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Introduction to Linear Functions

Understanding the concept of linear functions is fundamental in algebra and various applied mathematics fields. A linear function, H(x), is a function that graphs as a straight line on the coordinate plane. It is characterized by its constant rate of change, known as the slope, and its y-intercept, which is the point where the line crosses the y-axis.

The general form of a linear function is:

$$
H(x) = mx + b
$$

where:
- m is the slope of the line,
- b is the y-intercept.

Given specific points on this line, such as $H(7) = -27$ and $H(-4) = 28$, we can determine the exact formula for H(x). This process involves calculating the slope and then determining the y-intercept.

Understanding the Problem Statement

The problem provides two key points on the linear function H(x):

- When $x = 7$, $H(7) = -27$,
- When $x = -4$, $H(-4) = 28$.

Our goal:
- Find the specific linear function H(x) that passes through these points,
- Use this function to calculate $H(5)$.

This process involves three main steps:
1. Calculating the slope m,
2. Finding the y-intercept b,
3. Using the derived linear function to find $H(5)$.

Calculating the Slope (m)

The slope m measures how steep the line is. It can be calculated using the two points provided:

$$
m = \frac{\text{change in } y}{\text{change in } x} = \frac{H(x_2) - H(x_1)}{x_2 - x_1}
$$

Where:

- $(x_1, H(x_1)) = (7, -27)$,
- $(x_2, H(x_2)) = (-4, 28)$.

Calculating:

$$
m = \frac{28 - (-27)}{-4 - 7} = \frac{28 + 27}{-4 - 7} = \frac{55}{-11} = -5
$$

Result: The slope of the line is $m = -5$.

Finding the Y-Intercept (b)

Once the slope m is known, the next step is to find the y-intercept b.

Using the general linear form:

$$
H(x) = mx + b
$$

We can substitute one of the known points into this equation to solve for b.

Let's use $(7, -27)$:

$$
-27 = (-5) \times 7 + b

\]

\[
-27 = -35 + b
\]

\[
b = -27 + 35 = 8
\]

Result: The y-intercept is \(b = 8 \).

Constructing the Linear Function H(x)

Now that we have both \(m \) and \(b \), we can write the explicit formula for H(x):

\[
H(x) = -5x + 8
\]

This function models the line passing through the points \((7, -27) \) and \((-4, 28) \).

Calculating H(5)

With the linear function established, we can now find the value of \(H(5) \):

$$
H(5) = -5 \times 5 + 8 = -25 + 8 = -17
$$

Result: $H(5) = -17$.

Summary of the Process

To recap:

- Calculated the slope $m = -5$,
- Determined the y-intercept $b = 8$,
- Derived the linear function $H(x) = -5x + 8$,
- Computed $H(5) = -17$.

This process showcases how to determine a linear function from two points and use it for further calculations.

Additional Insights and Applications

Understanding how to find a linear function from points is fundamental in many real-world applications, such as:

- Economics: Modeling cost versus production,
- Physics: Describing constant velocity motion,

- Biology: Growth rates analysis,
- Data Analysis: Fitting straight-line models to data points.

Common Methods for Finding Linear Functions

- Using Two Points: As demonstrated, calculating slope and intercept from two known points.
- Using Point-Slope Form: $y - y_1 = m(x - x_1)$,
- Using Slope-Intercept Form: Directly plugging in known values once slope is known.

Importance of Accuracy

Always verify calculations by substituting the points back into the derived function:

$$
H(7) = -5 \times 7 + 8 = -35 + 8 = -27 \quad \checkmark
$$

$$
H(-4) = -5 \times (-4) + 8 = 20 + 8 = 28 \quad \checkmark
$$

Since both points satisfy the function, the derivation is correct.

Conclusion

Understanding how to find a linear function from given points is a key algebraic skill. In this case, starting with the points $(7, -27)$ and $(-4, 28)$, we deduced the slope and y-intercept, resulting in the function $H(x) = -5x + 8$. Using this function, we determined that $H(5) = -17$. This process highlights the power of algebra in modeling real-world data and solving mathematical problems efficiently.

FAQs

1. **Q:** What is a linear function?
2. **A:** A linear function is a function that graphs as a straight line, typically expressed as $y = mx + b$, where m is the slope and b is the y-intercept.
3. **Q:** How do I find the slope between two points?
4. **A:** Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, substituting the known points.
5. **Q:** How can I verify my linear function?
6. **A:** Substitute the known points into the derived function to see if they satisfy the equation.

7. **Q:** Why is understanding linear functions important?

8. **A:** They are foundational in understanding relationships between variables in science, economics, engineering, and many other fields.

By mastering the steps outlined above, students and professionals can confidently determine linear functions from data points and apply this knowledge to solve practical problems efficiently.

Frequently Asked Questions

How do you find the linear function H(x) given H(7) = -27 and H(-4) = 28?

First, find the slope m = (H(7) - H(-4)) / (7 - (-4)) = (-27 - 28) / (7 + 4) = -55 / 11 = -5. Then, use point-slope form with one point, for example H(7) = -27: H(x) = -5(x - 7) + (-27) = -5x + 35 - 27 = -5x + 8.

What is the linear function H(x) given H(7) = -27 and H(-4) = 28?

The linear function is H(x) = -5x + 8.

How do you calculate H(5) once you have the linear function H(x) = -5x + 8?

Substitute x = 5 into the function: H(5) = -5(5) + 8 = -25 + 8 = -17.

What is the value of H(5) for the given linear function?

H(5) = -17.

Why is the slope of the linear function H(x) = -5x + 8?

Because the slope m is calculated as -5 based on the change in H over the change in x between the two points.

Can you verify the linear function H(x) = -5x + 8 using the given points?

Yes, plugging in x=7: H(7) = -5(7) + 8 = -35 + 8 = -27, and x=-4: H(-4) = -5(-4) + 8 = 20 + 8 = 28. Both points satisfy the function.

What is the importance of using the two points to find the linear function?

Using two points allows calculation of the slope and determination of the precise linear function passing through both points.

Is the function H(x) = -5x + 8 linear?

Yes, it is a linear function because it is in the form y = mx + b.

How does the slope affect the shape of the linear function H(x)?

The slope of -5 indicates the function decreases by 5 units for each increase of 1 in x, reflecting a downward sloping line.

Additional Resources

Understanding the Problem: Finding a Linear Function H Given Two Points and Evaluating H(5)

When encountering a problem involving a linear function H, given certain points, the core objective is to determine the explicit form of the function and then evaluate it at a specific input. This task involves understanding the fundamental properties of linear functions, applying algebraic methods to find the slope and intercept, and finally computing the desired value. Let's explore this process in depth, breaking down each step and discussing relevant concepts thoroughly.

Foundations of Linear Functions

Before tackling the problem directly, it's essential to review what constitutes a linear function and the key properties that define it.

Definition of a Linear Function

A linear function H is a function of the form:

$$H(x) = mx + b$$

where:
- m is the slope of the line, indicating the rate of change of the output relative to the input.
- b is the y-intercept, the value of $H(x)$ when $x = 0$.

This form is called the slope-intercept form and is straightforward for analyzing and graphing linear functions.

Key Properties of Linear Functions

- The graph of a linear function is a straight line.
- The slope \(m \) is constant across the entire domain.
- Knowing two points on the line allows us to determine both \(m \) and \(b \).

Given Data and What We Need to Find

The problem states:

- \(H(7) = -27 \)
- \(H(-4) = 28 \)

Our goals are:

1. Find the explicit form of the linear function \(H(x) \).
2. Use this form to compute \(H(5) \).

The key here is that two points are given, which enables us to determine the slope \(m \) and then find \(b \).

Step 1: Find the Slope \(m \)

The slope of a line passing through two points \((x_1, y_1) \) and \((x_2, y_2) \) is given by:

\[

m = \frac{y_2 - y_1}{x_2 - x_1}

$$

Applying the given points:

- $(x_1, y_1) = (7, -27)$
- $(x_2, y_2) = (-4, 28)$

Calculating m:

$$
m = \frac{28 - (-27)}{-4 - 7} = \frac{28 + 27}{-4 - 7} = \frac{55}{-11} = -5
$$

Result: The slope $m = -5$.

This indicates that for every increase of 1 in x, the value of $H(x)$ decreases by 5.

Step 2: Find the y-intercept b

Having the slope $m = -5$, we now determine b by substituting one of the known points into the equation $H(x) = mx + b$.

Choose $(7, -27)$:

$$
-27 = (-5)(7) + b
$$

Calculate:

$$
-27 = -35 + b
$$

Solve for b:

$$
b = -27 + 35 = 8
$$

Result: The y-intercept $b = 8$.

Step 3: Write the explicit form of $H(x)$

Now that we have both the slope and the intercept, the linear function is:

$$
H(x) = -5x + 8
$$

This is the explicit formula for H.

Step 4: Verify the function with the second point

It's good practice to verify that the formula satisfies the second point $(-4, 28)$.

Calculate:

$$
H(-4) = -5(-4) + 8 = 20 + 8 = 28
$$

Which matches the given data, confirming the correctness of our function.

Step 5: Find $H(5)$

Now, to compute $H(5)$:

$$
H(5) = -5(5) + 8 = -25 + 8 = -17
$$

Final answer: $H(5) = -17$.

Deeper Insights and Broader Context

While the steps above provide a straightforward solution, it's valuable to explore broader concepts related to linear functions, their properties, and potential pitfalls.

Alternative Methods for Finding the Function

- Using point-slope form: Instead of directly solving for b, you could write the equation as:

$$
H(x) - y_1 = m(x - x_1)
$$

and then substitute the known point and slope to find the equation.

- Graphical approach: Plotting the two points and drawing the line can provide a visual confirmation of the slope and intercept, especially when graphing tools are available.

Implications of the Slope and Intercept

- The negative slope indicates a decreasing function—a typical scenario in real-world contexts like depreciation, temperature decrease over time, or inverse relationships.
- The intercept $b = 8$ tells us that when $x=0$, $H(x) = 8$. This could represent an initial value or baseline in applied contexts.

Potential Pitfalls and Common Mistakes

- Using incorrect point order: When calculating the slope, ensure you subtract y-values in the same order as the x-values.
- Miscalculating signs: Pay attention to signs, especially when subtracting negative numbers.
- Assuming linearity without confirmation: Always verify that two points indeed define a linear function, especially if additional points are involved.

Extensions and Related Topics

- Finding the equation of a line with more than two points: For more points, confirm they are collinear before concluding the line's equation.
- Linear regression: When dealing with data points that are approximations, statistical methods are used to find a best-fit linear function.
- Piecewise linear functions: Sometimes functions are linear only within certain domains; understanding the context helps determine the appropriate model.

Conclusion: Summarizing the Process and Insights

In this comprehensive exploration, we've:

- Recognized the importance of understanding the structure of linear functions.
- Used the two given points to calculate the slope m confidently.
- Derived the y-intercept b by substituting a known point into the slope-intercept form.
- Validated the function with the second point.
- Finally, evaluated the function at $x=5$ to find $H(5) = -17$.

This structured approach exemplifies how algebraic techniques can be systematically applied to solve real-world problems involving linear functions. Recognizing the significance of each step not only yields the correct answer but deepens understanding of the mathematical principles involved. Whether in academic settings or practical applications, mastering these methods is essential for analyzing relationships modeled by lines.

In essence, the linear function H is:

$$
\boxed{H(x) = -5x + 8}
$$

and the value at $x=5$ is:

$$
\boxed{H(5) = -17}
$$

This problem encapsulates core algebra skills and highlights the importance of methodical problem-solving approaches in mathematics.

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