Evaluate Each Infinite Geometric Series. 1-1/5 +1/25 - 1/125+ . . . . .

Evaluate Each Infinite Geometric Series. 1 - 1/5 + 1/25 - 1/125 + ...

Understanding how to evaluate infinite geometric series is a fundamental skill in mathematics, particularly in the fields of calculus, algebra, and mathematical analysis. The series 1 - 1/5 + 1/25 - 1/125 + ... is a classic example that demonstrates the power of geometric series formulas and their applications. This article aims to thoroughly explore the methods to evaluate such series, explain the concepts involved, and provide step-by-step guidance to mastering the evaluation process.

Introduction to Infinite Geometric Series

An infinite geometric series is a sum of an infinite sequence of terms where each term is obtained by multiplying the previous term by a fixed, non-zero constant called the common ratio. These series are prevalent in various mathematical and real-world contexts, such as calculating interest, analyzing algorithms, and modeling natural phenomena.

The general form of an infinite geometric series is:

$$ S = a + ar + ar^2 + ar^3 + \dots $$

where:
- a is the first term,
- r is the common ratio (ratio between consecutive terms).

Key Point: Infinite geometric series converge to a finite sum only if the absolute value of the common ratio is less than 1 ($|r| < 1$).

Understanding the Series: 1 - 1/5 + 1/25 - 1/125 + ...

Let's analyze the given series:

$$ S = 1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \dots $$

Step 1: Identify the first term and the common ratio.

- First term, $a = 1$.
- To find the common ratio, divide the second term by the first:

$$ r = \frac{-\frac{1}{5}}{1} = -\frac{1}{5} $$

Check if this ratio holds for subsequent terms:

$$ \frac{\frac{1}{25}}{-\frac{1}{5}} = -\frac{1}{5} $$
$$ \frac{-\frac{1}{125}}{\frac{1}{25}} = -\frac{1}{5} $$

Since the ratio between successive terms is consistently $ -\frac{1}{5} $, the series is geometric with:

- $ a = 1 $
- $ r = -\frac{1}{5} $

Step 2: Verify convergence.

Because $ |r| = \frac{1}{5} < 1 $, the series converges, and we can find its sum.

Formula for the Sum of an Infinite Geometric Series

The sum $ S $ of an infinite geometric series with first term $ a $ and common ratio $ r $ (where $ |r| < 1 $) is:

$$ S = \frac{a}{1 - r} $$

Applying this to our series:

$$ S = \frac{1}{1 - (-\frac{1}{5})} = \frac{1}{1 + \frac{1}{5}} $$

Simplify the denominator:

$$ 1 + \frac{1}{5} = \frac{5}{5} + \frac{1}{5} = \frac{6}{5} $$

Therefore:

$$ S = \frac{1}{\frac{6}{5}} = \frac{5}{6} $$

Final answer:

$$
\boxed{
\text{Sum of the series} = \frac{5}{6}
}
$$

This means the infinite sum of the series 1 - 1/5 + 1/25 - 1/125 + ... converges to $ \frac{5}{6} $.

Step-by-Step Breakdown of the Evaluation Process

Evaluating an infinite geometric series involves several systematic steps:

Step 1: Identify the first term a

- Look at the initial term of the series.
- For our series, $a = 1$.

Step 2: Determine the common ratio r

- Divide the second term by the first:

$$r = \frac{\text{second term}}{\text{first term}}$$

- Confirm that this ratio applies to subsequent terms to ensure the series is geometric.

Step 3: Check for convergence

- Confirm that $|r| < 1$. If true, the series converges; otherwise, it diverges.

Step 4: Apply the sum formula

- Use:

$$S = \frac{a}{1 - r}$$

- Plug in the values of a and r.

Step 5: Simplify to get the sum

- Simplify the resulting expression to obtain the sum in lowest terms.

Additional Examples of Infinite Geometric Series

To deepen understanding, consider the following examples:

- **Series 1:** $3 + 1.5 + 0.75 + \dots$
  - First term: $a = 3$
  - Common ratio: $r = 0.5$
  - Sum: $S = \frac{3}{1 - 0.5} = \frac{3}{0.5} = 6$

- **Series 2:** $2 - 1 + \frac{1}{2} - \frac{1}{4} + \dots$

- First term: $a = 2$
- Common ratio: $r = -\frac{1}{2}$
- Sum: $S = \frac{2}{1 - (-\frac{1}{2})} = \frac{2}{1 + \frac{1}{2}} = \frac{2}{\frac{3}{2}} = \frac{2 \times 2}{3} = \frac{4}{3}$

These examples illustrate how to apply the process in different contexts.

Applications of Infinite Geometric Series

Infinite geometric series are not just theoretical constructs but have practical applications across various fields:

1. Financial Mathematics

- Calculating the present value of perpetuities, where payments continue indefinitely at a constant rate.

2. Computer Science

- Analyzing algorithms with recursive structures or exponential decay.

3. Physics and Engineering

- Modeling phenomena like radioactive decay, capacitor discharge, or wave attenuation.

4. Signal Processing

- Understanding filters and transfer functions.

Example: Calculating the total accumulated interest on a perpetuity or the sum of an infinite series of decreasing signals.

Common Mistakes to Avoid When Evaluating Infinite Geometric Series

To ensure accurate evaluation, be mindful of the following pitfalls:

- **Incorrect ratio calculation:** Always verify the common ratio by dividing successive terms and confirm consistency.
- **Ignoring convergence criteria:** Remember, the series converges only if \(|r| < 1 \). If \(|r| \geq 1 \), the series diverges.
- **Misapplication of the formula:** The formula \(S = \frac{a}{1 - r} \) applies only when the series converges. Do not use it otherwise.
- **Simplification errors:** Carefully perform algebraic manipulations to avoid errors in the final sum.

Conclusion

Evaluating the infinite geometric series 1 - 1/5 + 1/25 - 1/125 + ... reveals not only the power of geometric series formulas but also highlights important concepts such as convergence and ratio analysis. By systematically identifying the first term and common ratio, verifying convergence criteria, and applying the sum formula, one can confidently determine the sum of such series.

The series in question converges to \(\frac{5}{6} \), showcasing how an infinite sum—initially seemingly unbounded—can be precisely calculated. Mastery of these techniques is essential for advanced mathematics, financial modeling, engineering, and numerous scientific applications.

Remember: Always verify the common ratio and convergence before applying the sum formula. With practice, evaluating infinite geometric series becomes an intuitive and valuable skill in your mathematical toolkit.

Frequently Asked Questions

What is the common ratio of the infinite geometric series 1 - 1/5 + 1/25 - 1/125 + ...?

The common ratio is -1/5, since each term is multiplied by -1/5 to get the next term.

How do you determine if an infinite geometric series converges?

An infinite geometric series converges if the absolute value of the common ratio is less than 1, i.e., |r| < 1.

What is the sum of the infinite geometric series 1 - 1/5 + 1/25 - 1/125 + ...?

Using the formula for the sum of an infinite geometric series, S = a / (1 - r), the sum is 1 / (1 - (-1/5)) = 1 / (1 + 1/5) = 1 / (6/5) = 5/6.

Why does the series 1 - 1/5 + 1/25 - 1/125 + ... converge to 5/6?

Because the common ratio is within the convergence interval (|r| < 1), and applying the sum formula yields 5/6, which confirms convergence.

Can the sum of this geometric series be approximated by adding a finite number of terms?

Yes, summing a finite number of terms approximates the series, and as more terms are added, the sum gets closer to the infinite sum of 5/6.

Additional Resources

Evaluating Each Infinite Geometric Series: 1 - 1/5 + 1/25 - 1/125 + ...

Understanding how to evaluate infinite geometric series is fundamental in mathematics, especially in calculus, algebra, and various applied sciences. The series 1 - 1/5 + 1/25 - 1/125 + ... offers a classic example of an alternating geometric series, which provides insight into convergence, sum calculation, and the underlying principles guiding such series.

Introduction to Infinite Geometric Series

An infinite geometric series is a sum of infinitely many terms where each term is obtained by multiplying the previous term by a constant ratio r. The general form of an infinite geometric series is:

$$
S = a + ar + ar^2 + ar^3 + \dots
$$

where:
- a is the first term,
- r is the common ratio.

Key Point: The series converges (approaches a finite sum) if and only if $|r| < 1$. If $|r| \geq 1$, the series diverges.

Analyzing the Given Series

The series to evaluate is:

$$
1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \dots
$$

Let's identify its components:

- First term $a = 1$
- Common ratio r can be found by dividing the second term by the first, or any subsequent term by its predecessor.

Calculating the ratio:

$$
r = \frac{-\frac{1}{5}}{1} = -\frac{1}{5}
$$

Check consistency:

$$
\frac{1}{25} \div -\frac{1}{5} = -\frac{1}{5}
$$

which confirms the ratio is consistent throughout.

This is an alternating geometric series because of the negative ratio, which causes the signs to alternate between positive and negative.

Convergence of the Series

Since:

$$
|r| = \frac{1}{5} < 1
$$

the series converges; therefore, we can compute its sum.

Implication: The terms decrease in magnitude, and the partial sums approach a finite limit as the number of terms increases.

Calculating the Sum of the Series

The sum S of an infinite geometric series with first term a and common ratio r, where $|r| < 1$, is given by:

$$
S = \frac{a}{1 - r}
$$

Applying this to our series:

$$
a = 1, \quad r = -\frac{1}{5}
$$

we get:

$$
S = \frac{1}{1 - (-\frac{1}{5})} = \frac{1}{1 + \frac{1}{5}} = \frac{1}{\frac{6}{5}} = \frac{5}{6}
$$

Result:

$$
\boxed{
\text{Sum} = \frac{5}{6}
}
$$

Thus, the infinite series converges to $\frac{5}{6}$.

Deeper Exploration: Why Does the Sum Converge?

The convergence of an infinite geometric series hinges on the behavior of the terms as the number of terms tends to infinity. For this series:

- The absolute value of the ratio $r = -\frac{1}{5}$ is less than 1.
- The terms alternate in sign but their magnitudes tend to zero.

Mathematically, the n-th term t_n is:

$$
t_n = a r^{n-1}
$$

As $n \to \infty$:

$$
|t_n| = |a| |r|^{n-1} \to 0
$$

because $|r| < 1$. This guarantees the partial sums approach a finite limit.

Implications and Applications of Infinite Geometric Series

Understanding how to evaluate such series is vital across multiple disciplines:

- Mathematics and Calculus: Series summation techniques underpin many integral and differential calculus concepts.
- Physics: Modeling decay processes, wave phenomena, and signal processing often involves infinite series.
- Finance: Calculating present values of perpetuities uses geometric series.
- Computer Science: Recursive algorithms and analysis of algorithms sometimes involve geometric sums.

Step-by-Step Methodology for Series Evaluation

To systematically evaluate an infinite geometric series like the one in question, follow these steps:

1. Identify the first term a:
For the series $1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \dots$,
$a = 1$.

2. Determine the common ratio r:
Divide the second term by the first:

$$
r = \frac{-\frac{1}{5}}{1} = -\frac{1}{5}
$$

3. Verify convergence:
Check if $|r| < 1$:

$$
|\,-\frac{1}{5}\,| = \frac{1}{5} < 1
$$

Since true, the series converges.

4. Apply the sum formula:

$$
S = \frac{a}{1 - r}
$$

Substitute:

$$
S = \frac{1}{1 - (-\frac{1}{5})} = \frac{1}{1 + \frac{1}{5}} = \frac{1}{\frac{6}{5}} = \frac{5}{6}
$$

5. Interpretation of result:

The entire infinite series sums to $\frac{5}{6}$. This means that adding all infinitely many terms results in a finite value, despite the series' infinite nature.

Alternative Perspectives and Mathematical Insights

While the sum formula provides a quick route to the answer, deeper insights can be gained through various approaches:

1. Partial Sums:

The partial sum S_n of the first n terms is:

$$
S_n = a \frac{1 - r^n}{1 - r}
$$

For our series:

$$
S_n = 1 \times \frac{1 - \left(-\frac{1}{5}\right)^n}{1 + \frac{1}{5}} = \frac{1 - \left(-\frac{1}{5}\right)^n}{\frac{6}{5}} = \frac{5}{6} \left[1 - \left(-\frac{1}{5}\right)^n \right]
$$

As $n \to \infty$:

$$
\left(-\frac{1}{5}\right)^n \to 0
$$

which confirms the sum approaches $\frac{5}{6}$.

2. Graphical Interpretation:

Plotting partial sums illustrates how the sum oscillates around the limit $\frac{5}{6}$, getting closer as n increases. The alternating signs cause the partial sums to "bounce" around the final value, with the amplitude decreasing geometrically.

3. Connection to Power Series:

The geometric series can be viewed as a power series:

$$
\sum_{n=0}^\infty r^n = \frac{1}{1 - r}
$$

for $|r| < 1$. Our series aligns with this concept, emphasizing the importance of the geometric series in function expansions and analysis.

Extensions and Variations

Understanding evaluation of this specific series opens doors to analyzing many similar series:

- Different ratios: For example, series like $2 + 2r + 2r^2 + \dots$
- Alternating series with different initial terms: Such as $a - br + cr^2 - dr^3 + \dots$
- Series with non-constant ratios: Leading into more complex convergence tests.

Common Mistakes and Pitfalls

When evaluating infinite series, students and practitioners often encounter errors:

- Ignoring the convergence condition: Applying the sum formula when $|r| \geq 1$ leads to incorrect conclusions.
- Misidentifying the ratio r: For alternating series, sign mistakes can occur.
- Assuming convergence without verification: Not checking $|r| < 1$ can result in divergence.

Summary and Final Thoughts

The series:

$$
1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \dots
$$

is a textbook example of an infinite geometric series with an alternating sign pattern. Its convergence is guaranteed by the ratio $r = -\frac{1}{5}$, which has an absolute value less than 1. Applying the geometric series sum formula yields a finite sum

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