11.find The Values Of C That Satisty The Mean Value Theorem. Show Steps

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Understanding the Mean Value Theorem (MVT) is fundamental in calculus, especially when analyzing the behavior of functions. The process of finding the specific values of c that satisfy the theorem helps in comprehending how functions behave between two points. In this article, we will explore the steps to find these values of c that satisfy the Mean Value Theorem, complete with detailed explanations and examples to solidify your understanding.

What Is the Mean Value Theorem?

The Mean Value Theorem states that if a function f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists at least one point c in (a, b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

This theorem essentially guarantees that at some point c, the instantaneous rate of change (the derivative) equals the average rate of change over $[a, b]$.

Steps to Find Values of c That Satisfy the MVT

Finding the specific c values involves a systematic process. Let’s break down the steps:

Step 1: Verify the Conditions of the MVT

Before proceeding, ensure that:

- The function f is continuous on the closed interval $[a, b]$.
- The function f is differentiable on the open interval (a, b).

If these conditions are not satisfied, the MVT does not apply, and you cannot find such c.

Step 2: Calculate the Average Rate of Change

Compute the difference quotient:

$$\frac{f(b) - f(a)}{b - a}$$

This value represents the average rate of change of f from a to b.

Step 3: Find the Derivative $f'(x)$

Differentiate the function $f(x)$ to obtain $f'(x)$. This derivative represents the instantaneous rate of change at any point x.

Step 4: Set $f'(c)$ Equal to the Average Rate of Change

Solve the equation:

$$ f'(c) = \frac{f(b) - f(a)}{b - a} $$

for c. This involves substituting c into the derivative $f'(x)$.

Step 5: Solve for c in the Interval (a, b)

Find all solutions c that satisfy the equation within the interval (a, b). These are the points where the tangent line is parallel to the secant line connecting $(a, f(a))$ and $(b, f(b))$.

Example: Find c for a Specific Function

Let's apply these steps to a concrete example for clarity.

Suppose $f(x) = x^2 + 2x$, with $a = 1$ and $b = 3$.

Step 1: Verify Conditions

- $f(x) = x^2 + 2x$ is a polynomial, which is continuous and differentiable everywhere.
- Therefore, MVT applies on $[1, 3]$.

Step 2: Calculate the Average Rate of Change

$$
f(3) = 3^2 + 2 \times 3 = 9 + 6 = 15 \\
f(1) = 1^2 + 2 \times 1 = 1 + 2 = 3
$$

$$
\frac{f(3) - f(1)}{3 - 1} = \frac{15 - 3}{2} = \frac{12}{2} = 6
$$

Step 3: Find $f'(x)$

$$
f'(x) = 2x + 2
$$

Step 4: Set $f'(c) = 6$ and solve for c

$$
2c + 2 = 6 \\
2c = 4 \\
c = 2
$$

Step 5: Verify c is in (a, b)

$$
a = 1, \quad b = 3, \quad c = 2
$$

Since $1 < 2 < 3$, the value $c = 2$ satisfies the Mean Value Theorem.

Additional Tips for Finding c Values

- Always double-check the conditions of the theorem before proceeding.
- When solving $f'(c) =$ average rate of change, carefully manipulate the algebraic expressions.
- Remember that the solution for c must lie strictly within (a, b), so discard any solutions outside this interval.
- For more complex functions, consider using numerical methods or graphing tools to estimate solutions.

Common Mistakes to Avoid

- Applying MVT to functions that are not continuous or differentiable on the interval.
- Forgetting to verify the interval for the solution c.
- Mistakenly solving for c outside the open interval (a, b).
- Miscalculating derivatives or the average rate of change.

Conclusion

Finding the values of c that satisfy the Mean Value Theorem involves a clear, step-by-step process: verify the theorem's conditions, compute the average rate of change, differentiate the function, set the derivative equal to this value, and solve within the interval. Mastering this process enables you to analyze functions' behavior between two points confidently and is essential for higher-level calculus topics.

Whether working with simple polynomial functions or more complex ones, these steps serve as a reliable guide to identify the points where the function's instantaneous rate matches its average rate over an interval. Practice with various functions to become proficient in applying the Mean Value Theorem effectively.

Frequently Asked Questions

What is the Mean Value Theorem (MVT) in calculus?

The Mean Value Theorem states that if a function is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists at least one point c in (a, b) such that $f'(c)$ equals the average rate of change over $[a, b]$, i.e., $f'(c) = (f(b) - f(a)) / (b - a)$.

How do you set up the equation to find the value of C satisfying the Mean Value Theorem?

First, compute the average rate of change: $(f(b) - f(a)) / (b - a)$. Then, find the derivative $f'(x)$. To satisfy MVT, solve the equation $f'(c) = (f(b) - f(a)) / (b - a)$ for c within (a, b).

Given the function f(x) = x^2 + 3x + 2 on [1, 4], how do I find the value of C that satisfies MVT?

First, calculate the average rate of change: $(f(4) - f(1)) / (4 - 1) = ((16 + 12 + 2) - (1 + 3 + 2)) / 3 = (30 - 6) / 3 = 24 / 3 = 8$. Next, find $f'(x) = 2x + 3$. Set $2c + 3 = 8$, so $2c = 5$, thus $c = 2.5$, which lies within $(1, 4)$.

What steps are involved in solving for C when given a specific function and interval?

Steps include: 1) Calculate the average rate of change over the interval; 2) Find the derivative of the function; 3) Set the derivative equal to the average rate of change; 4) Solve for c; 5) Verify that c lies within the interval (a, b).

Can C be outside the interval [a, b] when satisfying the Mean Value Theorem?

No, according to the Mean Value Theorem, the value c must lie within the open interval (a, b). If the solution for c is outside this interval, it does not satisfy the theorem.

What common mistakes should I avoid when finding C that satisfies MVT?

Common mistakes include: forgetting to verify that the function is continuous and differentiable on the interval, miscalculating the average rate of change, solving for C outside the interval, or incorrectly solving the derivative equation.

How does the value of C relate to the function's behavior on the interval?

The value of C corresponds to a point where the tangent line to the curve is parallel to the secant line connecting the endpoints. It indicates a point where the instantaneous rate of change equals the average rate of change.

Is it necessary to find all possible values of C that satisfy MVT?

It depends on the context. Typically, if the derivative is not monotonic, multiple values of C may satisfy the theorem. Identifying all such points provides a complete understanding of the function's behavior on the interval.

Additional Resources

Find the Values of C That Satisfy the Mean Value Theorem. Show Steps

Understanding the Mean Value Theorem (MVT) is fundamental in calculus, as it bridges the gap between derivatives and function behaviors over an interval. This theorem not only provides insight into the slope of a tangent line but also guarantees the existence of at least one point within an interval where the instantaneous rate of change equals the average rate of change over that interval. In this comprehensive guide, we will explore the process of finding the specific values of c that satisfy the MVT, illustrating each step with clarity and detail.

Introduction to the Mean Value Theorem

The Mean Value Theorem states that if a function f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists at least one point $c \in (a, b)$ such that:

$$
f'(c) = \frac{f(b) - f(a)}{b - a}
$$

This equation states that the instantaneous rate of change (the derivative at c) equals the average rate of change over the interval. The theorem is pivotal in understanding function behaviors, proving other theorems, and solving real-world problems involving rates.

Step-by-Step Process for Finding the Values of c

Applying the MVT involves a systematic approach:

1. Verify the Conditions of the MVT

Before proceeding, ensure the function satisfies the prerequisites:

- Continuity on $[a, b]$: The function must have no breaks, jumps, or asymptotes within the interval.
- Differentiability on (a, b): The function must be smooth enough (no sharp corners or vertical tangent lines) within the interval.

Example: For $f(x) = x^2$ over $[1, 3]$, the function is both continuous and differentiable everywhere, satisfying the conditions.

2. Calculate the Average Rate of Change

Compute:

$$
\frac{f(b) - f(a)}{b - a}
$$

This gives the slope of the secant line connecting the points $(a, f(a))$ and $(b, f(b))$.

Example: For $f(x) = x^2$, over $[1, 3]$:

$$
\frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = 4
$$

3. Find the Derivative $f'(x)$

Differentiate the function:

Example: $f(x) = x^2 \Rightarrow f'(x) = 2x$

4. Set the Derivative Equal to the Average Rate of Change

Solve for c:

$$
f'(c) = \frac{f(b) - f(a)}{b - a}
$$

Example: $2c = 4 \Rightarrow c = 2$

5. Verify c Lies Within (a, b)

Ensure the solution for c is strictly within the open interval:

In our example: $c = 2$, and since $1 < 2 < 3$, it satisfies the condition.

Applying the Process to Specific Functions

Let's delve into detailed examples to illustrate the entire process.

Example 1: Polynomial Function

Problem: Find all c satisfying MVT for $f(x) = x^3 - 3x + 2$ over $[1, 3]$.

Step 1: Verify conditions

- f is a polynomial, so continuous and differentiable everywhere.

- Interval $[1, 3]$ is closed, and the function is smooth in $(1, 3)$.

Step 2: Calculate the average rate of change

$$
f(3) = 27 - 9 + 2 = 20
$$
$$
f(1) = 1 - 3 + 2 = 0
$$
$$
\text{Average rate} = \frac{20 - 0}{3 - 1} = \frac{20}{2} = 10
$$

Step 3: Find the derivative

$$
f'(x) = 3x^2 - 3
$$

Step 4: Set derivative equal to the average rate

$$
3c^2 - 3 = 10
$$
$$
3c^2 = 13
$$
$$
c^2 = \frac{13}{3}
$$
$$
c = \pm \sqrt{\frac{13}{3}} \approx \pm 2.08
$$

Step 5: Verify c within $(1, 3)$

- $c \approx 2.08$, which lies between 1 and 3.
- $c \approx -2.08$, which is outside the interval, so discard.

Conclusion: The value $c \approx 2.08$ satisfies the MVT.

Example 2: Trigonometric Function

Problem: Find c for $f(x) = \sin x$ over $[0, \pi]$.

Step 1: Verify conditions

- Continuous and differentiable everywhere.
- Interval is closed and the function is smooth.

Step 2: Calculate average rate

$$
f(\pi) = 0
$$
\[

$$
f(0) = 0
$$

$$
\text{Average rate} = \frac{0 - 0}{\pi - 0} = 0
$$

Step 3: Find derivative

$$
f'(x) = \cos x
$$

Step 4: Set derivative equal to average rate

$$
\cos c = 0
$$

$$
c \in (0, \pi)
$$

$$
\cos c = 0 \Rightarrow c = \frac{\pi}{2}
$$

Step 5: Verify c within $(0, \pi)$

- $\frac{\pi}{2}$ is within the interval.

Conclusion: $c = \frac{\pi}{2}$ is the point satisfying the MVT.

Features and Pros/Cons of the Method

Features:

- Systematic approach: The method involves clear steps—verify conditions, compute average rate, differentiate, set equal, and solve.
- Broad applicability: Works with polynomial, trigonometric, exponential, and other functions satisfying the theorem's conditions.
- Geometric interpretation: Helps visualize the point where the tangent slope matches the secant slope.

Pros:

- Provides guaranteed existence of at least one point c.
- Useful in proving other theorems or solving real-world problems involving rates.
- Clarifies the link between average and instantaneous rates.

Cons:

- Does not specify how many such points exist—only guarantees at least one.
- Requires functions to meet specific conditions; not applicable otherwise.
- Sometimes the solutions for c may lie outside the interval, necessitating careful verification.

Special Cases and Common Mistakes

- Functions not meeting conditions: Applying MVT to functions with discontinuities or non-differentiability points leads to invalid results.
- Multiple solutions: Some functions may have multiple points \(c \) satisfying the derivative condition; only those within \((a, b)\) are valid.
- Ignoring interval boundaries: The theorem guarantees points within the open interval; boundary points are not considered.

Conclusion

Finding the values of \(c \) that satisfy the Mean Value Theorem involves a methodical process rooted in understanding the function's behavior, calculating the average rate of change, derivatives, and solving equations accurately. Mastering this process enhances one's ability to analyze function behavior, prove related theorems, and solve practical problems involving rates of change. Remember to always verify the conditions of the theorem before applying it, and carefully interpret the solutions within the context of the interval.

By practicing with various functions and intervals, students can develop a robust intuition for when and how the MVT applies, ultimately strengthening their calculus toolkit for more advanced studies and real-world applications.

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