Evaluate The Expression-4-4i /6i And Write The Result In The Form A + Bi.

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Understanding how to evaluate complex expressions, such as $\frac{-4 - 4i}{6i}$, is fundamental in complex number algebra. In this comprehensive guide, we'll explore the step-by-step process to simplify this expression and write the result in the standard form $A + Bi$. This article is designed to help students, math enthusiasts, and professionals grasp the concepts and techniques necessary for solving similar problems with confidence.

Introduction to Complex Numbers and Their Standard Form

What Are Complex Numbers?

Complex numbers are numbers that consist of a real part and an imaginary part, expressed in the form $A + Bi$, where:
- A is the real component.
- B is the coefficient of the imaginary component.
- i is the imaginary unit, defined as $\sqrt{-1}$.

Standard Form of a Complex Number

The standard form of a complex number is written as:
$$
A + Bi
$$
where:
- A and B are real numbers.
- i represents the imaginary unit.

Expressing complex numbers in this form allows for easier addition, subtraction, multiplication, and division.

Understanding the Given Expression

The expression we are evaluating is:

$$
\frac{-4 - 4i}{6i}
$$

Our goal is to simplify this expression into the form $A + Bi$.

Step-by-Step Solution Approach

Step 1: Recognize the division of complex numbers

Dividing a complex number by another often involves rationalizing the denominator, especially when it contains i.

Step 2: Rationalize the denominator

To eliminate the imaginary unit from the denominator, multiply numerator and denominator by the conjugate of the denominator. Since the denominator is $6i$, and i is purely imaginary, we can manipulate the expression accordingly.

Step 3: Simplify numerator and denominator

Perform algebraic operations to simplify the resulting expression.

Detailed Calculation Process

Step 1: Write the original expression

$$
\frac{-4 - 4i}{6i}
$$

Step 2: Multiply numerator and denominator by the conjugate of the denominator

Since the denominator is $6i$, and conjugates are used primarily for binomials, in this case, we can manipulate directly because the denominator is purely imaginary.

Alternatively, a more straightforward approach is to multiply numerator and denominator by i to rationalize.

Method: Multiply numerator and denominator by \(i\):
\[
\frac{-4 - 4i}{6i} \times \frac{i}{i} = \frac{(-4 - 4i) \times i}{6i \times i}
\]

Step 3: Simplify numerator

Calculate \((-4 - 4i) \times i\):

\[
-4 \times i = -4i
\]
\[
-4i \times i = -4i^2
\]

Recall that \(i^2 = -1\), so:

\[
-4i^2 = -4 \times (-1) = 4
\]

Adding these together:

\[
-4i + 4
\]

So, numerator simplifies to:

\[
4 - 4i
\]

Step 4: Simplify denominator

Calculate \(6i \times i\):

\[
6i \times i = 6i^2 = 6 \times (-1) = -6
\]

Step 5: Write the simplified expression

Now, the expression becomes:

\[
\frac{4 - 4i}{-6}

\]

Final Simplification and Writing in \(A + Bi\) Form

Step 6: Divide numerator by denominator

Divide each term in the numerator by \(-6\):

\[
A = \frac{4}{-6} = -\frac{2}{3}
\]
\[
B = \frac{-4i}{-6} = \frac{2i}{3}
\]

Expressed in standard form:

\[
-\frac{2}{3} + \frac{2}{3}i
\]

Summary of the Result

The simplified form of the original expression \(\frac{-4 - 4i}{6i}\) is:

\[
\boxed{
-\frac{2}{3} + \frac{2}{3}i
}
\]

where:
- \(A = -\frac{2}{3}\),
- \(B = \frac{2}{3}\).

Additional Tips for Simplifying Complex Expressions

- Always rationalize denominators when they contain imaginary units to simplify the expression.
- Use the property $i^2 = -1$ to simplify powers of i.
- Separate real and imaginary parts after simplification to write the result in $A + Bi$ form.
- Double-check calculations by substituting the simplified form back into the original expression to verify correctness.

Common Mistakes to Avoid

- Forgetting to multiply numerator and denominator by the conjugate or an appropriate expression.
- Mishandling the powers of i, especially $i^2 = -1$.
- Confusing the signs when distributing or simplifying.
- Not simplifying fractions properly after division.

Practical Applications of Complex Number Division

Understanding how to evaluate and simplify complex expressions like $\frac{-4 - 4i}{6i}$ has numerous applications, including:

- Signal processing and electrical engineering.
- Quantum physics calculations.
- Control systems analysis.
- Fractal and chaos theory visualizations.

Mastering these skills enhances problem-solving capabilities across various scientific and engineering disciplines.

Conclusion

Evaluating the complex expression $\frac{-4 - 4i}{6i}$ and expressing the result in the form $A + Bi$ involves understanding complex number properties, rationalizing denominators, and careful algebraic manipulation. The final answer, $-\frac{2}{3} + \frac{2}{3}i$, provides a clear and standardized representation of the complex division. Practice with similar problems will strengthen your proficiency in complex algebra and prepare you for more advanced mathematical concepts.

Meta Description:
Learn how to evaluate the complex expression \(\frac{-4 - 4i}{6i}\) and write the result in standard form \(A + Bi\). Step-by-step guidance, tips, and applications included.

Frequently Asked Questions

How do you evaluate the complex expression (4 - 4i) divided by 6i?

To evaluate (4 - 4i) / 6i, multiply numerator and denominator by the conjugate of the denominator or simplify directly by dividing numerator and denominator by i. The simplified result in the form A + Bi is 2/3 - 2/3i.

What is the step-by-step process to write (4 - 4i) / 6i in the form A + Bi?

First, rewrite the division as (4 - 4i) / 6i. Multiply numerator and denominator by i to eliminate the imaginary unit in the denominator: ((4 - 4i) i) / (6i i). Simplify numerator: 4i - $4i^2$ = 4i + 4 (since i^2 = -1). Denominator: 6i i = $6i^2$ = -6. So, the expression becomes (4i + 4)/(-6). Separate real and imaginary parts: (4 / -6) + (4i / -6) = -2/3 - 2/3i.

Why do we multiply numerator and denominator by i when simplifying (4 - 4i) / 6i?

Multiplying numerator and denominator by i helps to eliminate the imaginary unit from the denominator, making it easier to simplify the expression into the form A + Bi. Since i i = -1, this step simplifies the division process.

What is the final simplified form of (4 - 4i) / 6i in A + Bi format?

The final simplified form is -2/3 - 2/3i.

Can you verify the result of (4 - 4i) / 6i by plugging in the values?

Yes, by substituting the simplified form -2/3 - 2/3i back into the original expression or by performing the division step-by-step, you can confirm that the calculation is correct. The result matches the simplified form obtained through algebraic manipulation.

Additional Resources

Evaluate The Expression-4-4i / 6i and Write the Result in the Form A + Bi

When faced with complex number expressions such as -4 - 4i / 6i, understanding the principles of complex number division becomes essential. This type of problem is a common exercise in algebra and complex number theory, requiring a clear grasp of how to manipulate imaginary units and simplify expressions into the standard form A + Bi. The process involves rationalizing the denominator, simplifying the resulting expression, and expressing the final answer in a format that clearly distinguishes the real and imaginary parts. In this article, we will explore step-by-step how to evaluate the given expression, discuss key concepts involved, and provide a detailed explanation suitable for students and enthusiasts alike.

Understanding Complex Numbers and Their Forms

Before diving into the calculation, it's important to understand what complex numbers are and how they are expressed.

Definition and Standard Form

- A complex number is a number that can be written in the form A + Bi, where:
- A is the real part
- B is the imaginary part
- i is the imaginary unit, satisfying $i^2 = -1$

- For example, 3 + 4i has a real part of 3 and an imaginary part of 4.

Operations with Complex Numbers

- Addition and subtraction are straightforward: combine like terms.
- Multiplication involves distributing the terms and simplifying using $i^2 = -1$.
- Division requires rationalizing the denominator, often by multiplying numerator and denominator by the conjugate of the denominator.

Step-by-Step Evaluation of -4 - 4i / 6i

The expression is:

$$
\frac{-4 - 4i}{6i}
$$

Our goal is to simplify this expression and write it in the form A + Bi.

Step 1: Recognize the Structure

- The numerator is a complex number: -4 - 4i
- The denominator is purely imaginary: 6i

To simplify, we need to eliminate the imaginary unit from the denominator, which is achieved by multiplying numerator and denominator by the conjugate of the denominator.

Step 2: Multiply numerator and denominator by the conjugate of the denominator

- The conjugate of 6i is -6i because multiplying by the conjugate helps to remove the imaginary component from the denominator.
- Alternatively, since the denominator is purely imaginary, we can multiply numerator and denominator by -i to simplify.

Let's choose to multiply numerator and denominator by -i:

$$
\frac{-4 - 4i}{6i} \times \frac{-i}{-i} = \frac{(-4 - 4i) \times (-i)}{6i \times (-i)}
$$

Step 3: Simplify the denominator

$$
6i \times (-i) = 6 \times i \times (-i) = -6i^2
$$

Using $i^2 = -1$:

$$
-6 \times (-1) = 6
$$

So, the denominator simplifies to 6.

Step 4: Simplify the numerator

$$
(-4 - 4i) \times (-i) = (-4) \times (-i) + (-4i) \times (-i)
$$

Calculate each term:

- $-4 \times -i = 4i$

- $-4i \times -i = 4i^2$

Recall $i^2 = -1$:

$$
4i^2 = 4 \times (-1) = -4
$$

Adding these:

$$
4i - 4
$$

Thus, the numerator becomes $4i - 4$

Step 5: Write the simplified fraction

Putting numerator and denominator together:

$$
\frac{4i - 4}{6}
$$

Separate into real and imaginary parts:

$$
\frac{-4}{6} + \frac{4i}{6}
$$

Simplify fractions:

$$
-\frac{2}{3} + \frac{2i}{3}
$$

Final result:

$$
A + Bi = -\frac{2}{3} + \frac{2}{3}i
$$

Features and Insights

Evaluating complex expressions like this showcases several key features and steps that are essential for mastering complex number operations.

Features of the Process

- Rationalization Technique: Multiplying numerator and denominator by a conjugate or an appropriate complex number to eliminate imaginary units from the denominator.
- Simplification of Fractions: Breaking down complex fractions into real and imaginary parts for clarity.
- Use of $i^2 = -1$: Applying fundamental properties of the imaginary unit to simplify expressions involving powers of i.
- Expressing in Standard Form: Presenting the result in A + Bi for easy interpretation and further calculations.

Pros of this Approach

- Provides a systematic method for handling complex division.
- Ensures the denominator is real, simplifying interpretation.
- Reinforces the understanding of conjugates and their role in complex arithmetic.

Cons or Challenges

- Can be confusing for beginners unfamiliar with conjugate multiplication.
- Requires careful algebraic manipulation to avoid sign errors.
- In more complex expressions, the steps can become lengthy and prone to mistakes.

Additional Tips and Common Mistakes

- Always double-check the multiplication, especially signs when applying conjugates.
- Remember that multiplying numerator and denominator by the conjugate keeps the value unchanged.
- Simplify fractions at the end to their lowest terms.
- Be cautious with negative signs and multiplying through.

Conclusion

Evaluating the expression -4 - 4i / 6i demonstrates the importance of complex conjugates, algebraic manipulation, and a clear understanding of the properties of imaginary numbers. By rationalizing the denominator and carefully simplifying, we obtained the final answer:

\[
\boxed{-\frac{2}{3} + \frac{2}{3}i}

\]

Expressing complex numbers in the form A + Bi is fundamental for clarity in mathematical communication, especially when dealing with complex arithmetic. Mastery of these techniques enhances problem-solving skills and lays a solid foundation for more advanced topics in complex analysis, engineering, and physics. Whether you are a student or a professional, understanding how to evaluate such expressions accurately is a valuable skill that will serve you across various scientific and mathematical disciplines.

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