

Consider Angle In Quadrant II, Where $\cos = -12/13$. What's The Value Of $\sin$

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Understanding the relationships between trigonometric functions is fundamental in mathematics, especially in the context of the unit circle. When given specific values for one trigonometric function, such as cosine, it becomes essential to determine the corresponding sine value, considering the angle's quadrant. This article explores the process of finding the sine of an angle situated in the second quadrant, where the cosine value is known to be $-12/13$. We'll delve into the principles of trigonometry, review relevant formulas, and step through a detailed solution to this problem.

Understanding the Context: Trigonometric Functions and the Unit Circle

What Are Trigonometric Functions?

Trigonometric functions—sine, cosine, tangent, and their reciprocals—are fundamental in analyzing angles and their relationships within right-angled triangles and the unit circle. These functions relate the angles to the ratios of sides in right triangles or coordinates on the circle.

- Sine ($\sin$): Opposite side over hypotenuse
- Cosine ($\cos$): Adjacent side over hypotenuse
- Tangent ($\tan$): Opposite side over adjacent side

The Unit Circle and Quadrants

The unit circle, a circle with a radius of 1 centered at the origin of a coordinate plane, provides a geometric representation of these functions. The coordinates of a point on the circle, (x, y) , correspond to $(\cos \theta, \sin \theta)$.

The circle is divided into four quadrants:

1. Quadrant I (0° to 90°): Both sine and cosine are positive.
2. Quadrant II (90° to 180°): Sine is positive; cosine is negative.
3. Quadrant III (180° to 270°): Both sine and cosine are negative.
4. Quadrant IV (270° to 360°): Sine is negative; cosine is positive.

Knowing the quadrant helps to determine the signs of the trigonometric functions.

Given Data and Problem Restatement

The problem states:

> Consider angle in Quadrant II, where $\cos \theta = -12/13$. What's the value of $\sin \theta$?

Key points:

- Quadrant: II
- Cosine value: $-12/13$
- Question: Find sine value, $\sin \theta$

Since cosine is negative and the angle is in Quadrant II, the signs align with the properties of the unit circle.

Step-by-Step Solution to Find Sin θ

Step 1: Recall the Pythagorean Identity

The fundamental identity relating sine and cosine is:

$$\sin^2 \theta + \cos^2 \theta = 1$$

This identity holds for all angles θ .

Step 2: Plug in the Known Cosine Value

Given:

$$\cos \theta = -\frac{12}{13}$$

Substitute into the identity:

$$\sin^2 \theta + \left(-\frac{12}{13}\right)^2 = 1$$

Calculating:

$$\sin^2 \theta + \frac{144}{169} = 1$$

Step 3: Solve for $\sin^2 \theta$

Subtract $\frac{144}{169}$ from both sides:

$$\sin^2 \theta = 1 - \frac{144}{169}$$

Express 1 as $\frac{169}{169}$:

$$\sin^2 \theta = \frac{169}{169} - \frac{144}{169} = \frac{25}{169}$$

Step 4: Take the Square Root to Find $\sin \theta$

$$\sin \theta = \pm \sqrt{\frac{25}{169}}$$

$$\sin \theta = \pm \frac{5}{13}$$

Step 5: Determine the Sign of $\sin \theta$ Based on the Quadrant

In Quadrant II:

- Sine is positive.
- Cosine is negative.

Since the angle is in Quadrant II, the sine value must be positive:

$$\sin \theta = \frac{5}{13}$$

Summary of the Result

Final Answer:

$$\boxed{\sin \theta = \frac{5}{13}}$$

This value complements the cosine value of $-\frac{12}{13}$ for an angle located in the second quadrant.

Additional Insights and Related Problems

Understanding Sign Conventions in Different Quadrants

It's crucial to remember the signs of trigonometric functions based on the quadrant:

- Quadrant I: $(\sin \theta > 0, \cos \theta > 0)$
- Quadrant II: $(\sin \theta > 0, \cos \theta < 0)$
- Quadrant III: $(\sin \theta < 0, \cos \theta < 0)$
- Quadrant IV: $(\sin \theta < 0, \cos \theta > 0)$

This knowledge guides the determination of the correct sign when solving for sine or cosine.

Related Problems and Applications

- Finding tangent given sine and cosine: Use $(\tan \theta = \frac{\sin \theta}{\cos \theta})$.
- Solving for angles in different quadrants: Use inverse trigonometric functions, considering the signs and ranges.
- Real-world applications: Engineering, physics, and navigation often require calculating angles using known trigonometric ratios.

Conclusion: Mastering Trigonometric Values in Different Quadrants

Determining the sine of an angle given the cosine value involves fundamental trigonometric identities and an understanding of the unit circle's quadrant-based sign conventions. In this case, with $(\cos \theta = -\frac{12}{13})$ and θ in the second quadrant, the sine value is positive, $(\frac{5}{13})$. This problem exemplifies the importance of combining algebraic manipulation with geometric insights to solve trigonometric problems effectively.

By mastering these techniques, students and professionals can confidently handle a wide range of problems involving angles and their trigonometric functions, essential skills in mathematics, physics, engineering, and related fields.

Meta Description for SEO:

Learn how to find the sine of an angle in Quadrant II when cosine is $(-12/13)$. Step-by-step explanation of trigonometric identities, quadrant signs, and solving for sine. Perfect for students and math enthusiasts seeking a comprehensive understanding of trigonometry problems.

Frequently Asked Questions

In Quadrant II, where $\cos \theta = -12/13$, what is the value of $\sin \theta$?

$\sin \theta = 5/13$

Given $\cos \theta = -12/13$ in Quadrant II, how do you find $\sin \theta$?

Use the Pythagorean identity: $\sin \theta = \sqrt{1 - \cos^2 \theta}$. Since θ is in Quadrant II, $\sin \theta$ is positive. So, $\sin \theta = 5/13$.

Why is $\sin \theta$ positive when $\cos \theta = -12/13$ in Quadrant II?

In Quadrant II, sine values are positive, even though cosine values are negative.

What is the Pythagorean relationship used to find $\sin \theta$ when $\cos \theta$ is known?

The identity is $\sin^2 \theta + \cos^2 \theta = 1$. Plugging in $\cos \theta = -12/13$, $\sin \theta = \sqrt{1 - (-12/13)^2} = 5/13$.

If $\cos \theta = -12/13$ in Quadrant II, what is the exact value of $\sin \theta$?

$\sin \theta = 5/13$

How does the sign of sine and cosine differ in Quadrant II for the given values?

In Quadrant II, sine is positive and cosine is negative, aligning with the given $\cos \theta = -12/13$.

Can you confirm $\sin \theta$ for $\cos \theta = -12/13$ in Quadrant II without a calculator?

Yes. Using Pythagoras, $\sin \theta = 5/13$.

What is the reference angle corresponding to $\cos \theta = -12/13$ in Quadrant II?

The reference angle θ' satisfies $\cos \theta' = 12/13$, so $\theta' = \cos^{-1}(12/13)$.

What is the significance of the negative value of $\cos$ in

Quadrant II?

A negative cos value indicates the angle is in Quadrant II, where cosine is negative and sine is positive.

How do the values of sine and cosine relate for angles in Quadrant II with given cosine?

For $\cos \theta = -12/13$ in Quadrant II, $\sin \theta = 5/13$, maintaining the Pythagorean relationship and the sign conventions of the quadrant.

Additional Resources

Consider Angle In Quadrant II, Where Cos = $-12/13$. What's The Value Of Sin?

Understanding the relationship between the sine and cosine functions is fundamental in trigonometry, especially when working with angles in different quadrants. The problem, "Consider an angle in Quadrant II where $\cos \theta = -12/13$. What's the value of $\sin \theta$?" provides a classic example of applying the Pythagorean identity and understanding the properties of angles within specific quadrants. This type of question is common not only in academic settings but also in practical applications such as physics, engineering, and computer graphics, where precise angle measurements are critical.

Understanding the Context: Quadrant II and Trigonometric Sign Conventions

Quadrant II Overview

Quadrant II is the section of the Cartesian plane where the x-coordinates are negative, and the y-coordinates are positive. On the coordinate plane, angles in this quadrant are measured from the positive x-axis, moving counterclockwise, typically between 90° and 180° , or $\pi/2$ and π radians.

Sign conventions in Quadrant II:

- Sine ($\sin \theta$) is positive because the y-coordinate is positive.
- Cosine ($\cos \theta$) is negative because the x-coordinate is negative.
- Tangent ($\tan \theta$) is negative, being the ratio of sine to cosine.

Knowing these conventions helps in determining the signs of the trigonometric functions once the angle's location is established.

Relevance of the Given Cosine Value

Given $\cos \theta = -12/13$, the negative value confirms that θ is in either Quadrant II or Quadrant III, where

cosine is negative. The problem specifies the angle is in Quadrant II, aligning with the positive sine value and the negative cosine.

Mathematical Approach to Find Sin θ

Step 1: Recall the Pythagorean Identity

The fundamental Pythagorean identity relates sine and cosine:

$$\sin^2 \theta + \cos^2 \theta = 1$$

This identity holds true for all angles θ .

Step 2: Substitute the Known Cosine Value

Given:

$$\cos \theta = -\frac{12}{13}$$

Squaring both sides:

$$\cos^2 \theta = \left(-\frac{12}{13}\right)^2 = \frac{144}{169}$$

Step 3: Solve for $\sin^2 \theta$

Using the identity:

$$\sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{144}{169} = \frac{169}{169} - \frac{144}{169} = \frac{25}{169}$$

Step 4: Find $\sin \theta$

Taking the square root:

$$\sin \theta = \pm \sqrt{\frac{25}{169}} = \pm \frac{5}{13}$$

Determining the sign:

Since θ is in Quadrant II, where sine is positive:

$$\sin \theta = +\frac{5}{13}$$

Summary of the Solution

- Given $\cos \theta = -\frac{12}{13}$ in Quadrant II.
- Using the Pythagorean identity, $\sin \theta = +\frac{5}{13}$.

Final answer:

$$\boxed{\sin \theta = \frac{5}{13}}$$

This calculation demonstrates the power of fundamental identities in trigonometry and the importance of understanding the sign conventions associated with different quadrants.

Deeper Insights and Conceptual Understanding

Why is the signs important?

Correctly identifying the signs of sine and cosine in each quadrant is crucial for solving trigonometric problems accurately. Misinterpretation can lead to incorrect answers, especially in applications like vector components, oscillations, and wave analysis.

Geometric interpretation

Visualizing the problem on a right triangle:

- The hypotenuse is 13.
- The adjacent side (x) is -12 (since cosine = adjacent/hypotenuse).
- The opposite side (y) can be found using the Pythagorean theorem:
$$|y| = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$$

Since the angle θ is in Quadrant II:

- x (adjacent side) is negative.
- y (opposite side) is positive.

Thus, $\sin \theta = y/\text{hypotenuse} = 5/13$, aligning with our earlier calculation.

Additional Considerations and Applications

Applications in Physics and Engineering

Knowing the sine value allows for computing components of vectors, analyzing oscillatory motion, and solving problems involving angles of elevation or depression.

Pros:

- Precise calculation of components.
- Simplifies complex problems into manageable parts.

Cons:

- Assumes perfect right triangles; real-world data might involve measurement errors.
- Requires understanding of quadrants and signs to avoid mistakes.

Use in Trigonometric Equations and Identities

The value of $\sin \theta$ can be used in solving various identities:

- Double angle formulas
- Half-angle formulas
- Sum and difference identities

Common Pitfalls and Misunderstandings

- Confusing quadrant signs: Remember sine is positive in Quadrants I and II.
- Forgetting to take the correct square root sign.
- Misidentifying the quadrant based solely on the cosine value.

Conclusion

The question "Consider angle in Quadrant II where $\cos \theta = -12/13$. What's the value of $\sin$?" exemplifies fundamental trigonometry principles—using the Pythagorean identity, understanding quadrant signs, and visualizing geometric relationships. The derived sine value, $\frac{5}{13}$, aligns perfectly with the properties of right triangles and the sign conventions of the second quadrant.

Mastering such problems enhances one's ability to navigate the broader landscape of trigonometry, supporting applications across sciences and engineering. Whether you're solving for unknown angles, analyzing vectors, or working on wave functions, understanding how to determine sine from cosine (and vice versa) is an essential skill. Remember, the key is to always consider the quadrant and apply identities carefully to arrive at the correct, meaningful solutions.

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