Find All Real Values Of X That Satisfy The Equation $(x^2 - 82)^2 = 324$

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Understanding how to solve algebraic equations involving squares is fundamental in algebra. The equation $(x^2 - 82)^2 = 324$ presents a quadratic expression inside a squared term, equated to a constant. To find all real solutions for x, we need to methodically analyze and manipulate the equation, applying algebraic principles such as square roots, quadratic solutions, and factoring. This problem is a classic example of how to approach equations involving quadratic and squared expressions, which are common in algebraic problem-solving and serve as foundational skills for more complex mathematical topics.

In this comprehensive guide, we will walk through the process of solving the equation step by step, providing clear explanations and tips along the way. We will also explore related concepts such as the properties of squares, solving quadratic equations, and verifying solutions to ensure they satisfy the original equation. Whether you're a student preparing for exams or someone interested in sharpening your algebra skills, this article will equip you with the tools necessary to tackle similar problems confidently.

Understanding the Equation $(x^2 - 82)^2 = 324$

Before diving into solving the equation, it's essential to understand its structure and the mathematical principles involved.

Breaking Down the Equation

The given equation is:

$$
(x^2 - 82)^2 = 324
$$

This involves a squared binomial on the left side, which equals a constant (324). Recognizing that both sides involve squares suggests that taking the square root of both sides will be a key step in solving.

Key Concepts Involved

- Square Roots: Since both sides are squares, the primary approach involves taking square roots, remembering that this introduces both positive and negative solutions.
- Quadratic Expressions: The term x^2 indicates a quadratic nature, which may lead to quadratic equations after substitution or rearrangement.
- Equality and Sign Considerations: When dealing with squares, remember that:

$$
A^2 = B^2 \implies A = \pm B
$$

- Domain of Real Numbers: Because we're solving for real values, any solutions that involve taking the square root must be checked to ensure they satisfy the original equation.

Step-by-Step Solution Process

Now, let's proceed through the systematic steps to find all real values of x that satisfy the equation.

Step 1: Take the Square Root of Both Sides

Starting with:

$$
(x^2 - 82)^2 = 324
$$

Applying the square root to both sides gives:

$$
x^2 - 82 = \pm \sqrt{324}
$$

Since $\sqrt{324} = 18$, we have:

$$
x^2 - 82 = \pm 18
$$

This results in two separate cases:

1. $x^2 - 82 = 18$
2. $x^2 - 82 = -18$

Step 2: Solve Each Case for x^2

Case 1:

$$
x^2 - 82 = 18 \implies x^2 = 18 + 82 = 100
$$

Case 2:

$$
x^2 - 82 = -18 \implies x^2 = -18 + 82 = 64
$$

Now, we have two quadratic equations:

- $x^2 = 100$
- $x^2 = 64$

Step 3: Find the Values of x

Recall that:

$$
x^2 = a \implies x = \pm \sqrt{a}
$$

Applying this to each:

- For $x^2 = 100$:

$$
x = \pm \sqrt{100} = \pm 10
$$

- For $x^2 = 64$:

$$
x = \pm \sqrt{64} = \pm 8
$$

Hence, the potential solutions are:

$$
x = 10, -10, 8, -8
$$

Verifying the Solutions

While the above steps give potential solutions, it's crucial to verify each in the original equation to ensure they satisfy it, especially since taking square roots can introduce extraneous solutions.

Step 4: Plug Each Value Back into the Original Equation

Recall the original equation:

$$
(x^2 - 82)^2 = 324
$$

Let's verify each solution:

For $x = 10$:

$$
(10^2 - 82)^2 = (100 - 82)^2 = (18)^2 = 324 \quad \checkmark
$$

For $x = -10$:

$$
((-10)^2 - 82)^2 = (100 - 82)^2 = 18^2 = 324 \quad \checkmark
$$

For $x = 8$:

$$
(8^2 - 82)^2 = (64 - 82)^2 = (-18)^2 = 324 \quad \checkmark
$$

For $x = -8$:

$$
((-8)^2 - 82)^2 = (64 - 82)^2 = (-18)^2 = 324 \quad \checkmark
$$

All four solutions satisfy the original equation, confirming they are valid.

Summary of Solutions

The solutions to the equation $(x^2 - 82)^2 = 324$ are:

- $x = 10$
- $x = -10$
- $x = 8$
- $x = -8$

These are all real solutions, and the process demonstrates how algebraic manipulation, square root properties, and verification work together to solve quadratic equations involving squares.

Additional Tips for Solving Similar Equations

When approaching equations like $(x^2 - c)^2 = k$, consider the following strategies:

- Always take the square root carefully: Remember to include both positive and negative roots.
- Simplify step-by-step: Break down the equation into manageable parts.
- Check solutions: Substitute solutions back into the original to verify they are valid.
- Be aware of extraneous solutions: Especially when squaring both sides, some solutions may not satisfy the original equation.
- Use substitution if helpful: For complex equations, substituting $y = x^2$ can simplify solving.

Related Concepts and Practice Problems

To deepen understanding, explore these related topics and practice problems:

Related Concepts:

- Solving quadratic equations
- Properties of square roots
- Discriminant and quadratic formula
- Solving equations with absolute values

Practice Problems:

1. Solve for x: $(x^2 + 3)^2 = 100$
2. Find all real solutions to $(2x^2 - 5)^2 = 49$

3. Determine the solutions of $(x^2 - 4x + 3)^2 = 16$
4. Solve for x: $(x^2 + 1)^2 = 0$
5. Find all real x satisfying $(x^2 - 10)^2 = 0$

Working through these problems will reinforce the techniques discussed and improve your algebraic problem-solving skills.

Conclusion

Solving equations like $(x^2 - 82)^2 = 324$ involves recognizing the properties of squares, applying square roots carefully, and verifying solutions. The key steps include isolating the squared term, considering both positive and negative roots, solving resulting quadratic equations, and validating solutions within the original equation. Mastering these steps not only solves the problem at hand but also builds a solid foundation for tackling more complex algebraic equations. Whether you're preparing for tests or simply seeking to strengthen your math skills, understanding the process outlined here ensures confidence in solving similar equations in the future.

Frequently Asked Questions

What are the steps to solve the equation $(x^2 - 82)^2 = 324$ for all real values of x?

First, take the square root of both sides to get $|x^2 - 82| = 18$. Then, set up two equations: $x^2 - 82 = 18$ and $x^2 - 82 = -18$. Solve each quadratic separately to find all real solutions for x.

How do you solve the equation $(x^2 - 82)^2 = 324$ for real numbers?

Rewrite as $|x^2 - 82| = 18$, then consider two cases: $x^2 - 82 = 18$ and $x^2 - 82 = -18$. Solve each for x by isolating x^2 and taking square roots.

What are the solutions to $x^2 - 82 = 18$?

Add 82 to both sides: $x^2 = 100$. Taking square roots gives $x = \pm 10$.

What are the solutions to $x^2 - 82 = -18$?

Add 82 to both sides: $x^2 = 64$. Taking square roots gives $x = \pm 8$.

What is the complete set of real solutions for (x^2 - 82)^2 = 324?

The solutions are x = -10, 10, -8, and 8.

Are there any extraneous solutions when solving (x^2 - 82)^2 = 324?

No, all solutions obtained from solving the quadratic equations are valid because they satisfy the original equation.

Can the solutions x = ±10 and x = ±8 be verified directly in the original equation?

Yes, substituting each back into (x^2 - 82)^2 confirms they satisfy the equation, as they yield the value 324.

What is the graph of the function y = (x^2 - 82)^2, and how does it relate to the solutions?

The graph is a parabola opening upwards with its minimum at x=0, y= (0 - 82)^2 = 6724. The solutions x = ±8 and ±10 correspond to the points where the graph intersects y=324.

Is the equation (x^2 - 82)^2 = 324 solvable over complex numbers?

Yes, but the question specifically asks for real solutions. Over complex numbers, solutions would include complex roots, but in this context, only the real solutions x = ±8 and x = ±10 are relevant.

How can understanding this equation help in solving higher-degree polynomial equations?

This equation illustrates the method of taking square roots and solving quadratic equations, which are fundamental techniques applicable in solving more complex polynomial equations by substitution and factorization.

Additional Resources

Finding All Real Values of x That Satisfy the Equation \((x^2 - 82)^2 = 324\)

When approaching algebraic equations involving squares and quadratic expressions, the key is to understand the properties of squares and how they influence the solution set. The equation \((x^2 - 82)^2 = 324\) is a perfect example of a quadratic in disguise, and solving it requires careful manipulation and attention to detail. In this in-depth exploration, we will examine every aspect of this equation, from initial interpretation to detailed solution steps, ensuring clarity and thorough understanding.

Understanding the Equation

Before jumping into the solution process, it's essential to understand the structure and nature of the given equation:

$$
(x^2 - 82)^2 = 324
$$

Key observations include:

- The left side is a square of a binomial expression involving x^2.
- The right side is a positive constant (324).

Since the square of any real number is non-negative, and here it equals 324, which is positive, we anticipate two possible cases:

$$
\text{Either } x^2 - 82 = \pm \sqrt{324}
$$

As $\sqrt{324} = 18$, the solutions will branch into two pathways based on the $\pm$ sign.

Rewriting the Equation

To solve $(x^2 - 82)^2 = 324$, the standard approach involves taking the square root of both sides, remembering the principle of square roots:

$$
|x^2 - 82| = \sqrt{324} = 18
$$

This absolute value equation splits into two linear equations:

$$
x^2 - 82 = 18 \quad \text{and} \quad x^2 - 82 = -18
$$

This step simplifies the problem from a quadratic in x^2 into two quadratic equations, which can be solved individually.

Breaking Down the Solution: Two Main Cases

Let's analyze each case separately:

Case 1: $x^2 - 82 = 18$

$$
x^2 = 18 + 82 = 100
$$

This simplifies to:

$$
x^2 = 100
$$

which leads to:

$$
x = \pm \sqrt{100} = \pm 10
$$

Case 2: $x^2 - 82 = -18$

$$
x^2 = -18 + 82 = 64
$$

which simplifies to:

$$
x^2 = 64
$$

and thus:

$$
x = \pm \sqrt{64} = \pm 8
$$

Summary of solutions:

$$
x = \pm 10, \quad x = \pm 8
$$

\]

Verification of Solutions

It's essential to verify solutions, especially for equations involving absolute values and squares, to ensure no extraneous solutions have been introduced.

Verification process:

- Substitute each candidate solution into the original equation:

$$
(x^2 - 82)^2 = 324
$$

- Check for each x:

x	x^2	$x^2 - 82$	$(x^2 - 82)^2$	Is it equal to 324?
10	100	100 - 82 = 18	$18^2 = 324$	Yes
-10	100	100 - 82 = 18	$18^2 = 324$	Yes
8	64	64 - 82 = -18	$(-18)^2=324$	Yes
-8	64	64 - 82 = -18	$(-18)^2=324$	Yes

All solutions satisfy the original equation, confirming their validity.

Domain Considerations and Real Solutions

Because the original equation involves squaring, it’s important to confirm that all solutions are within the domain of real numbers:

- The expressions involve x^2, which is defined for all real x.
- No restrictions on x emerge from the original equation, as squares are always non-negative and the constants involved are real.

Therefore, all solutions derived (± 8 and ± 10) are valid within the real number system.

Alternative Methods and Insight

While the direct approach of taking square roots and solving two quadratic equations is straightforward, it's instructive to understand alternative perspectives and deeper insights into the problem.

Method 1: Substitution Technique

Let's define:

$$
u = x^2
$$

which transforms the original equation into:

$$
(u - 82)^2 = 324
$$

This is a quadratic in u:

$$
u^2 - 164u + (82)^2 = 324
$$

but more simply, as shown earlier, solving directly for u:

$$
u - 82 = \pm 18
$$

which leads to the same solutions:

$$
u = 100 \quad \text{or} \quad u = 64
$$

then back to x:

$$
x = \pm \sqrt{u}
$$

This substitution clarifies the process and demonstrates that the problem reduces to solving a quadratic in u, simplifying the approach.

Method 2: Graphical Interpretation

Plotting the functions:

- $y = (x^2 - 82)^2$
- $y = 324$

would visually reveal the intersection points corresponding to solutions.

- The graph of $y = (x^2 - 82)^2$ is a parabola opening upwards with its minimum at $x=0$, where $y = (0 - 82)^2 = 6724$.
- The horizontal line $y=324$ intersects this parabola at points where the equation holds true.

From the algebra, solutions occur at $x = \pm 8, \pm 10$, which can be checked graphically as well.

Summary and Final Solutions

After detailed analysis, the solutions to the equation $(x^2 - 82)^2 = 324$ are:

$$
\boxed{
x = -10, \quad x = 10, \quad x = -8, \quad x = 8
}
$$

These four real solutions are all valid, satisfy the original equation, and are derived through a systematic approach involving splitting the absolute value, solving quadratics, and verifying solutions.

Additional Considerations and Extensions

While this particular problem has straightforward solutions, similar equations can be more complex, such as:

- Equations involving higher powers or multiple variables.
- Equations where the quadratic expression is nested within more complicated functions.
- Cases where solutions might involve complex numbers if the radicand becomes negative.

In such situations, the methods outlined here—substitution, factoring, and graphical analysis—remain fundamental tools in the mathematician’s toolkit.

Conclusion

The process of solving $(x^2 - 82)^2 = 324$ exemplifies core algebraic principles:

- Recognizing the structure of the equation (square of a binomial).
- Applying the absolute value property to split into two cases.
- Solving resulting quadratics systematically.
- Verifying solutions to avoid extraneous roots.
- Understanding the domain and the nature of the solutions.

By carefully dissecting the problem, transforming it appropriately, and validating the solutions, one gains not only the correct answer but also a deeper appreciation for algebraic problem-solving techniques. The solutions $x = \pm 8, \pm 10$ form the complete set of real solutions, providing a clear and comprehensive answer to the original problem.

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