(c) Find The Length Of The Curve- Y = X Y = {x3/2 +4, *3/2 + 4, 0 SX S1.

(c) Find The Length Of The Curve- Y = X Y = {x^{3/2} + 4, ^{3/2} + 4, 0 □ x □ 1}

Understanding how to find the length of a curve is a fundamental aspect of calculus, especially in the context of applications involving arc lengths, physics, and engineering. In this article, we will delve into the process of determining the length of the specific curve defined by the functions y = x^{3/2} + 4 and y = ^{3/2} + 4 over the interval 0 □ x □ 1. We will explore the mathematical principles involved, step-by-step calculations, and the interpretation of the results, providing a comprehensive guide to this problem.

Understanding the Problem Statement

Interpreting the Curve Definitions

The notation given, y = x^{3/2} + 4 and y = ^{3/2} + 4, appears to involve a typographical or formatting issue. Typically, in calculus problems, a curve is defined by a function y = f(x). Here, the two functions seem to be similar, perhaps representing the same function or two related functions over a specified domain. The notation "^{3/2}" might be a placeholder or misprint, but given the context, it is likely intended to be y = x^{3/2} + 4, with x in the interval [0,1].

Assuming that the problem asks for the length of the curve $y = x^{3/2} + 4$ over the interval $x \in [0,1]$, the task becomes calculating the arc length of this function within these bounds. If there is an alternative interpretation, such as multiple functions or parametric equations, the method would adapt accordingly.

Clarifying the Interval and Domain

- **Interval:** $0 \leq x \leq 1$
- **Function:** $y = x^{3/2} + 4$

This simplifies our problem to finding the length of a single, well-defined curve over a finite interval.

Mathematical Foundations of Curve Length Calculation

Arc Length Formula for a Function y = f(x)

The length L of a curve y = f(x), from x = a to x = b, is given by the integral:

$$
L = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx
$$

where $\frac{dy}{dx}$ is the derivative of y with respect to x.

Applicability to Our Function

- Our function: $y = x^{3/2} + 4$
- Domain: $x \in [0,1]$

The process involves:

1. Calculating the derivative $\frac{dy}{dx}$
2. Plugging into the arc length formula
3. Evaluating the integral over the given interval

Step-by-Step Solution

Step 1: Find the Derivative $\frac{dy}{dx}$

Given:

$$
y = x^{3/2} + 4
$$

Derivative with respect to x:

$$
\frac{dy}{dx} = \frac{d}{dx} \left(x^{3/2} \right) + 0 = \frac{3}{2} x^{1/2}
$$

Note: The derivative of a constant (4) is zero.

Step 2: Set Up the Arc Length Integral

Using the formula:

$$
L = \int_{0}^{1} \sqrt{1 + \left(\frac{3}{2} x^{1/2} \right)^2} \, dx
$$

Simplify inside the square root:

$$
L = \int_{0}^{1} \sqrt{1 + \frac{9}{4} x} \, dx
$$

Step 3: Simplify the Integral

The integral becomes:

$$
L = \int_{0}^{1} \sqrt{1 + \frac{9}{4} x} \, dx
$$

Let’s write:

$$
L = \int_{0}^{1} \sqrt{1 + k x} \, dx \quad \text{where} \quad k = \frac{9}{4}
$$

This is a standard integral form.

Step 4: Solve the Integral

The integral:

$$
I = \int \sqrt{1 + k x} \, dx
$$

has the solution:

$$
I = \frac{2}{3k} (1 + k x)^{3/2} + C
$$

Derivation of the integral:

- Let $u = 1 + k x$
- Then, $du = k dx$ or $dx = \frac{du}{k}$

The integral becomes:

$$
I = \int \sqrt{u} \frac{du}{k} = \frac{1}{k} \int u^{1/2} du = \frac{1}{k} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{3k} u^{3/2} + C
$$

Back to x:

$$
I = \frac{2}{3k} (1 + k x)^{3/2} + C
$$

Applying the limits from 0 to 1:

$$
L = \left[\frac{2}{3k} (1 + k x)^{3/2} \right]_0^1
$$

where $k = \frac{9}{4}$

Calculations:

$$
L = \frac{2}{3 \times \frac{9}{4}} \left[(1 + \frac{9}{4} \times 1)^{3/2} - (1 + 0)^{3/2} \right]
$$

Simplify denominator:

$$
3 \times \frac{9}{4} = \frac{27}{4}
$$

So,

$$
L = \frac{2}{\frac{27}{4}} \left[\left(1 + \frac{9}{4}\right)^{3/2} - 1^{3/2} \right]
$$

Compute the prefactor:

$$
\frac{2}{\frac{27}{4}} = 2 \times \frac{4}{27} = \frac{8}{27}
$$

Compute inside the brackets:

$$
1 + \frac{9}{4} = \frac{4}{4} + \frac{9}{4} = \frac{13}{4}
$$

Raise to the power $3/2$:

$$
\left(\frac{13}{4} \right)^{3/2} = \left(\sqrt{\frac{13}{4}} \right)^3
$$

Calculate $\sqrt{\frac{13}{4}}$:

$$
\sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}
$$

Then,

$$
\left(\frac{\sqrt{13}}{2} \right)^3 = \frac{ (\sqrt{13})^3 }{ 2^3 } = \frac{ 13 \sqrt{13} }{8}
$$

Putting it all together:

$$
L = \frac{8}{27} \left(\frac{13 \sqrt{13}}{8} - 1 \right)
$$

Simplify:

$$
L = \frac{8}{27} \times \left(\frac{13 \sqrt{13} - 8}{8} \right) = \frac{13 \sqrt{13} - 8}{27}
$$

Final expression for the length:

$$
\boxed{
L = \frac{13 \sqrt{13} - 8}{27}
}
$$

Interpretation of the Result

The computed arc length L provides the exact length of the curve $y = x^{3/2} + 4$ from $x = 0$ to $x = 1$. Numerically, we can approximate:

- $\sqrt{13} \approx 3.6056$
- $13 \times 3.6056 \approx 46.8728$

Thus,

$$
L \approx \frac{46.8728 - 8}{27} = \frac{38.8728}{27} \approx 1.439
$$

This indicates that the length of the curve over the specified interval is approximately 1.439 units.

Extensions and Additional Considerations

Parametric and Polar Forms

- If the problem involved parametric equations or polar coordinates, the approach would involve different formulas,

Frequently Asked Questions

What is the main goal when finding the length of the curve $y = x^{3/2} + 4$?

The main goal is to compute the arc length of the curve between two points, which involves integrating the square root of 1 plus the derivative squared over the given interval.

How do you find the derivative of $y = x^{3/2} + 4$?

The derivative $y' = (3/2) x^{1/2}$, obtained by differentiating $x^{3/2}$ with respect to x and noting that the constant 4 vanishes upon differentiation.

What is the formula for the length of a curve $y = f(x)$ from $x = a$ to $x = b$?

The arc length L = ∫ from a to b of √(1 + (dy/dx)^2) dx.

How do you set up the integral for the length of $y = x^{3/2} + 4$ from

x = 0 to x = 1?

L = □□^{1} □(1 + [(3/2) x^{1/2}]^2) dx, which simplifies to □□^{1} □(1 + (9/4) x) dx.

How do you evaluate the integral □ □(1 + (9/4) x) dx?

Use substitution u = 1 + (9/4) x, then du = (9/4) dx, and rewrite the integral as (4/9) □ □u du, which can be integrated using standard power rules.

What is the final formula for the length of the curve y = x^{3/2} + 4 from 0 to 1?

L = (4/9) (2/3) [u^{3/2}] evaluated from u=1 to u=1 + (9/4)1, resulting in a numerical value after computing the definite integral.

Can the arc length formula be applied to any function y = f(x)?

Yes, as long as the function is continuous and differentiable on the interval, allowing the use of the arc length integral formula.

What are common challenges when calculating the length of complex curves?

Challenges include integrating complicated expressions, handling functions with difficult derivatives, and ensuring the correct limits are used for the interval.

How does understanding the derivative help in finding the curve's length?

The derivative provides the slope of the tangent at each point, which is essential for calculating the infinitesimal segments that sum up to the total arc length.

Additional Resources

(c) Find The Length Of The Curve - Y = X Y = {x^{3/2} + 4, 3/2 + 4, 0 □ x □ 1}

Understanding the precise length of a curve is a fundamental aspect of calculus that bridges geometric intuition with mathematical rigor. In this article, we delve into the process of calculating the length of a specific curve defined by the function $y = x^{3/2} + 4$ over the interval from $x = 0$ to $x = 1$. Whether you're a student brushing up on calculus concepts or a math enthusiast exploring the intricacies of curve lengths, this comprehensive guide aims to make the process clear, detailed, and accessible.

Introduction: Setting the Stage for Curve Length Calculation

The problem at hand involves finding the length of a curve described by the function:

$$y = x^{3/2} + 4 \quad \text{for} \quad 0 \leq x \leq 1.$$

Understanding this task requires familiarity with the concept of arc length, a measure of the distance along a curve between two points. Unlike the straight-line distance, calculating the length of a curve involves integrating the infinitesimal segments that make up the path.

The general formula for the length L of a function $y = f(x)$ over $[a, b]$ is:

$$L = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx$$

Applying this formula to our specific function involves several steps: differentiating y with respect to x, squaring the derivative, simplifying the expression under the square root, and performing the integration.

Understanding the Curve: The Function $y = x^{3/2} + 4$

Before diving into the calculus, it's beneficial to analyze the properties of the given function:

- Domain: x ranges from 0 to 1.
- Range: Since $x^{3/2}$ is non-negative over this interval, the function's output starts at $y(0) = 0 + 4 = 4$ and increases to $y(1) = 1^{3/2} + 4 = 1 + 4 = 5$.
- Shape: The function is monotonically increasing and concave upward, reflecting the fractional power's growth behavior.

Graphically, the curve starts at the point $(0, 4)$ and gently rises to $(1, 5)$, with the rate of increase governed by the derivative dy/dx.

Step 1: Differentiating the Function

The first key step is to find the derivative dy/dx. Given:

$$
y = x^{3/2} + 4
$$

The derivative is:

$$
\frac{dy}{dx} = \frac{d}{dx} (x^{3/2}) + \frac{d}{dx}(4) = \frac{3}{2} x^{(3/2) - 1} + 0 = \frac{3}{2} x^{1/2}
$$

This simplifies to:

$$
\frac{dy}{dx} = \frac{3}{2} \sqrt{x}
$$

Understanding this derivative is crucial because it describes the slope of the tangent to the curve at any point x.

Step 2: Setting Up the Arc Length Integral

Applying the arc length formula:

$$
L = \int_{0}^{1} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx
$$

Substituting dy/dx:

$$
L = \int_{0}^{1} \sqrt{1 + \left(\frac{3}{2} \sqrt{x} \right)^2} \, dx
$$

Simplify the expression inside the square root:

$$
\left(\frac{3}{2} \sqrt{x} \right)^2 = \left(\frac{3}{2} \right)^2 x = \frac{9}{4} x
$$

Therefore, the integrand becomes:

$$
\sqrt{1 + \frac{9}{4} x}
$$

The integral for the total length simplifies to:

$$
L = \int_{0}^{1} \sqrt{1 + \frac{9}{4} x} \, dx
$$

Step 3: Simplifying and Computing the Integral

To evaluate:

$$
L = \int_{0}^{1} \sqrt{1 + \frac{9}{4} x} \, dx
$$

we can approach by substitution to handle the square root.

Substitution:

Let:

$$
u = 1 + \frac{9}{4} x
$$

Then:

$$
du = \frac{9}{4} dx \quad \Rightarrow \quad dx = \frac{4}{9} du
$$

When $x = 0$:

$$
u = 1 + 0 = 1
$$

When $x = 1$:

$$
u = 1 + \frac{9}{4} \times 1 = 1 + \frac{9}{4} = \frac{4}{4} + \frac{9}{4} = \frac{13}{4}
$$

Rewriting the integral:

$$
L = \int_{u=1}^{u=13/4} \sqrt{u} \times \frac{4}{9} du
$$

Factor out constants:

$$
L = \frac{4}{9} \int_{1}^{13/4} u^{1/2} \, du
$$

The integral of $u^{1/2}$ is straightforward:

$$
\int u^{1/2} \, du = \frac{2}{3} u^{3/2} + C
$$

Applying the limits:

$$
L = \frac{4}{9} \times \frac{2}{3} \left[u^{3/2} \right]_{1}^{13/4}
$$

Simplify constants:

$$
L = \frac{8}{27} \left[\left(\frac{13}{4} \right)^{3/2} - 1^{3/2} \right]
$$

Since $1^{3/2} = 1$, the length is:

$$
L = \frac{8}{27} \left[\left(\frac{13}{4} \right)^{3/2} - 1 \right]
$$

Step 4: Simplifying the Expression for Final Result

To express the answer more clearly, evaluate $\left(\frac{13}{4} \right)^{3/2}$.

Recall:

$$

$$
\left(\frac{13}{4} \right)^{3/2} = \left(\left(\frac{13}{4} \right)^{1/2} \right)^3
$$

First, compute $\left(\frac{13}{4} \right)^{1/2}$:

$$
\left(\frac{13}{4} \right)^{1/2} = \frac{\sqrt{13}}{\sqrt{4}} = \frac{\sqrt{13}}{2}
$$

Now, cube this:

$$
\left(\frac{\sqrt{13}}{2} \right)^3 = \frac{ (\sqrt{13})^3 }{ 2^3 } = \frac{ (\sqrt{13})^2 \times \sqrt{13} }{8} = \frac{ 13 \times \sqrt{13} }{8 }
$$

Therefore:

$$
\left(\frac{13}{4} \right)^{3/2} = \frac{13 \sqrt{13}}{8}
$$

Substituting back into the length expression:

$$
L = \frac{8}{27} \left(\frac{13 \sqrt{13}}{8} - 1 \right)
$$

Distribute $\frac{8}{27}$:

$$

$$
L = \frac{8}{27} \times \frac{13 \sqrt{13}}{8} - \frac{8}{27} \times 1
$$

Simplify:

$$
L = \frac{13 \sqrt{13}}{27} - \frac{8}{27}
$$

Expressed as a single fraction:

$$
L = \frac{13 \sqrt{13} - 8}{27}
$$

Final Result: The Length of the Curve

The length of the curve $y = x^{3/2} + 4$ from $x=0$ to $x=1$ is:

$$
\boxed{
L = \frac{13 \sqrt{13} - 8}{27}
}
$$

This exact expression provides a precise measure of the arc length, combining both radical and rational components.

Practical Implications and Visualization

Understanding the length of this curve offers insight into how fractional powers influence the shape and

[C Find The Length Of The Curve Y X Y X3 2 4 3 2 4 0 Sx S1]

C Find The Length Of The Curve Y X Y X3 2 4 3 2 4 0 Sx S1

Related Articles

- Juxtapose (point Out The Opposites Between) The Zulu And Xhosa Tribes.
- In What 3 Ways Does Whitmans Poem Match The Time In Which He Was Writing?
- Is Nato Passing A Resolution Against Iran An Example Of Arbitration?

Back to Home