What Is Three Less Than Twice A Number X Is Less Than Twenty ?

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Understanding algebraic expressions and inequalities is fundamental in mathematics. One common type of problem involves translating words into mathematical expressions and then solving for the unknown. The phrase "What is three less than twice a number x is less than twenty?" is a classic example of such a problem. It requires careful interpretation of the language to create an inequality that can be solved step-by-step. This article will explore how to interpret, formulate, and solve this type of inequality, providing detailed explanations and helpful tips along the way.

Interpreting the Problem Statement

Before jumping into calculations, it’s essential to understand what the problem is asking. The statement involves several key components:

- "Three less than twice a number x"
- "is less than twenty"

Let's break down these parts:

1. "Twice a number x"
This refers to multiplying x by 2, which gives us 2x.

2. "Three less than twice a number x"
This phrase indicates subtracting 3 from 2x, resulting in the expression 2x - 3.

3. "is less than twenty"
This suggests an inequality where the expression 2x - 3 is less than 20.

Formulating the Inequality

Using the interpretations above, we can now translate the word problem into a mathematical inequality:

$$
2x - 3 < 20
$$

This inequality reads as: "Three less than twice x is less than twenty."

Solving the Inequality Step-by-Step

To find the range of values for x that satisfy this inequality, follow these steps:

Step 1: Isolate the term containing x

Add 3 to both sides of the inequality to move constants to the other side:

$$
2x - 3 + 3 < 20 + 3
$$

Simplifies to:

$$
2x < 23
$$

Step 2: Solve for x

Divide both sides of the inequality by 2:

$$
x < \frac{23}{2}
$$

which simplifies to:

$$
x < 11.5
$$

Result: The set of all real numbers x less than 11.5 satisfies the original inequality.

Understanding the Solution

The inequality $x < 11.5$ implies that any x value less than 11.5 will satisfy the original statement.

If you’re working with real numbers, the solution set includes all numbers approaching 11.5 from below, but not including 11.5 itself.

Summary of the solution:

- Solution set: $\{ x \in \mathbb{R} \mid x < 11.5 \}$
- Interpretation: The value of x must be less than 11.5.

Graphical Representation

Visualizing inequalities can often help in understanding the solution space.

Number line illustration:

- Draw a horizontal line representing real numbers.
- Mark the point 11.5.
- Use an open circle at 11.5 to indicate that x cannot equal 11.5.
- Shade the region to the left of 11.5 to show all x less than 11.5.

This visual aid confirms that all points to the left of 11.5 are solutions.

Key Concepts and Tips in Solving Similar Problems

When approaching inequalities like "What is three less than twice a number x is less than twenty," keep these concepts in mind:

1. Carefully Read the Language

- Pay attention to words such as "less than," "more than," "equal to," etc.
- Identify phrases indicating operations (e.g., "twice" means multiply by 2).

2. Convert Word Phrases into Mathematical Expressions

- "Twice" → multiply by 2
- "Three less than" → subtract 3
- "Is less than" → <

3. Formulate the Inequality

- Combine the expressions logically based on the sentence structure.
- Confirm that the inequality matches the intended meaning.

4. Solve Step-by-Step

- Isolate the variable term.
- Use inverse operations (addition/subtraction, multiplication/division).
- Keep track of inequality direction (remember, multiplying or dividing by negative numbers reverses the inequality sign).

5. Interpret the Solution

- Express the solution set clearly (e.g., $x < 11.5$).
- Use interval notation if needed: $(-\infty, 11.5)$.

6. Graph the Solution

- Use number lines to visualize the range of solutions.
- Use open or closed circles depending on whether the boundary is included.

Additional Examples for Practice

To strengthen understanding, consider these similar problems:

Example 1:
What is five more than twice a number y is greater than ten?

Solution:
Translate: $2y + 5 > 10$
Solve:
$$
2y + 5 > 10 \Rightarrow 2y > 5 \Rightarrow y > \frac{5}{2} \Rightarrow y > 2.5
$$

Example 2:
The sum of three times a number z and four is less than or equal to twenty.

Solution:
Translate: $3z + 4 \leq 20$
Solve:
$$
3z \leq 16 \Rightarrow z \leq \frac{16}{3} \approx 5.33
$$

Common Mistakes to Avoid

While solving inequalities, be cautious of these pitfalls:

- Incorrectly reversing the inequality sign when multiplying or dividing both sides by a negative number.
- Misinterpreting the language, such as confusing "less than" with "less than or equal to."
- Forgetting to include the boundary point when the inequality is non-strict ($\leq$ or $\geq$).
- Not simplifying the inequality fully before solving, which can lead to errors.

Conclusion

Understanding and solving the inequality derived from the phrase "What is three less than twice a number x is less than twenty" is straightforward when approached systematically. By carefully translating the words into a mathematical inequality, solving step-by-step, and interpreting the solution, you gain clarity on the range of values that satisfy the condition. Remember to visualize the solutions on a number line for better comprehension and to avoid common mistakes. Mastery of these skills enhances your ability to tackle diverse algebraic inequalities and strengthens your overall mathematical reasoning.

Summary:

- The inequality is $2x - 3 < 20$.
- Solution: $x < 11.5$.
- The set of solutions includes all real numbers less than 11.5.
- Visualization and understanding of inequalities are vital skills in algebra.

With practice and careful attention to detail, solving such problems can become an intuitive part of your mathematical toolkit.

Frequently Asked Questions

What does the phrase 'Three less than twice a number X' mean mathematically?

It means taking twice the number X and subtracting three from it, which can be expressed as $2X - 3$.

How do you interpret the inequality 'Three less than twice a number X is less than twenty'?

It translates to the inequality $2X - 3 < 20$, meaning twice the number minus three is less than twenty.

What is the solution to the inequality $2X - 3 < 20$?

Adding 3 to both sides gives $2X < 23$; dividing both sides by 2 results in $X < 11.5$.

What does the inequality $X < 11.5$ tell us about the possible values of X?

It indicates that any number less than 11.5 satisfies the inequality, so X can be any real number less than 11.5.

Can you give an example of a number that satisfies the inequality $2X - 3 < 20$?

Yes, for example, $X = 10$: $2(10) - 3 = 20 - 3 = 17$, which is less than 20, so $X = 10$ satisfies the inequality.

Additional Resources

Mathematical Expression Analysis: "Three Less Than Twice a Number X Is Less Than Twenty"

Understanding mathematical expressions is fundamental in developing problem-solving skills, critical thinking, and logical reasoning. One such intriguing expression is "Three less than twice a number x is less than twenty." This phrase encapsulates a comparison involving a variable, constants, and an inequality, which can be deciphered into a precise algebraic statement. In this comprehensive review, we will explore this problem in depth, dissect its components, interpret its meaning, and demonstrate how to solve it systematically. Whether you're a student, educator, or enthusiast, this detailed explanation aims to clarify all aspects of this expression.

Deciphering the Phrase: Breaking Down the Language

Before diving into the algebraic formulation, it’s essential to understand the language used in the phrase:

"Three less than twice a number x is less than twenty."

This sentence contains several key components:

- "Twice a number x": This indicates two times the variable x, represented mathematically as $2x$.
- "Three less than": This phrase suggests subtracting three from the preceding quantity, i.e., $2x - 3$.
- "Is less than twenty": This indicates an inequality where the expression is strictly smaller than 20.

By analyzing these parts, we identify that the phrase describes an inequality involving x.

Translating the Phrase into an Algebraic Expression

The critical step in understanding the problem is translating the verbal statement into a formal algebraic inequality.

Step 1: Identify the core expression

- "Twice a number x" translates to 2x.
- "Three less than twice a number x" translates to 2x - 3.

Step 2: Express the comparison

- The phrase "is less than twenty" indicates a strict inequality:

$$ 2x - 3 < 20 $$

This is the core inequality that models the statement precisely.

Understanding the Inequality: "2x - 3 < 20"

This inequality encapsulates the relationship described. To comprehend its significance, let's analyze each component:

- Left side (LHS): 2x - 3 — representing "three less than twice a number x."
- Right side (RHS): 20 — the boundary value.

The inequality less than signifies that the expression 2x - 3 takes on values strictly less than 20.

Solving the Inequality Step-by-Step

To find the range of x satisfying the inequality, we perform algebraic operations, maintaining the inequality's direction.

Step 1: Isolate x

Starting with:

$$ 2x - 3 < 20 $$

Add 3 to both sides:

$$2x - 3 + 3 < 20 + 3$$
$$2x < 23$$

Step 2: Divide both sides by 2

Since 2 is positive, dividing preserves the inequality:

$$\frac{2x}{2} < \frac{23}{2}$$
$$x < \frac{23}{2}$$

Expressed as a mixed number:

$$x < 11.5$$

Interpreting the Solution: What Does $x < 11.5$ Mean?

The solution indicates that any real number less than 11.5 satisfies the original phrase. This gives us a comprehensive understanding of the range of x.

Key Points:

- The set of solutions is all real numbers less than 11.5, excluding 11.5 itself.
- In interval notation, this is:

$$(-\infty, 11.5)$$

- If considering integer solutions, x can be any integer less than or equal to 11.

Practical implications:

- For instance, $x = 10$, $x = 0$, or $x = -5$ are solutions.
- $x = 11.5$ is not a solution because it makes the expression exactly 20, not less than 20.
- Values greater than or equal to 11.5 do not satisfy the inequality.

Graphical Representation of the Solution

Visualizing inequalities enhances understanding. The solution $x < 11.5$ can be graphically represented on a number line:

- Draw a number line with marks at key points, especially at 11.5.
- Use an open circle at 11.5 to indicate that it is not included.
- Shade all points to the left of 11.5, representing all numbers less than 11.5.

This visual aid confirms that the solution set extends infinitely to the left, encompassing all real numbers smaller than 11.5.

Extensions and Related Concepts

Understanding this inequality opens doors to broader topics and more complex problems.

1. Solving for Other Inequalities

Similar inequalities can be constructed by changing constants or the variable:

- "Three less than twice a number x is greater than twenty":
$$ 2x - 3 > 20 $$
Solution: $x > 11.5$.

- "Three less than twice a number x is equal to twenty":
$$ 2x - 3 = 20 $$
Solution: $x = 11.5$.

2. Compound Inequalities

Combining multiple inequalities, e.g.,
$$ 10 < 2x - 3 < 20 $$,
which would require solving both inequalities to find the range of x that satisfies the compound statement.

3. Application in Word Problems

This type of inequality is common in real-world contexts such as budgeting, measurements, or constraints in optimization problems.

Key Takeaways for Learners and Practitioners

- Accurate translation from verbal to algebraic form is crucial.
- Step-by-step algebraic manipulation ensures precise solutions.
- Understanding the solution set helps in practical applications.
- Graphical visualization reinforces comprehension.

Summary

The phrase "Three less than twice a number x is less than twenty" translates into the inequality:

$$2x - 3 < 20$$

which simplifies to:

$$x < 11.5$$

This solution indicates that any real number less than 11.5 meets the criteria expressed by the phrase. The process involves careful linguistic translation, algebraic manipulation, and interpretation of the solution set. Mastery of such inequalities not only enhances mathematical skills but also sharpens problem-solving acumen applicable in various fields like engineering, economics, and everyday decision-making.

Final Thoughts:
Understanding the nuances of inequalities stemming from verbal descriptions is a foundational skill in mathematics. Whether solving simple inequalities or tackling complex systems, the clarity gained from dissecting language and translating it into formal expressions equips learners with confidence and precision. The key is to approach each problem methodically, ensuring every step logically follows the previous, leading to insightful solutions and a deeper appreciation for the elegance of algebraic reasoning.

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