A= 131221311Find The Basis For: A) RS(A)=CS(A T) B) NS L(A)=NS R(A T)

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Understanding the concepts of bases in linear algebra is essential for analyzing linear transformations and matrix properties. In this article, we will explore how to find the basis for the subspaces related to the matrix \(A = 131221311 \), specifically focusing on the following problems:

- A) Find the basis for \(RS(A) = CS(A^T) \).
- B) Find the basis for \(NS L(A) = NS R(A^T) \).

By the end of this guide, you'll gain a clear understanding of the relationships between row spaces, column spaces, null spaces, and how to determine their bases effectively. Let's begin by clarifying the key concepts involved.

Understanding Key Concepts: Row Space, Column Space, Null Space, and Transpose

Before diving into the calculations, it's crucial to understand the fundamental subspaces associated with a matrix.

Row Space (RS(A))

- The row space of matrix \(A \) is the subspace spanned by its row vectors.
- It is a subset of \(\mathbb{R}^n \) where \(n \) is the number of columns.
- The row space is important because it represents all possible linear combinations of the rows of \(A \).

Column Space (CS(A))

- The column space of \(A \) is the subspace spanned by its column vectors.
- It is a subset of \(\mathbb{R}^m \), where \(m \) is the number of rows.
- The column space indicates all possible linear combinations of the columns of \(A \).

Null Space (NS(A))

- The null space of A (denoted as $NS(A)$) consists of all vectors x such that $Ax = 0$.
- It describes solutions to the homogeneous system and reflects the degrees of freedom in the solutions.

Relationships Between Spaces and Transpose

- The row space of A is equal to the column space of A^T: $RS(A) = CS(A^T)$.
- Conversely, the null space of A is the orthogonal complement of the row space, and similarly for their transposes.

Part A: Finding the Basis for $RS(A) = CS(A^T)$

This part involves understanding the relationship between the row space of A and the column space of its transpose A^T. It is a fundamental property in linear algebra that:

$$
RS(A) = CS(A^T)
$$

which means that the row space of A is the same as the column space of A^T.

Step 1: Representing the Matrix A

Given the matrix:

$$
A = \begin{bmatrix}
1 & 3 & 1 \\
2 & 2 & 1 \\
3 & 1 & 1
\end{bmatrix}
$$

This is a 3x3 matrix with entries:

- Row 1: [1, 3, 1]
- Row 2: [2, 2, 1]
- Row 3: [3, 1, 1]

Step 2: Find the Row Space Basis of \(A \)

The row space is spanned by the linearly independent rows of \(A \). To identify these, perform row operations to find the row echelon form.

Row Reduction Process:

1. Start with matrix \(A \):

\[
\begin{bmatrix}
1 & 3 & 1 \\
2 & 2 & 1 \\
3 & 1 & 1
\end{bmatrix}
\]

2. Use the first row as the pivot to eliminate entries below:

- \(R_2 \leftarrow R_2 - 2 R_1 \):

\[
\begin{bmatrix}
1 & 3 & 1 \\
0 & -4 & -1 \\
3 & 1 & 1
\end{bmatrix}
\]

- \(R_3 \leftarrow R_3 - 3 R_1 \):

\[
\begin{bmatrix}
1 & 3 & 1 \\
0 & -4 & -1 \\
0 & -8 & -2
\end{bmatrix}
\]

3. Simplify \(R_2 \):

- \(R_2 \leftarrow -\frac{1}{4} R_2 \):

\[
\begin{bmatrix}
1 & 3 & 1 \\
0 & 1 & \frac{1}{4} \\
0 & -8 & -2
\end{bmatrix}
\]

4. Use R_2 to eliminate entry in R_3:

- $R_3 \leftarrow R_3 + 8 R_2$:

$$
\begin{bmatrix}
1 & 3 & 1 \\
0 & 1 & \frac{1}{4} \\
0 & 0 & 0
\end{bmatrix}
$$

The last row is zero, indicating the rank is 2, with two independent rows.

Basis for RS(A):

Select the original rows corresponding to the leading ones:

$$
\boxed{
\text{Basis for } RS(A) = \left\{ \begin{bmatrix} 1 & 3 & 1 \end{bmatrix},
\begin{bmatrix} 2 & 2 & 1 \end{bmatrix} \right\}
}
$$

or equivalently, the rows:

$$
\left\{ (1, 3, 1), (2, 2, 1) \right\}
$$

Step 3: Find the Column Space Basis of A^T

Since $CS(A^T) = RS(A)$, the basis for $CS(A^T)$ is formed by the columns of A^T, which are the rows of A.

Therefore, the basis for $CS(A^T)$ is:

$$
\left\{ (1, 3, 1)^T, (2, 2, 1)^T \right\}
$$

which are the transposes of the original row vectors of A.

Part B: Finding the Basis for $NS L(A) = NS R(A^T)$

This section focuses on null spaces, specifically:

$$
NS L(A) = NS R(A^T)
$$

In linear algebra, the null space of A, denoted $NS(A)$, consists of all solutions to $Ax = 0$. Likewise, the null space of A^T, $NS(A^T)$, contains all solutions to $A^T y = 0$.

By the fundamental theorem of linear algebra:

$$
NS(A) = (CS(A^T))^\perp
$$
$$
NS(A^T) = (RS(A))^\perp
$$

which means they are orthogonal complements of the corresponding row and column spaces.

Note: The notation $NS L(A)$ and $NS R(A^T)$ likely refer to the null spaces associated with the left and right null spaces, respectively.

Calculating $NS(A)$ and $NS(A^T)$

Step 1: Find $NS(A)$

Solve $Ax = 0$.

Given A:

$$
\begin{bmatrix}
1 & 3 & 1 \\
2 & 2 & 1 \\
3 & 1 & 1
\end{bmatrix}
$$

Set $x = (x_1, x_2, x_3)^T$, then:

$$
\begin{cases}
x_1 + 3x_2 + x_3 = 0 \\
2x_1 + 2x_2 + x_3 = 0 \\
3x_1 + x_2 + x_3 = 0
\end{cases}
$$

Use the row-reduced form for simplicity:

$$
\begin{bmatrix}
1 & 3 & 1 \\
0 & -4 & -1 \\
0 & 0 & 0
\end{bmatrix}
$$

Corresponds to:

- $x_1 + 3x_2 + x_3 = 0$ (Equation 1)
- $-4x_2 - x_3 = 0$ (Equation 2)

From Equation 2:

$$
-4x_2 - x_3 = 0 \Rightarrow x_3 = -4x_2
$$

Substitute into Equation 1:

$$
x_1 + 3x_2 + (-4x_2) = 0 \
$$

Frequently Asked Questions

What is the main goal when finding the basis for RS(A) and CS(A^T)?

The main goal is to determine the sets of vectors that span the row space and column space of matrix A, respectively, which form bases for these subspaces.

How do you find RS(A) and CS(A^T) for a given

matrix?

RS(A) is found by determining the row space from the rows of A after row reduction, while CS(A^T) is the column space of the transpose of A, which corresponds to the row space of A transposed. Both involve identifying linearly independent vectors.

What is the significance of CS(A^T) in relation to A's column space?

CS(A^T) is the same as the row space of A, which is spanned by the rows of A. It provides an alternative perspective to analyze the column space of A.

How is the null space NS L(A) related to NS R(A^T)?

The null space NS L(A) (left null space) consists of vectors that satisfy A^T x = 0, which is equivalent to NS R(A^T). These spaces are orthogonal complements of the row spaces.

What steps are involved in finding the basis for NS L(A)?

To find the basis for NS L(A), solve the homogeneous equation A^T x = 0, typically by row reducing A^T and identifying the free variables to find the basis vectors.

Why is understanding the basis of these subspaces important in linear algebra?

Understanding these bases helps in analyzing the structure of the matrix, solving systems of equations, and understanding the rank, nullity, and the relationships between different subspaces.

Given the matrix A=131221311, how do you determine RS(A) and CS(A^T)?

First, write A in matrix form, perform row reduction to find the row space basis for RS(A), then transpose A to find A^T and determine the basis for its row space, which corresponds to CS(A^T).

What does the notation NS R(A) and NS L(A) refer to?

NS R(A) refers to the null space of A, i.e., solutions to A x = 0, while NS L(A) refers to the left null space, i.e., solutions to A^T x = 0.

How are the row space and column space related through the transpose of A?

The row space of A is equal to the column space of A^T, and vice versa. Transposing swaps these subspaces, which helps in understanding their bases and dimensions.

What are common methods to verify the correctness of the found bases for these subspaces?

Common methods include checking linear independence of the basis vectors, ensuring they span the respective subspace, and verifying that they satisfy the defining equations (e.g., A x = 0 for null spaces).

Additional Resources

Comprehensive Analysis of the Basis in Matrix Operations: Exploring RS(A) = CS(AT) and NS L(A) = NS R(AT)

Introduction

In advanced linear algebra and matrix theory, understanding the foundational subspaces associated with a given matrix is crucial for numerous applications, ranging from solving systems of equations to understanding transformations and their properties. The concepts of row space, column space, null space, and their relationships with matrix transpose operations are fundamental. The question at hand involves exploring the basis structures of specific subspaces and establishing the equality of certain subspace bases:

- A) RS(A) = CS(AT)
- B) NS L(A) = NS R(AT)

Here, the notation presumes standard definitions, though the symbols may vary in different contexts. In this discussion, we'll interpret:

- RS(A): The row space of matrix A
- CS(AT): The column space of AT
- NS L(A): The null space of the matrix A (or possibly the left null space, depending on notation)
- NS R(AT): The null space of AT (the transpose of A)

Our goal is to identify the basis for these subspaces, analyze their equivalence, and understand the implications of these relationships.

Fundamental Concepts and Definitions

Before diving into the specific relationships, it’s essential to clarify the core concepts involved:

1. Row Space (RS(A))

- Definition: The row space of an $m \times n$ matrix A is the subspace of $\mathbb{R}^n$ spanned by the row vectors of A.
- Properties:
- It is a subspace of $\mathbb{R}^n$.
- Its dimension equals the rank of A.
- Basis: The set of linearly independent rows of A.

2. Column Space (CS(A))

- Definition: The column space of A is the subspace of $\mathbb{R}^m$ spanned by the column vectors of A.
- Properties:
- It is a subspace of $\mathbb{R}^m$.
- Its dimension equals the rank of A.
- Basis: The set of linearly independent columns.

3. Null Space (NS(A))

- Definition: The null space of A is the set of all vectors $x \in \mathbb{R}^n$ such that $A x = 0$.
- Properties:
- It is a subspace of $\mathbb{R}^n$.
- Dimension: Nullity of A.
- Basis: The set of vectors that span the null space, found via methods like Gaussian elimination.

4. Left Null Space (NS L(A))

- Definition: The null space of AT, consisting of all vectors y such that AT y = 0.
- Properties:
- It is a subspace of $\mathbb{R}^m$.
- Corresponds to the orthogonal complement of the column space of A.

Relationship Between Subspaces and Transposes

An essential aspect of matrix theory involves the relationships between these subspaces:

- Row space of A is the same as the column space of AT:

\[

$$
RS(A) = CS(A^{T})
$$

- Null space of A is orthogonal complement to the row space:

$$
NS(A) \perp RS(A)
$$

- Null space of AT (or the left null space) is orthogonal complement to the column space:

$$
NS(A^{T}) \perp CS(A)
$$

Exploring the Statements

Now, let's analyze the two core statements:

A) RS(A) = CS(AT)

- Claim: The row space of A is equal to the column space of AT.

- Analysis:

By definition, the row vectors of A form a subspace in $\mathbb{R}^n$, and the columns of AT are in $\mathbb{R}^m$. However, carefully examining the dimensions:

- The row space of A is spanned by the row vectors, which are in $\mathbb{R}^n$.
- The column space of AT is spanned by the columns of AT, which are in $\mathbb{R}^m$.

But AT is an $n \times m$ matrix, so:

- RS(A): Subspace of $\mathbb{R}^n$
- CS(AT): Subspace of $\mathbb{R}^n$ (since columns of AT are in $\mathbb{R}^n$)

Therefore, both RS(A) and CS(AT) are subspaces of $\mathbb{R}^n$, making their equality plausible.

- Key insight:

The row space of A is exactly the column space of AT because the rows of A are the columns of AT.

- Implication:

RS(A) = CS(AT) is always true.

- Basis for RS(A):

To find a basis:

1. Perform row operations to find the set of linearly independent rows.
2. The non-zero rows in the row echelon form form a basis.

- Basis for CS(AT):

1. The columns of AT that are linearly independent form a basis.
2. Equivalently, the set of linearly independent columns of AT.

- Conclusion:

Since the rows of A are the columns of AT, the basis for RS(A) is the set of linearly independent rows of A, which corresponds directly to the basis of CS(AT).

B) NS L(A) = NS R(AT)

- Claim: The null space of A equals the null space of AT.

- Analysis:

Clarifying notation:

- NS L(A): Likely the null space of A.
- NS R(AT): The null space of AT.

The null space of A is:

$$
\{ x \in \mathbb{R}^n : A x = 0 \}
$$

The null space of AT is:

$$
\{ y \in \mathbb{R}^m : A^{T} y = 0 \}
$$

These two are generally different subspaces of different dimensions unless under specific conditions.

- When are they equal?

It is only true that:

\[

$$
NS(A) \perp CS(A^{T})
$$

and

$$
NS(A^{T}) \perp CS(A)
$$

and these spaces are orthogonal complements, not equal.

- Special case:

For square, symmetric matrices (A = AT), the null spaces coincide:

$$
NS(A) = NS(A^{T})
$$

But for general matrices, these are distinct.

- Key Point:

The null space of A is a subspace of $\mathbb{R}^n$, and the null space of AT is a subspace of $\mathbb{R}^m$.

- They are subspaces of different ambient spaces, hence cannot be equal unless in trivial or special cases.

- Conclusion:

In general, NS L(A) ≠ NS R(AT). Therefore, the statement is not true unless under specific conditions (e.g., symmetric matrices).

Basis Construction for Each Subspace

Having clarified the core relationships, let’s explore how to find the bases explicitly:

Basis for RS(A)

1. Method:

- Perform row operations to reduce A to row echelon form.
- Identify the non-zero rows.
- These non-zero rows form a basis for RS(A).

2. Example:

```
Suppose:

\[
A = \begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{bmatrix}
\]

- Row reduce to find independent rows.
- The first two rows are linearly independent; the third is dependent.

- Basis for RS(A): The first two rows.

Basis for CS(AT)

1. Method:

- Find the rank of A.
- Identify linearly independent columns of AT (which correspond to
independent rows of A).

2. Connection:
```

A 131221311find The Basis For A Rs A Cs A T B Ns L A Ns R A T

A 131221311find The Basis For A Rs A Cs A T B Ns L A Ns R A T

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