6. (5 Points) Find The Amplitude, Period, And Phase Shift Of Y = 4sin(3x).

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The task of analyzing a trigonometric function such as $Y = 4sin(3x)$ involves extracting key features that describe its shape and position on the graph. These features include the amplitude, period, and phase shift. Understanding these properties allows mathematicians and students to graph the function accurately and interpret the behavior of sinusoidal functions in various applications, from physics to engineering. In this article, we will explore how to determine each of these parameters systematically, with detailed explanations and illustrative examples.

Understanding the General Form of Sine Functions

The Standard Sine Function

The basic sine function is written as:

- **Y = sin(x)**

which has the following characteristics:

1. **Amplitude:** 1
2. **Period:** 2π
3. **Phase shift:** 0
4. **Vertical shift:** 0

Transformations of the Sine Function

When the basic sine function undergoes transformations, its general form can be written as:

- **$Y = A \sin(Bx - C) + D$**

where:

1. **Amplitude (A):** The height from the midline to the maximum or minimum point.
2. **Period:** The length of one complete cycle, calculated as $2\pi / |B|$.
3. **Phase shift:** The horizontal shift, calculated as C / B.
4. **Vertical shift (D):** The upward or downward shift of the graph.

Our task is to analyze the function $Y = 4\sin(3x)$ within this framework.

Step 1: Find the Amplitude

Definition of Amplitude

The amplitude of a sine function indicates the maximum distance from the midline (equilibrium position) to the peak (maximum) or trough (minimum). It reflects how "tall" or "short" the wave is.

Calculation of Amplitude for Y = 4sin(3x)

In the given equation, the coefficient of the sine function is 4, which directly represents the amplitude. Therefore:

- Amplitude $= |A| = |4| = 4$

Interpretation

This means the graph of Y = 4sin(3x) oscillates between 4 and -4, with its midline at y=0. The maximum value of the function is 4, and the minimum value is -4.

Step 2: Find the Period

Understanding the Period

The period of a sinusoidal function is the length of the interval over which the function completes one full cycle. For the basic sine function, the period is 2π, but transformations involving B modify this value.

Calculating the Period for Y = 4sin(3x)

Recall that the period for a sine function of the form $Y = A\ sin(Bx)$ is given by:

- **Period = 2π / |B|**

In our case, B = 3, so:

- Period = 2π / 3

Interpretation

This indicates that the function completes one full cycle over an interval of length $2\pi/3$ along the x-axis. As B increases, the function oscillates more rapidly, resulting in a shorter period.

Step 3: Find the Phase Shift

Understanding Phase Shift

The phase shift refers to the horizontal translation of the graph from its standard position. It indicates how far the graph has shifted left or right along the x-axis.

Calculating the Phase Shift for Y = 4sin(3x)

In the general form $Y = A\ sin(Bx - C) + D$, the phase shift is:

- **Phase shift = C / B**

In our case, the function is $Y = 4sin(3x)$, which can be viewed as $Y = 4sin(3x - 0)$. Here, C = 0, so:

- Phase shift = 0 / 3 = 0

Interpretation

This means the graph is not shifted horizontally; it starts at the standard position with no phase shift. If there were a C term, say in a more general form, the calculation would be similar, and the result could be a positive or negative shift depending on the sign of C.

Summary of Key Features

Based on the above calculations, the sinusoidal function $Y = 4sin(3x)$ has the following key properties:

- **Amplitude:** 4
- **Period:** $2\pi / 3$
- **Phase shift:** 0

Additional Considerations and Variations

Vertical Shifts and Changes in Phase

If the function included a vertical shift D or a phase shift C, the properties would be adjusted accordingly. For example:

- Y = 4sin(3x - π/2) + 2

would have:

1. Amplitude: 4
2. Period: $2\pi / 3$
3. Phase shift: $(\pi/2) / 3 = \pi/6$
4. Vertical shift: +2

Visualizing the Graph

Understanding the mathematical properties aids in sketching the graph accurately. For $Y = 4\sin(3x)$:

- The graph oscillates between 4 and -4.
- It completes one cycle every $2\pi/3$ units along the x-axis.
- It begins at the origin (0,0), moving upward to its maximum at $x = \pi/6$, then downward to its minimum at $x = \pi/2$, and repeats every $2\pi/3$.

Real-World Applications of These Parameters

The amplitude, period, and phase shift are critical in various fields:

- **Physics:** Analyzing wave motion, oscillations, and vibrations.
- **Engineering:** Designing systems with periodic signals.
- **Biology:** Modeling circadian rhythms or other cyclical processes.
- **Economics:** Analyzing seasonal trends in data.

Conclusion

In summary, the function *Y = 4sin(3x)* exhibits an amplitude of 4, a period of *2π / 3*, and no phase shift. These features collectively define the shape, size, and position of the sinusoidal graph. Understanding how to derive these parameters from a given function is fundamental in the study of trigonometry, enabling accurate graphing and interpretation of periodic phenomena across various disciplines.

Frequently Asked Questions

What is the amplitude of the function y = 4sin(3x)?

The amplitude is 4, which is the coefficient of the sine function.

How do you determine the period of y = 4sin(3x)?

The period is given by 2π divided by the coefficient of x inside the sine, so period = 2π / 3.

What is the phase shift of the function y = 4sin(3x)?

Since there is no horizontal shift term added or subtracted, the phase shift is 0.

Calculate the period of y = 4sin(3x).

The period is 2π / 3, approximately 2.094 radians.

How does the amplitude affect the graph of y = 4sin(3x)?

The amplitude determines the maximum and minimum values of the graph, which are 4 and -4 respectively.

If the function were y = 4sin(3x + π/2), what would be the phase shift?

The phase shift would be -π/6, shifting the graph to the left by π/6.

What is the general form of a sine function used to find amplitude, period, and phase shift?

The general form is y = A sin(Bx - C) + D, where A is amplitude, 2π / B is period, and C/B is phase shift.

Why is it important to identify the amplitude, period, and phase shift of a trigonometric function?

These parameters help to understand the graph's shape, position, and how it oscillates over its cycle.

Can the phase shift of y = 4sin(3x) be non-zero? Why or why not?

Yes, if there were a horizontal translation term added inside the function, but in this case, since there's no such term, the phase shift is zero.

What is the significance of the coefficient 3 inside the sine function y = 4sin(3x)?

The coefficient 3 affects the period of the function, making it shorter than the standard sine wave.

Additional Resources

Understanding the Amplitude, Period, and Phase Shift of the Function y = 4sin(3x)

When analyzing sinusoidal functions, such as y = 4sin(3x), understanding their fundamental properties—including amplitude, period, and phase shift—is essential. These characteristics determine the shape and position of the graph, providing insights into the behavior of the function and how it models real-world phenomena like sound waves, tides, or oscillations. This comprehensive review aims to explore these aspects in detail, empowering you to interpret, graph, and manipulate sinusoidal functions effectively.

Introduction to Sinusoidal Functions

A sinusoidal function is any function that exhibits a wave-like pattern, repeating periodically over a certain interval. The basic form of a sine function is:

$$ y = A \sin(Bx + C) + D $$

where:

- A is the amplitude,
- B affects the period,
- C shifts the graph horizontally (phase shift),

- D shifts the graph vertically (vertical shift).

In our specific function:

$$ y = 4 \sin(3x) $$

- The coefficient 4 modifies the amplitude.
- The coefficient 3 inside the sine function influences the period.
- There is no C or D term present, indicating no phase shift or vertical shift.

Decoding the Amplitude of y = 4sin(3x)

What is Amplitude?

The amplitude of a sinusoidal wave refers to the maximum distance it reaches from its central axis (equilibrium position). It essentially measures the "height" of the wave's peaks and the depth of its troughs.

Mathematically, the amplitude A is the absolute value of the coefficient in front of the sine function:

$$ \text{Amplitude} = |A| $$

Calculating the Amplitude for y = 4sin(3x)

Given the function:

$$ y = 4 \sin(3x) $$

- The coefficient A is 4.

Therefore:

$$ \boxed{\text{Amplitude} = 4} $$

This means that the graph of $y = 4\sin(3x)$:

- Rises from the midline to a maximum of +4,

- Falls to a minimum of -4,
- Oscillates symmetrically about the midline (which is at $y = 0$, since there is no vertical shift).

Implications of the Amplitude

- The larger the amplitude, the taller the wave.
- An amplitude of 4 indicates a relatively high oscillation compared to functions with smaller amplitudes.
- The amplitude influences the energy or intensity in physical systems modeled by the function.

Determining the Period of y = 4sin(3x)

What is the Period?

The period of a sinusoidal function is the length of the smallest interval over which the function repeats itself. For the sine function, the standard period is 2π. When the function is transformed via a coefficient B inside the argument, the period adjusts accordingly.

The general formula for the period P of a sine function:

$$ P = \frac{2\pi}{|B|} $$

where:

- B is the coefficient of x inside the sine function.

Calculating the Period for y = 4sin(3x)

Given:

$$ B = 3 $$

Applying the formula:

$$ P = \frac{2\pi}{|3|} = \frac{2\pi}{3} $$

Therefore:

$$\boxed{\text{Period} = \frac{2\pi}{3}}$$

Interpreting the Period

- The function completes one full cycle over an interval of $\frac{2\pi}{3}$.
- This is a shorter period than the standard 2π, indicating the wave oscillates more rapidly.
- For example, within the interval from $x = 0$ to $x = \frac{2\pi}{3}$, the sine function goes from 0 to maximum, back to 0, to minimum, and back to 0 again.

Graphical Implications

- The wave repeats more frequently due to the larger value of B.
- The increased frequency results in a "compressed" wave along the x-axis.
- This is critical for applications where the frequency or speed of oscillations matters, such as in signal processing.

Understanding the Phase Shift of y = 4sin(3x)

What is Phase Shift?

The phase shift of a sinusoidal function reflects a horizontal translation along the x-axis. It indicates how far the graph has been moved from its standard position.

The general form involving phase shift:

$$y = A \sin(Bx + C) + D$$

where:

- C affects the phase shift,
- The phase shift ϕ is calculated as:

$$\phi = -\frac{C}{B}$$

Assessing the Phase Shift for y = 4sin(3x)

In our function:

$$y = 4 \sin(3x)$$

- The term C is 0 (since there is no horizontal shift term added).

Thus,

$$\phi = -\frac{0}{3} = 0$$

Result: There is no phase shift; the graph is in its standard position.

Implications of Zero Phase Shift

- The wave starts at the standard sine position, crossing the origin at $x = 0$.
- The maximum, minimum, and midline points are aligned with the standard sine curve.
- If a phase shift had been present (say, $y = 4 \sin(3x - \frac{\pi}{2})$), the graph would shift to the right by $\frac{\pi}{6}$, since:

$$\phi = -\frac{-\frac{\pi}{2}}{3} = \frac{\pi}{6}$$

Summary of Key Characteristics of y = 4sin(3x)

Property	Value	Explanation
Amplitude	4	The maximum displacement from the midline
Period	$\frac{2\pi}{3}$	The length of one complete cycle
Phase Shift	0	No horizontal shift from the standard sine position
Vertical Shift	0	No vertical displacement; midline at $y = 0$

Deeper Insights and Applications

Physical Significance

- Amplitude: In physics, amplitude reflects the energy or intensity of oscillations; for example, louder sounds or higher tides.
- Period: Determines the frequency of the wave—how often oscillations occur per unit of time or distance.
- Phase Shift: Can model delays or advances in wave phenomena, such as the time lag in signals or phase differences in waves.

Graphing Tips

- Start by plotting the midline at $y = 0$.
- Mark the maximum and minimum at +4 and -4, respectively.
- Use the period \(\frac{2\pi}{3} \) to locate key points: maxima, minima, and zeros.
- Since phase shift is zero, the sine wave begins at the origin, ascending initially.

Transformations and Variations

- Changing the amplitude A scales the wave vertically.
- Modifying B affects the frequency and period.
- Adding a phase shift C shifts the wave horizontally.
- Adding D shifts the wave vertically.

Real-World Modeling

- Sound waves: amplitude correlates with loudness.
- Tides: amplitude relates to high and low tide heights.
- Mechanical vibrations: amplitude reflects energy levels.
- Signal processing: frequency (related to period) determines the rate of oscillations.

Conclusion

The function y = 4sin(3x) exemplifies a classic sinusoidal wave with specific, identifiable properties. Its amplitude of 4 indicates significant oscillation height, while its period of $\frac{2\pi}{3}$ reveals rapid oscillations compared to the standard sine wave. The absence of a phase shift means the wave starts at the origin, following the standard sine pattern. Recognizing these key features allows for accurate graphing, interpretation, and application in various scientific and engineering contexts.

Mastering the analysis of such properties enhances your understanding of sinusoidal functions, enabling more advanced work in mathematics, physics, and related fields. Whether modeling real-world oscillations or exploring mathematical theories, grasping the amplitude, period, and phase shift is fundamental to working confidently with wave phenomena.

6 5 Points Find The Amplitude Period And Phase Shift Of Y 4sin 3x

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