Determine If The Following Integral Converges Or Diverges. S. (1+2x)e* Da

Determine If The Following Integral Converges Or Diverges. S. (1+2x)e Da is a fundamental problem in calculus, involving the analysis of whether a given integral converges or diverges. This type of problem is central to understanding the behavior of functions over infinite intervals or near points where the function may become unbounded. In this article, we will explore the methods used to evaluate the convergence or divergence of integrals, specifically focusing on the integral of the form S. (1+2x)e Da, which appears to be a notation for an integral involving an exponential function multiplied by a polynomial expression. We will clarify the integral's form, analyze its convergence properties, and provide step-by-step approaches to determine its behavior.

Understanding the Integral: Clarifying the Expression

Before delving into convergence tests, it is crucial to interpret the integral's notation and form correctly. The given expression, "S. (1+2x)e Da," appears to be shorthand or a notation that needs clarification.

Interpreting the Integral Notation

- Likely, the integral is of the form *∫ (1 + 2x) e dx*, where the asterisk () indicates an exponential function with some base or exponent.
- Commonly, in calculus, integrals involving exponential functions are written as *∫ (1 + 2x)* e^{ax} *dx*, where *a* is a constant.
- Since the notation "e" is ambiguous, it probably represents an exponential function, such as e^{ax}.
- The limits of integration are not specified, but typically, for convergence analysis, we consider integrals over infinite intervals, such as from *0 to* ∞ or -∞ *to* ∞.

Assumed Integral for Analysis

Based on common problems in calculus, we will assume the integral to be:

$$\int_1^\infty (1 + 2x)\ e^{ax}\ dx$$

where *a* is a real number. The reason for choosing this form is because integrals involving exponential functions multiplied by polynomials are classic examples used to analyze convergence.

Methods to Determine Convergence or Divergence

Once the integral's form is clarified, the next step is to analyze whether it converges or diverges. Several methods are used in calculus to assess the behavior of improper integrals.

1. Comparison Test

The comparison test involves comparing the integral of interest with another integral whose convergence properties are known.

- If the integrand $f(x)$ is less than or equal to a function $g(x)$ for large x, and $\int g(x)\,dx$ converges, then $\int f(x)\,dx$ also converges.
- Conversely, if $f(x)$ is greater than or equal to a divergent function $g(x)$, then $\int f(x)\,dx$ diverges.

2. Limit Comparison Test

This test compares the integrand to a known benchmark function, often polynomial or exponential, by taking the limit of their ratio as $x \to \infty$.

3. Direct Evaluation and Asymptotic Behavior

For integrals involving exponential functions, analyzing the asymptotic behavior for large x helps determine convergence.

Analyzing the Integral $\int_1^\infty (1 + 2x)\, e^{ax}\, dx$

Let's analyze the integral assuming the form:

$$I = \int_1^\infty (1 + 2x)\, e^{ax}\, dx$$

where a is a real number.

Case 1: When *a* < *0*

In this case, the exponential decay dominates, and the integrand behaves like a polynomial multiplied by an exponentially decreasing function.

- As $x \to \infty$, e^{ax} tends to 0 since $a < 0$.
- The polynomial term $(1 + 2x)$ grows linearly, but exponential decay outweighs polynomial growth.
- Thus, the integrand behaves like *polynomial × exponential decay*, which tends to 0 rapidly.

Conclusion:
The integral converges for $a < 0$ due to the exponential decay overpowering polynomial growth.

Case 2: When *a* = *0*

In this scenario, the integral reduces to:

$$I = \int_1^\infty (1 + 2x)\, dx$$

which is a polynomial integral.

- Since $(1 + 2x)$ grows without bound as $x \to \infty$, the integral diverges.

Conclusion:
The integral diverges when $a = 0$.

Case 3: When *a* > *0*

Here, the exponential term grows exponentially, and the integrand behaves like *polynomial × exponential* as $x \to \infty$.

- Since exponential growth dominates polynomial growth, the integrand tends to infinity.

- Therefore, the integral diverges due to unbounded growth of the integrand.

Conclusion:
The integral diverges for $a > 0$.

Explicit Evaluation for $a < 0$

When $a < 0$, the integral converges, and we can evaluate it explicitly using integration by parts.

Step-by-Step Calculation

Suppose we evaluate:

$$I = \int (1 + 2x)\ e^{ax}\ dx$$

Method: Integration by parts, with:

- Let $u = 1 + 2x$, then $du = 2\ dx$.
- Let $dv = eax\ dx$, then $v = (1/a)\ eax$.

Applying integration by parts:

$$I = u\ v - \int v\ du = (1 + 2x)\ (1/a)\ e^{ax} - \int (1/a)\ e^{ax}\ 2\ dx$$

Simplify:

$$I = (1 + 2x)\ (1/a)\ e^{ax} - (2/a) \int e^{ax}\ dx$$

The remaining integral:

$$\int e^{ax}\ dx = (1/a)\ e^{ax}$$

So,

$$I = (1 + 2x)\ (1/a)\ e^{ax} - (2/a)\ (1/a)\ e^{ax} + C$$
$$= e^{ax}\ [\ (1 + 2x)/a - 2/a^2\ ] + C$$

Thus, the indefinite integral is:

$$\int (1 + 2x)\, e^{ax}\, dx = e^{ax} [(1 + 2x)/a - 2/a^2] + C$$

Convergence of the improper integral from 1 to ∞:

Since *a < 0*, as *x* → ∞, *eax* tends to 0 exponentially.

Evaluate the definite integral:

$$I = \lim_{t\to\infty} [e^{ax} ((1 + 2x)/a - 2/a^2)] \text{ from } x=1 \text{ to } x=t$$

At the upper limit:

$$x = t \to e^{at} \to 0$$

Therefore,

$$I = 0 - e^{a\ 1} [(1 + 21)/a$$

Frequently Asked Questions

How do I determine if the integral of (1+2x)e^x converges or diverges as x approaches infinity?

To analyze the convergence, examine the behavior of (1+2x)e^x as x approaches infinity. Since e^x grows faster than any polynomial, the integral diverges because the integrand increases rapidly without bound.

Does the integral of (1+2x)e^x from 0 to infinity converge?

No, the integral diverges because as x approaches infinity, (1+2x)e^x tends to infinity, making the improper integral divergent.

Can the integral of (1+2x)e^x be evaluated explicitly?

Yes, the indefinite integral can be found using integration by parts, but for convergence purposes, analyzing the limit behavior is more relevant.

What is the key factor in determining whether the integral of (1+2x)e^x converges?

The key factor is the exponential term e^x, which dominates polynomial growth, leading to divergence of the integral over infinite limits.

Is the integral of (1+2x)e^x from negative infinity to some finite value convergent?

Yes, since e^x approaches zero as x approaches negative infinity, the integral from negative infinity to a finite value converges.

How does the growth rate of the integrand affect the convergence of the integral?

If the integrand grows exponentially (like e^x), the integral tends to diverge over infinite limits unless the limits are finite or the integrand decays sufficiently fast.

Should I use comparison tests to determine the convergence of the integral involving (1+2x)e^x?

Yes, comparison tests are useful; for example, since (1+2x)e^x behaves similarly to e^x for large x, comparing with known divergent integrals helps determine divergence.

What is the behavior of the integrand (1+2x)e^x near infinity?

Near infinity, (1+2x)e^x behaves like 2x e^x, which tends to infinity rapidly, indicating divergence of the integral over an infinite interval.

Can the integral of (1+2x)e^x from 0 to infinity be considered convergent or divergent?

It is divergent because the integrand grows exponentially as x increases, causing the integral to diverge to infinity.

Additional Resources

Determine If The Following Integral Converges Or Diverges: S. (1+2x)e^x dx

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Introduction

The question of whether an integral converges or diverges is fundamental in calculus, especially when dealing with improper integrals over unbounded domains or those involving unbounded functions. Today, we will analyze the integral:

$$
\int (1 + 2x) e^x \, dx
$$

with a focus on its convergence properties as the limits approach infinity or negative infinity, depending on the context. To do this thoroughly, we need to understand the nature of the integrand, the techniques for evaluating integrals involving exponential functions, and the criteria for convergence.

This detailed review aims to lead you through the process of analyzing the integral's convergence, including:

- Understanding the structure of the integrand
- Determining whether the integral is definite or indefinite
- Applying relevant convergence tests
- Evaluating the integral explicitly
- Interpreting the results in the context of convergence or divergence

Let's dive in.

Understanding the Integral

The Form of the Integral

The integral in question appears to be:

$$
\int (1 + 2x) e^x \, dx
$$

which suggests a need for indefinite integration, unless specific bounds are provided.

Key Points:

- The integrand is a product of a polynomial, $(1 + 2x)$, and an exponential function, e^x.
- Such integrals are commonly encountered in calculus, especially in integration by parts.

Context: Improper vs. Proper Integrals

Before analyzing convergence, it's essential to clarify whether the integral is:

- Definite: With specified limits, such as $\int_a^b (1 + 2x) e^x \, dx$
- Indefinite: Without limits, representing an antiderivative

Given the statement, "Determine if the following integral converges or diverges," it suggests considering an improper integral over an unbounded domain, typically:

$$
\int_{a}^{\infty} (1 + 2x) e^x \, dx
$$

or

$$
\int_{-\infty}^{a} (1 + 2x) e^x \, dx
$$

or possibly the indefinite integral. For comprehensive analysis, we will assume the integral over $[a, \infty)$ and $(-\infty, a]$.

Part 1: Formal Evaluation of the Integral

Integration Techniques

The integrand combines polynomial and exponential functions, making integration by parts the most appropriate method.

Recall: Integration by parts formula:

\[
\int u \, dv = uv - \int v \, du
\]

Step 1: Choose \(u\) and \(dv\):

- Let \(u = 1 + 2x\), then \(du = 2\, dx\)
- Let \(dv = e^x \, dx\), then \(v = e^x\)

Step 2: Apply integration by parts:

\[
\int (1 + 2x) e^x \, dx = (1 + 2x) e^x - \int 2 e^x \, dx
\]

Step 3: Simplify the remaining integral:

\[
\int 2 e^x \, dx = 2 e^x + C
\]

Step 4: Final expression:

\[
\boxed{
\int (1 + 2x) e^x \, dx = (1 + 2x) e^x - 2 e^x + C = e^x (1 + 2x - 2) + C = e^x (2x - 1) + C
}
\]

This explicit antiderivative simplifies the analysis of the integral's convergence over specific bounds.

Part 2: Analyzing Convergence of Improper Integrals

Now, considering the indefinite integral's convergence, the primary focus shifts to the behavior of the antiderivative at the limits of integration, especially as \(x \to \pm \infty\).

2.1: Behavior as \(x \to \infty\)

Evaluate the limit:

\[

\lim_{x \to \infty} e^x (2x - 1)
\]

- As \(x \to \infty\), \(e^x\) grows exponentially, which dominates any polynomial term \(2x - 1\).
- The product \(e^x (2x - 1)\) tends to infinity because exponential growth outpaces polynomial growth.

Conclusion: The antiderivative tends to infinity as \(x \to \infty\). Therefore, the improper integral over \([a, \infty)\):

\[
\int_a^{\infty} (1 + 2x) e^x \, dx
\]

diverges for any finite \(a\).

2.2: Behavior as \(x \to -\infty\)

Evaluate:

\[
\lim_{x \to -\infty} e^x (2x - 1)
\]

- As \(x \to -\infty\), \(e^x \to 0\).
- Polynomial \(2x - 1 \to -\infty\), but exponential decay dominates polynomial growth.

- The product:

\[
e^x (2x - 1) \to 0 \times (-\infty) \to 0
\]

since exponential decay toward zero outweighs unbounded polynomial growth in the negative direction.

Conclusion: The antiderivative tends to 0 as \(x \to -\infty\).

Part 3: Final Determination of Convergence

Based on the previous analysis:

- Over \([a, \infty)\):

Since the antiderivative \(\to \infty\) as \(x \to \infty\), the improper integral:

$$
\int_a^{\infty} (1 + 2x) e^x \, dx
$$

diverges.

- Over $(-\infty, b]$:

The integral:

$$
\int_{-\infty}^b (1 + 2x) e^x \, dx
$$

converges because the antiderivative tends to 0 at $-\infty$.

Summary:

Limits of Integration	Convergence / Divergence	Explanation
$[a, \infty)$	Diverges	Antiderivative tends to infinity as $x \to \infty$
$(-\infty, b]$	Converges	Antiderivative tends to 0 as $x \to -\infty$

Part 4: Additional Considerations

4.1: When Is the Integral Absolutely Convergent?

In integrals involving oscillatory functions, absolute convergence is a critical concept, but here, since the integrand involves exponential growth, absolute convergence over $[a, \infty)$ is impossible.

4.2: Practical Implications

- If the integral is over a bounded interval, it converges trivially.
- For unbounded intervals, the exponential growth dominates, leading to divergence over $[a, \infty)$.

Part 5: Summary of Key Concepts

- Exponential functions, like e^x, grow faster than any polynomial in x, influencing convergence.
- Integration by parts is an effective method for integrals involving polynomial and exponential products.
- The behavior at infinity determines convergence of improper integrals:

- If the antiderivative tends to infinity, the integral diverges.
- If it tends to a finite limit or zero, the integral converges.
- Limit analysis of the antiderivative at the bounds is crucial for assessing convergence.

Final Thoughts

Understanding the convergence or divergence of integrals is essential in various fields such as mathematical analysis, physics, and engineering. The integral:

$$
\int (1 + 2x) e^x \, dx
$$

has a straightforward antiderivative, which simplifies the process of analyzing its improper versions.

- Over the positive unbounded domain, the integral diverges due to exponential growth.
- Over the negative unbounded domain, the integral converges due to exponential decay.

In conclusion:

- The indefinite integral evaluates explicitly to:

$$
\boxed{
\int (1 + 2x) e^x \, dx = e^x (2x - 1) + C
}
$$

- The improper integral $\int_a^{\infty} (1 + 2x) e^x \, dx$ diverges.

- The improper integral $\int_{-\infty}^b (1 + 2x) e^x \, dx$ converges.

This comprehensive analysis underscores the importance of understanding the integrand's behavior at the bounds to determine convergence properties accurately.

References & Further Reading

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Apostol, T.

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