Find A Cartesian Equation For The Curve And Identify It. R = 10 Csc()

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Understanding the relationship between polar and Cartesian coordinates is fundamental in the study of conic sections and various curves in mathematics. The given polar equation, $R = 10 \csc(\theta)$, describes a specific curve whose Cartesian form and characteristics can be determined through systematic analysis. This article aims to guide you through the process of converting the polar equation into its Cartesian equivalent and identifying the nature of the curve.

Introduction to Polar and Cartesian Coordinates

Before delving into the specifics of the equation $R = 10 \csc(\theta)$, it is essential to understand the foundational concepts of polar and Cartesian coordinate systems.

Polar Coordinates

- In the polar coordinate system, a point in the plane is represented by (r, θ), where:
- r: the distance from the origin to the point.
- θ: the angle measured from the positive x-axis to the line segment connecting the origin to the point.
- The relationship between polar and Cartesian coordinates is given by:
- $x = r \cos(\theta)$
- $y = r \sin(\theta)$

Cartesian Coordinates

- In the Cartesian coordinate system, a point is represented by (x, y), which directly indicates its position relative to the x- and y-axes.
- Converting between polar and Cartesian coordinates involves the equations above.

Understanding the Given Equation: R = 10 Csc(θ)

The equation $R = 10 \csc(\theta)$ is a polar equation involving the cosecant function.

The Cosecant Function

- The cosecant function, $Csc(\theta)$, is the reciprocal of the sine function:

$Csc(\theta) = 1 / \sin(\theta)$

- It is undefined at $\theta = 0, \pi, 2\pi$, etc., where $\sin(\theta) = 0$.

Interpreting the Equation R = 10 Csc(θ)

- Replacing $Csc(\theta)$, the equation becomes:

$r = 10 / \sin(\theta)$

- Since $\sin(\theta) = y / r$, we can rewrite the equation as:

$r = 10 / (y / r)$

- Multiplying both sides by y / r:

$r y / r = 10$

Simplifies to:

$y = r$

- But this direct substitution requires a careful approach, so let's proceed systematically.

Converting the Polar Equation to Cartesian Coordinates

To convert $R = 10 Csc(\theta)$ into a Cartesian equation, follow these steps:

Step 1: Express R and Csc(θ) in terms of x and y

- Recall that:
- $r = \sqrt{(x^2 + y^2)}$
- $\sin(\theta) = y / r$
- Therefore:

$Csc(\theta) = 1 / \sin(\theta) = r / y$

Step 2: Substitute into the original equation

- The polar equation:

$r = 10 \, Csc(\theta)$

- becomes:

$r = 10 \, (r / y)$

Step 3: Simplify the equation

- Multiply both sides by y:

$r y = 10 r$

- As long as $r \neq 0$, divide both sides by r:

$y = 10$

This is a crucial step because it reveals the Cartesian form of the curve.

Resulting Cartesian Equation and Its Interpretation

From the above derivation, the Cartesian equation corresponding to $R = 10 \, Csc(\theta)$ is:

- **y = 10**

This is a straight horizontal line in the Cartesian plane, parallel to the x-axis, passing through $y = 10$.

Note on the Domain and Range

- Since $\sin(\theta)$ in the original equation cannot be zero (which would make $Csc(\theta)$ undefined), the original polar equation is undefined at $\theta = 0, \pi, 2\pi$, etc.
- In Cartesian coordinates, the line $y = 10$ extends infinitely in both directions along the x-axis, excluding any points where $y \neq 10$.

Identifying the Curve: What Is y = 10?

The Cartesian equation y = 10 represents a simple, infinite straight line located 10 units above the x-axis. Its characteristics include:

Properties of the Line y = 10

- Orientation: Horizontal line
- Position: Parallel to the x-axis
- Y-intercept: (0, 10)
- No curvature: It is a straight line with no bends or curves
- Unbounded: Extends infinitely in both the positive and negative x-directions

Graphical Representation

- When plotted on the Cartesian plane, this line appears as a straight, horizontal line crossing the y-axis at y = 10.
- It intersects the x-axis at all points where y = 10, regardless of the x-coordinate.

Additional Insights and Related Curves

While the equation y = 10 is straightforward, understanding its relation to polar equations and the significance of the original form can deepen your comprehension.

Relation to Other Conic Sections

- The line y = 10 is not a conic in itself but can be related to conic sections when considering other polar equations.
- For example, equations like $r = e d / (1 \pm e \cos(\theta))$ or $r = e d / (1 \pm e \sin(\theta))$ represent conics (ellipses, parabolas, hyperbolas), depending on eccentricity e.

Transformations and Symmetries

- The line y = 10 is symmetric with respect to the x-axis, which aligns with the symmetry properties of the polar equation involving cosecant.
- Understanding how transformations in polar and Cartesian coordinates reflect each other is key in advanced studies.

Applications of the Equation y = 10

Identifying that R = 10 Csc(θ) converts to y = 10 has practical implications in various fields:

- **Physics:** Modeling objects moving along a fixed height, such as a beam or a horizontal boundary in a physical system.
- **Engineering:** Designing components or systems where a specific height or level is maintained.
- **Mathematics Education:** Demonstrating the process of converting polar equations to Cartesian forms to facilitate easier graphing and analysis.

Conclusion

In conclusion, the polar equation R = 10 Csc(θ) simplifies to the Cartesian equation y = 10, representing a straight, horizontal line located 10 units above the x-axis. This transformation exemplifies how polar functions involving trigonometric reciprocals can correspond to simple geometric figures in the Cartesian plane. Recognizing these relationships enhances understanding of coordinate systems and the behavior of various curves. Whether you're analyzing conic sections, designing engineering systems, or studying mathematical properties, mastering the conversion process is invaluable.

Summary of Key Steps

1. Recall the definitions of polar and Cartesian coordinates.
2. Express Csc(θ) as r / y.
3. Substitute into the original polar equation.
4. Simplify to find the Cartesian form.
5. Interpret the resulting equation geometrically.

By following these steps, you can confidently convert a wide variety of polar equations into their Cartesian counterparts and identify the curves they represent.

Frequently Asked Questions

What is the Cartesian equation corresponding to the polar equation R = 10 Csc(θ)?

The Cartesian equation is $y = 10$.

How do you convert the polar equation R = 10 Csc(θ) into Cartesian coordinates?

Since $\csc(\theta) = 1 / \sin(\theta)$ and $R = \sqrt{x^2 + y^2}$, with $y = R \sin(\theta)$, substituting gives $y = 10$, which is a horizontal line.

What type of curve is represented by the equation y = 10?

It is a straight horizontal line located 10 units above the x-axis.

Is the given polar equation R = 10 Csc(θ) a circle, line, or other curve?

It represents a straight line, specifically $y = 10$, in the Cartesian plane.

What is the significance of the Csc(θ) function in the polar equation R = 10 Csc(θ)?

Csc(θ) relates the radius R to the y-coordinate, leading to the Cartesian equation $y = 10$.

Can the curve R = 10 Csc(θ) be expressed in terms of x and y only?

Yes, it simplifies to $y = 10$, involving only x and y coordinates.

What is the geometric interpretation of R = 10 Csc(θ)?

It describes a horizontal line at $y = 10$ in the Cartesian plane.

How does the value '10' in R = 10 Csc(θ) influence the position of the

curve?

It determines the distance from the x-axis where the line y = 10 is located.

Additional Resources

Understanding and Deriving the Cartesian Equation for the Polar Curve R = 10 Csc(θ)

Introduction

In the study of polar coordinates and their relationship to Cartesian coordinates, one of the fundamental tasks is transforming a given polar equation into its Cartesian counterpart and subsequently identifying the geometric nature of the curve. The equation in question, R = 10 Csc(θ), presents an interesting case that involves understanding reciprocal trigonometric functions and their geometric implications. This exploration aims to derive the Cartesian form of the curve, analyze its properties, and classify the type of curve it represents.

Fundamentals of Polar and Cartesian Coordinates

Before delving into the specific equation, it's essential to revisit the basics of the coordinate systems:

- Polar Coordinates: A point in the plane is represented as (r, θ) where:
- r: the distance from the origin to the point.
- θ: the angle measured counterclockwise from the positive x-axis.

- Cartesian Coordinates: A point is represented as (x, y), where:
- x: the horizontal coordinate.
- y: the vertical coordinate.

The relationship between these systems is given by:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$

and conversely:

$$
r = \sqrt{x^2 + y^2}, \quad \theta = \arctan\left(\frac{y}{x}\right)
$$

Analyzing the Given Equation: R = 10 Csc(θ)

The polar equation:

$$
r = 10 \csc \theta
$$

involves the cosecant function, which is the reciprocal of sine:

$$
\csc \theta = \frac{1}{\sin \theta}
$$

This implies:

$$
r = \frac{10}{\sin \theta}
$$

The goal is to express this relationship directly in Cartesian coordinates.

Step-by-Step Derivation of the Cartesian Equation

Step 1: Express $\sin \theta$ in terms of x and y

Recall:

$$
\sin \theta = \frac{y}{r}
$$

Substituting into the original equation:

$$
r = \frac{10}{\frac{y}{r}} = \frac{10 r}{y}
$$

Rearranging:

$$
r y = 10 r
$$

Assuming $r \neq 0$ (which is valid except at the origin), divide both sides by r:

$$
y = 10

$$
$$

Step 2: Interpret the resulting Cartesian equation

The simplified Cartesian form:

$$
\boxed{ y = 10 }
$$

indicates the curve is a horizontal line located at the vertical position $y = 10$.

Geometric Interpretation of the Curve $y = 10$

The derived Cartesian equation corresponds to a straight horizontal line parallel to the x-axis, crossing the y-axis at $y = 10$. This is a fundamental and simple geometric shape, but understanding its origin from the polar equation provides deeper insight into how polar equations map onto Cartesian geometry.

Confirming the Nature of the Curve: Does the Equation Cover All θ?

Given the polar equation:

$$
r = 10 \csc \theta
$$

let's analyze the domain and behavior:

- Domain of θ:
- Since $\csc \theta$ is undefined at $\theta = 0, \pi, 2\pi, \ldots$ where $\sin \theta = 0$, the equation is valid only where $\sin \theta \neq 0$.
- For all other θ, r is well-defined and positive if $\sin \theta > 0$, i.e., when $0 < \theta < \pi$.
- When $\sin \theta < 0$, r becomes negative, which in polar coordinates indicates the point is in the opposite direction (adding π to θ).

- Behavior of r:
- As $\theta \to 0^+$, $\sin \theta \to 0^+$, so $r \to +\infty$.
- As $\theta \to \pi^-$, $\sin \theta \to 0^+$, again $r \to +\infty$.
- When $\theta = \frac{\pi}{2}$, $\sin \theta = 1$, so:

$$
r = 10
$$

- For θ between 0 and π, r is positive, and the points lie along the line $y = 10$, confirming the earlier conclusion.

Critical Points and Behavior at Special Angles

- At $\theta = \frac{\pi}{2}$:
$$
r = 10 \csc \left(\frac{\pi}{2}\right) = 10 \times 1 = 10
$$
- The point is at $(x, y) = (r \cos \theta, r \sin \theta)$:
$$
x = 10 \times \cos \left(\frac{\pi}{2}\right) = 0, \quad y = 10 \times 1 = 10
$$
- This confirms the point lies directly on the line $y=10$ at the origin's vertical height.

- At $\theta \to 0^+$:
$$
r \to +\infty
$$
- The points extend infinitely far along the line $y=10$, approaching the x-axis asymptotically.

- At $\theta \to \pi^-$:
$$
r \to +\infty
$$
- The points again extend infinitely far along the same line.

Summary of the Geometric Nature

The polar equation $r = 10 \csc \theta$ describes a line $y=10$. The line extends infinitely in both directions, but from a polar perspective, the equation only defines points with positive r for $\theta \in (0, \pi)$. For negative r, the points are reflected across the origin, but since the line extends infinitely and is symmetric with respect to the origin, the entire line $y=10$ is represented.

Additional Insights and Generalizations

1. Connections to Other Polar Equations

- Equations of the form $r = a \csc \theta$ or $r = a \sec \theta$ typically represent lines in Cartesian coordinates:
- $r = a \csc \theta$ corresponds to the line $y = a$.
- $r = a \sec \theta$ corresponds to the line $x = a$.

2. Why Does $R = 10 \csc \theta$ Map to $y = 10$?

- The key is recognizing that $y = r \sin \theta$:

$$
y = r \sin \theta = (10 \csc \theta) \times \sin \theta = 10
$$

- This direct substitution confirms the line's position in the Cartesian plane.

3. Implications for Graphing

- When graphing polar equations of the form $r = a \csc \theta$, the graph will always be a straight line parallel to the x-axis at $y = a$.
- The asymptotic behavior at $\theta = 0, \pi$ indicates the points go off to infinity, reflecting the fact that the line is extended infinitely in the plane.

Summary of Key Points

- The polar equation $R = 10 \csc \theta$ simplifies to a Cartesian line $y = 10$.
- The derivation hinges on the fundamental relationships between polar and Cartesian coordinates, specifically $y = r \sin \theta$.
- The curve is a horizontal straight line at $y = 10$, extending infinitely in both the positive and negative x-directions.
- The domain of θ excludes points where $\sin \theta = 0$, i.e., $\theta = 0, \pi, 2\pi, \ldots$, where the polar equation is undefined due to division by zero.
- The graphical behavior reflects the asymptotic nature at the angles where $\sin \theta \to 0$.

Final Remarks: Broader Context and Applications

Understanding how polar equations translate into Cartesian equations is vital in various fields such as physics, engineering, and computer graphics. Specifically, recognizing that equations involving reciprocal trigonometric functions often correspond to lines or simple geometric shapes can simplify complex analyses.

In this particular case:

- The equation \(R = 10 \csc \theta \) exemplifies how a straightforward reciprocal trigonometric function can encode a fundamental geometric object—a line.
- Such insights allow for

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