What Is The Slope Of The Line That Passesthrough (0, 2) And (3, 0)?

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Understanding the slope of a line that passes through two specific points is a fundamental concept in algebra and coordinate geometry. When given points such as (0, 2) and (3, 0), calculating the slope helps us understand the line's inclination—how steep it is—and how it behaves across the coordinate plane. This article provides a comprehensive explanation of what the slope is, how to compute it for the given points, and why it’s essential in various mathematical and real-world applications.

What Is The Slope Of A Line?

Definition of Slope

The slope of a line is a measure of its steepness or incline. It indicates how much the y-coordinate (vertical change) of a point on the line changes for a corresponding change in the x-coordinate (horizontal change). Mathematically, the slope is often represented by the letter m.

In simple terms, the slope tells us how "tilted" the line is. If the slope is positive, the line rises as you move from left to right. If negative, it falls. A zero slope indicates a horizontal line, and an undefined slope corresponds to a vertical line.

Significance of Slope in Geometry and Algebra

- Graphing lines: The slope helps plot the line accurately given a point and the slope.
- Understanding relationships: In real-world data, the slope can represent rates such as speed, growth rate, or decline.
- Equation of a line: The slope is a critical component in the slope-intercept form $y = mx + b$.

How To Calculate The Slope Between Two Points

The Slope Formula

To find the slope of a line passing through two points (x_1, y_1) and (x_2, y_2), use the formula:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

This formula computes the ratio of the change in y-coordinates to the change in x-coordinates, often called the "rise over run."

Step-by-Step Calculation Using the Points (0, 2) and (3, 0)

Given points:
- Point 1: $(x_1, y_1) = (0, 2)$
- Point 2: $(x_2, y_2) = (3, 0)$

Applying the formula:

$$
m = \frac{0 - 2}{3 - 0} = \frac{-2}{3}
$$

Thus, the slope of the line passing through these two points is $-\frac{2}{3}$.

Interpreting the Slope $-\frac{2}{3}$

What Does a Slope of $-\frac{2}{3}$ Mean?

A slope of $-\frac{2}{3}$ indicates that for every 3 units moved horizontally to the right (positive x-direction), the line moves 2 units downward (negative y-direction). Conversely, if you move 3 units to the left, the line would rise 2 units.

Key points:
- The negative sign indicates the line declines as it moves from left to right.
- The magnitude $\frac{2}{3}$ reflects a relatively gentle slope, not too steep.

Graphical Representation

- The line passes through $(0, 2)$, which is the y-intercept.
- Using the slope, you can plot additional points to sketch the line accurately.

Equation of the Line Passing Through (0, 2) and (3, 0)

Using the Point-Slope Form

The point-slope form of a line is:

$$
y - y_1 = m(x - x_1)
$$

Choose point $(0, 2)$ and slope $m = -\frac{2}{3}$:

$$
y - 2 = -\frac{2}{3}(x - 0)
$$

which simplifies to:

$$
y - 2 = -\frac{2}{3}x
$$

Adding 2 to both sides:

$$
y = -\frac{2}{3}x + 2
$$

This is the slope-intercept form of the line.

Verifying the Equation with the Second Point

Plug in $x=3$:

$$
y = -\frac{2}{3} \times 3 + 2 = -2 + 2 = 0
$$

which matches the y-coordinate of the second point, confirming the equation's correctness.

Applications and Significance of the Slope in Real

Life

Educational Applications

- Teaching coordinate geometry concepts.
- Calculating rates of change in graphs.
- Graphing linear equations effectively.

Real-World Examples

- Physics: Determining velocity from position data.
- Economics: Calculating marginal cost or revenue.
- Biology: Analyzing growth rates over time.
- Engineering: Slope in design and structural analysis.

Practical Importance

Knowing how to compute the slope between two points is essential for solving problems involving linear relationships. It enables professionals and students to interpret data visually and mathematically, fostering better understanding of trends and behaviors.

Summary: Key Points About The Slope of the Line Through (0, 2) and (3, 0)

Key Takeaways:

1. The slope of a line passing through the points $(0, 2)$ and $(3, 0)$ is $-\frac{2}{3}$.
2. The negative slope indicates a declining line from left to right.
3. The line's equation in slope-intercept form is $y = -\frac{2}{3}x + 2$.
4. Calculating the slope involves the change in y over the change in x, which is vital for graphing and understanding linear relationships.
5. The concept of slope extends beyond mathematics, playing a crucial role in various scientific, economic, and engineering fields.

Conclusion

Understanding how to compute and interpret the slope of a line passing through any two points is a fundamental skill in mathematics. The specific example of points (0, 2) and (3, 0) illustrates the process clearly, resulting in a slope of \(-\frac{2}{3}\). Recognizing what this slope signifies helps in graphing the line, understanding the relationship between variables, and applying this knowledge in real-world situations. Whether you're a student, educator, or professional, mastering the concept of slope enhances your analytical capabilities and your ability to interpret data effectively.

Frequently Asked Questions

How do you find the slope of a line passing through two points?

You find the slope by subtracting the y-coordinates and dividing by the difference in the x-coordinates: slope = (y2 - y1) / (x2 - x1).

What is the slope of the line passing through (0, 2) and (3, 0)?

The slope is (0 - 2) / (3 - 0) = (-2) / 3 = -2/3.

Why is the slope important in understanding a line?

The slope indicates the steepness and direction of the line, showing how y changes with respect to x.

Can the slope be negative? What does a negative slope mean?

Yes, the slope can be negative, which means the line slopes downward from left to right.

How can I verify the slope calculation for these points?

You can verify by plugging the coordinates into the slope formula: (0 - 2) / (3 - 0) = -2/3, confirming the slope is -2/3.

Additional Resources

Slope: Unlocking the Steepness of a Line in Coordinate Geometry

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Introduction

In the world of mathematics, particularly in coordinate geometry, understanding the concept of a line's slope is fundamental. Whether you're a student tackling algebra problems or a professional applying these concepts to real-world data, knowing how to determine the slope of a line passing through specific points is an essential skill.

Imagine you're given two points on a plane: (0, 2) and (3, 0). The question arises—what is the slope of the line that passes through these points? This seemingly simple query unlocks a rich discussion about the geometric and algebraic principles underpinning lines on the Cartesian plane. In this comprehensive feature, we will explore the concept of slope in detail, analyze how to compute it between two points, interpret its significance, and see how it applies beyond theoretical mathematics.

Understanding the Key Concepts

1. The Coordinate Plane and Points

Before diving into the slope calculation, it's crucial to grasp the basic elements involved:

- Coordinate Plane: A two-dimensional surface divided into four quadrants by the x-axis (horizontal) and y-axis (vertical).
- Points: Locations on this plane expressed as ordered pairs (x, y). For example, (0, 2) indicates a point where x=0 and y=2.

In our case, the two points are:

- Point A: (0, 2)
- Point B: (3, 0)

These points are chosen specifically because they are simple, with one point on the y-axis (the y-intercept) and the other at a positive x-value, providing an excellent context for understanding the slope.

2. The Concept of Slope (m)

The slope of a line quantifies its steepness or inclination. It measures how much the y-value changes relative to the x-value between any two points on the line.

Mathematically, the slope (m) between two points (x_1, y_1) and (x_2, y_2) is defined as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

This formula calculates the ratio of the vertical change (rise) to the horizontal change (run). It is sometimes referred to as the rise over run.

Computing the Slope: Step-by-Step Analysis

Applying the slope formula to our specific points:

- Point A: (0, 2)
- Point B: (3, 0)

Step 1: Assign the points to the variables:

$$
(x_1, y_1) = (0, 2)
$$
$$
(x_2, y_2) = (3, 0)
$$

Step 2: Plug into the slope formula:

$$
m = \frac{0 - 2}{3 - 0}
$$

Step 3: Simplify numerator and denominator:

$$
m = \frac{-2}{3}
$$

Result: The slope of the line passing through points (0, 2) and (3, 0) is -2/3.

Interpreting the Slope: What Does -2/3 Mean?

The negative sign indicates that the line slopes downward from left to right. In terms of the graph:

- For every increase of 3 units along the x-axis (moving to the right), the y-value decreases by 2 units.
- Conversely, moving 2 units upward along the y-axis corresponds to a 3-unit increase in the x-direction, reflecting the reciprocal nature of the rise and run.

This negative slope signifies an inverse relationship between x and y along this line, which is characteristic of lines that descend as x increases.

Visualizing the Line

Plotting the points provides a visual confirmation of the slope:

- Start at point (0, 2), which is on the y-axis.
- From this point, moving horizontally 3 units to the right (x=3) takes us to (3, 2).
- Since the slope is -2/3, we expect the y-value to decrease by 2 units over this 3-unit increase in x:

$$
\text{Change in y} = -\frac{2}{3} \times 3 = -2
$$

- Therefore, from (0, 2), moving 3 units right, the y-value drops from 2 to 0, matching the second point (3, 0).

This confirms the accuracy of our calculation.

Significance and Applications of Slope

Understanding slope isn't just an academic exercise — it has practical applications across various fields:

- Engineering: Designing roads or ramps, where slope determines steepness.
- Economics: Analyzing cost functions or demand curves.
- Physics: Calculating velocity when analyzing displacement over time.
- Data Science: Understanding trends in datasets, such as regression lines.

Additional Insights

1. The Slope-Intercept Form of a Line

Once the slope is known, the line's equation can be formulated using the slope-intercept form:

$$
y = mx + b
$$

where b is the y-intercept (the point where the line crosses the y-axis). Since our line passes through (0, 2), the y-intercept b is 2.

Thus, the line's equation becomes:

$$
y = -\frac{2}{3}x + 2
$$

This equation encapsulates the line's entire behavior, allowing us to predict y-values for any x.

2. Verifying the Line Equation

Check if the second point (3, 0) satisfies the equation:

\[

y = -\frac{2}{3} \times 3 + 2 = -2 + 2 = 0
\]

Indeed, it does, confirming the correctness of our line's equation.

Practical Exercises for Mastery

To deepen understanding, consider the following exercises:

- Exercise 1: Find the slope of the line passing through points (1, 5) and (4, 1).
- Exercise 2: Write the equation of the line passing through (0, 3) with a slope of 4.
- Exercise 3: Determine the slope between points (-2, -3) and (2, 1).

Common Pitfalls and Clarifications

While calculating slopes is straightforward, some common mistakes can occur:

- Swapping points: Remember, the order of points affects the sign but not the magnitude of the slope.
- Incorrect subtraction: Always subtract the y-values in the numerator and x-values in the denominator separately, maintaining the order.
- Division by zero: If the two points have the same x-coordinate ($x_1 = x_2$), the slope is undefined, indicating a vertical line.

Final Thoughts

The calculation of the slope between points (0, 2) and (3, 0) reveals a slope of -2/3, illustrating a downward-sloping line. This simple ratio encapsulates the essence of the line’s inclination, offering insights into its geometric nature and real-world implications.

Mastering the concept of slope empowers you to analyze and interpret linear relationships across diverse disciplines, from engineering and physics to economics and data science. Whether visualizing the steepness of a hill or modeling economic trends, understanding how to compute and interpret slope remains an invaluable skill.

In conclusion, the slope of the line passing through (0, 2) and (3, 0) is -2/3 — a concise yet powerful descriptor of its orientation, steepness, and relationship between x and y coordinates.

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