension of the linear equation's structure. Whether using substitution or converting to slope-intercept form, the key is recognizing that intercepts are the simplest points to analyze and plot.

In conclusion, mastering the calculation of intercepts is essential for students and professionals working with linear equations. It enhances problem-solving skills, promotes a better grasp of algebraic and geometric concepts, and facilitates efficient graphing. As demonstrated with the equation $2x + 3y = 12$, intercepts are powerful tools that make the study of lines more accessible and intuitive.

In summary:

- The x-intercept of $2x + 3y = 12$ is $(6, 0)$.
- The y-intercept of $2x + 3y = 12$ is $(0, 4)$.
- These intercepts can be easily found by substitution or algebraic manipulation.
- Using intercepts simplifies graphing and enhances understanding of linear equations.

Understanding and applying the concept of intercepts will serve as a valuable skill in mathematics, enabling clearer analysis and visualization of lines across various contexts.

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