Find And Sketch The Domain Of The Function. F(x, Y) = Ln(16 X2 16y2)

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Introduction

Understanding the domain of a function is fundamental in calculus and analytical geometry because it delineates the set of all possible input values (x and y) for which the function is defined. In this article, we will analyze the function $F(x, y) = \ln(16x^{2} + 16y^{2})$. We will determine its domain comprehensively and provide guidance on sketching the region in the xy-plane where the function exists. This exploration involves understanding the properties of the natural logarithm, the conditions for its argument, and the geometric interpretation of the domain.

Understanding the Function $F(x, y) = \ln(16x^{2} + 16y^{2})$

Breakdown of the Function

The function involves the natural logarithm, $\ln$, applied to the expression $16x^{2} + 16y^{2}$. Recognizing the structure is crucial:
- The expression inside the logarithm is $16x^{2} + 16y^{2}$.
- This can be factored as $16(x^{2} + y^{2})$.

Properties of the Natural Logarithm

- The natural logarithm, $\ln(z)$, is defined only for $z > 0$.
- It is undefined when $z \leq 0$.

Applying this to our function, the argument $16x^{2} + 16y^{2}$ must be strictly positive:

$$
16x^{2} + 16y^{2} > 0
$$

Determining the Domain

Step 1: Simplify the Inequality

Since 16 is a positive constant, dividing both sides of the inequality by 16 preserves the inequality:

$$
x^{2} + y^{2} > 0
$$

Step 2: Interpret the Inequality

- The inequality $x^{2} + y^{2} > 0$ indicates that the point (x, y) cannot lie at the origin $(0, 0)$, because at the origin:

$$
0^{2} + 0^{2} = 0
$$

which makes the argument of the logarithm zero, and $\ln(0)$ is undefined.

- For all points where either $x \neq 0$ or $y \neq 0$, $x^{2} + y^{2} > 0$.

Step 3: Express the Domain

Thus, the domain D of the function is:

$$
D = \{ (x, y) \in \mathbb{R}^2 \mid (x, y) \neq (0, 0) \}
$$

or equivalently, all points in the xy-plane except the origin.

Geometric Interpretation of the Domain

The Set of All Points Except the Origin

- The domain corresponds to the entire xy-plane excluding the point $(0, 0)$.
- Geometrically, this is the plane with a single point removed, often referred to as $\mathbb{R}^2 \setminus \{ (0,0) \}$.

Relation to the Circle $x^{2} + y^{2} = 0$

- The locus where $x^{2} + y^{2} = 0$ is just the point $(0, 0)$.
- The domain is the complement of this point in the plane.

Sketching the Domain

Step 1: Draw the xy-plane

- Start with a coordinate plane with axes x and y.

Step 2: Identify the excluded point

- Mark the origin $(0, 0)$ explicitly.
- Shade or clearly indicate that this point is not part of the domain.

Step 3: Shade the rest of the plane

- The entire plane, except the origin, is part of the domain.
- You can use shading or labeling to emphasize the exclusion of the origin.

Optional: Use Polar Coordinates for Better Visualization

- In polar coordinates, $x = r \cos \theta$, $y = r \sin \theta$.
- The argument becomes $16r^{2}$.
- Since $r^{2} \geq 0$, the only problematic point is when $r = 0$ (the origin).
- The domain in polar coordinates is $r > 0$, $0 \leq \theta < 2\pi$.

Summary of the Domain

- The domain of $F(x, y) = \ln(16x^{2} + 16y^{2})$ is all points in the xy-plane except the origin.
- Mathematically expressed as:
$$
\boxed{
\text{Domain} = \{ (x, y) \in \mathbb{R}^2 \mid (x, y) \neq (0, 0) \}
}
$$
- Geometrically, this is the entire plane with a single point removed.

Additional Considerations and Related Concepts

Continuity and Differentiability

- The function F is continuous and differentiable everywhere in its domain.
- At points approaching the origin, the function tends toward negative infinity because $\ln(r^{2}) \to -\infty$ as $r \to 0$.

Behavior Near the Excluded Point

- As $(x, y) \to (0, 0)$, $16x^{2} + 16y^{2} \to 0^{+}$, and $F(x, y) \to -\infty$.
- This indicates a vertical asymptote at the origin in the graph of the function.

Implications for Graphing

- When sketching $F(x, y)$, note that the function is undefined at $(0, 0)$.
- The surface approaches negative infinity near this point, creating a "valley" or "cusp" at the origin in the 3D graph.

Conclusion

Determining the domain of the function $F(x, y) = \ln(16x^{2} + 16y^{2})$ hinges on understanding the properties of the natural logarithm and the geometric nature of the quadratic form $x^{2} + y^{2}$. The domain consists of all points in the xy-plane except the origin. Visualizing this, the entire plane is included, with the single point $(0, 0)$ excluded. This analysis provides a clear understanding of where the function is defined, how it behaves near the excluded point, and how to sketch its domain effectively.

Frequently Asked Questions

What is the domain of the function $F(x, y) = \ln(16x^2 - 16y^2)$?

The domain consists of all points (x, y) where the argument of the natural logarithm is positive: $16x^2 - 16y^2 > 0$, which simplifies to $x^2 - y^2 > 0$. Therefore, the domain is all points where $x^2 > y^2$, excluding the lines where $x^2 = y^2$.

How do you find the domain of $F(x, y) = \ln(16x^2 - 16y^2)$?

To find the domain, set the argument of the ln function greater than zero: $16x^2 - 16y^2 > 0$. Simplify to $x^2 - y^2 > 0$. The domain includes all (x, y) satisfying $|x| > |y|$, meaning points outside the lines $y = \pm x$.

What is the geometric interpretation of the domain for $F(x, y) = \ln(16x^2 - 16y^2)$?

Geometrically, the domain is the set of points in the plane where the absolute value of x exceeds that of y, i.e., the regions outside the lines $y = x$ and $y = -x$, excluding these lines themselves.

Can the domain include points where $x^2 = y^2$ for $F(x, y) = \ln(16x^2 - 16y^2)$?

No, the domain excludes points where $x^2 = y^2$ because at those points, the argument of the logarithm becomes zero, and $\ln(0)$ is undefined.

How would you sketch the domain of the function $F(x, y) = \ln(16x^2 - 16y^2)$?

To sketch the domain, plot the regions where $|x| > |y|$, which are the areas outside the lines $y = x$ and $y = -x$, and shade these regions while excluding the lines themselves. The domain consists of two open regions: one where $x > y$ and $x > -y$, and another where $x < y$ and $x < -y$.

Additional Resources

Find And Sketch The Domain Of The Function. $F(x, y) = \ln(16x^2 + 16y^2)$

Introduction

Mathematics, especially calculus, often challenges students and scholars alike to understand the fundamental concepts of functions, their behaviors, and their domains. The domain of a function is crucial because it defines the set of all possible input values (x and y, in this case) for which the function is defined and yields real results. In this detailed analysis, we explore the function:

$$F(x, y) = \ln(16x^2 + 16y^2)$$

Our goal is to determine its domain thoroughly and provide insights into how to visualize and sketch this domain effectively. The natural logarithm function, $\ln(z)$, is defined only for positive real numbers ($z > 0$), which leads to restrictions on the input variables x and y.

Understanding the Function

The Nature of the Function

The function:

$$F(x, y) = \ln(16x^2 + 16y^2)$$

can be simplified to:

$$ F(x, y) = \ln(16(x^2 + y^2)) $$

This form reveals that the argument of the natural logarithm is proportional to the sum of squares of x and y, scaled by 16.

Key Properties

- The expression inside the logarithm, $16(x^2 + y^2)$, is always non-negative because squares are non-negative.
- The natural logarithm is only defined for positive real numbers; thus, the argument must be strictly greater than zero:

$$ 16(x^2 + y^2) > 0 $$

which simplifies to

$$ x^2 + y^2 > 0 $$

- The function is undefined at the point where $x^2 + y^2 = 0$, which occurs only at the origin $(0, 0)$.

Determining the Domain

Step 1: Identify where the function is defined

Since:

$$ F(x, y) = \ln(16x^2 + 16y^2) $$

the domain is all (x, y) such that:

$$ 16x^2 + 16y^2 > 0 $$

Dividing both sides by 16:

$$ x^2 + y^2 > 0 $$

This is a fundamental inequality. The sum of squares is zero only at $(0, 0)$; everywhere else, it is positive.

Step 2: Express the domain explicitly

Thus, the domain, D, can be written as:

$$ D = \{ (x, y) \in \mathbb{R}^2 \mid x^2 + y^2 > 0 \} $$

In words:

> The domain consists of all points in the xy-plane except the origin.

Summary of the Domain

- The domain is the entire plane $\mathbb{R}^2$ minus the point $(0, 0)$.
- It is an open set because the inequality is strict ($>$ rather than $\geq$).
- The boundary point $(0,0)$ is excluded since the argument of the logarithm would be zero, making the function undefined.

Visualizing and Sketching the Domain

Step 1: Understanding the geometric interpretation

The condition $x^2 + y^2 > 0$ describes all points except the origin.

- The set of points where $x^2 + y^2 = r^2$ (a circle of radius r) is a circle centered at the origin.
- The exclusion of $(0, 0)$ means the domain includes all points outside the circle of radius zero, i.e., everywhere except the center point.

Step 2: Sketching the domain

- Draw the xy-plane.
- Mark the origin $(0, 0)$ with a distinct point.
- Shade the entire plane except the origin.
- To depict the exclusion explicitly, draw a small circle around the origin and leave it unshaded or marked as excluded, indicating the domain is all points outside this circle.

Step 3: Visual features

- The domain is open, with no boundary points included.
- The boundary is the singleton point at the origin, which is not part of the domain.
- Since the argument of the logarithm is proportional to the distance from the origin, the domain comprises all points except at the origin, where the function tends to $-\infty$.

Deeper Analysis: Behavior Near the Boundary

Behavior as $(x, y) \to (0, 0)$

- As (x, y) approaches $(0, 0)$, the argument of the logarithm approaches zero:

$$ 16x^2 + 16y^2 \to 0^+ $$

- The natural logarithm $\ln(z)$ tends to $-\infty$ as $z \to 0^+$.

Thus,

$$ \lim_{(x,y) \to (0,0)} F(x,y) = -\infty $$

which confirms the function's unbounded behavior near the excluded point.

Behavior at points far from the origin

- As $\sqrt{x^2 + y^2} \to \infty$,

$$ F(x, y) = \ln(16(x^2 + y^2)) \to \infty $$

indicating the function grows without bound as we move further away from the origin.

Extensions and Related Concepts

The Domain in Polar Coordinates

Expressing the domain in polar coordinates:

$$ x = r \cos \theta, \quad y = r \sin \theta $$

then:

$$ x^2 + y^2 = r^2 $$

and the domain becomes:

$$ r > 0, \quad \theta \in [0, 2\pi) $$

This confirms that the domain is the entire plane, excluding the origin, in polar coordinates.

Domain Summary

Aspect	Description
Set	$\{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 > 0 \}$
Geometric shape	The entire plane minus the origin
Boundary	The point $(0, 0)$ (not included)
Nature	Open set

Sketching the Domain: Step-by-Step Approach

1. Draw the coordinate axes: x and y.

2. Mark the origin: point $(0,0)$.

3. Indicate the excluded point: leave the origin unshaded or marked differently.

4. Shade the entire plane excluding the origin: the domain encompasses all other points.

5. Optional: draw a small circle around the origin to illustrate the boundary that is not included.

6. Label the domain: "All points in $\mathbb{R}^2$ except $(0, 0)$."

Practical Applications of the Domain

Understanding the domain of this logarithmic function is essential in various fields such as physics, engineering, and economics, where such functions model phenomena like:

- Radial symmetry problems.
- Electric potential fields.
- Distance-based cost functions.

Knowing the domain ensures accurate modeling and prevents undefined or non-real results.

Conclusion

The function $F(x, y) = \ln(16x^2 + 16y^2)$ has a straightforward yet fundamental domain: all points in the xy-plane except the origin. This is because the natural logarithm is only defined for positive arguments, and the expression inside the logarithm is zero only at $(0, 0)$.

By analyzing the geometric properties, behavior at the boundary, and employing coordinate transformations, we gain a comprehensive understanding of this function's domain. The visualization underscores the idea that the domain is an open, infinite region surrounding the origin, excluding the point itself. This insight not only aids in graphing but also enhances the understanding of the function's behavior, particularly near the boundary, where it exhibits unbounded negative values.

References

1. Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
2. Larson, R., & Edwards, B. H. (2013). Calculus. Brooks Cole.
3. Anton, H., Bivens, I., & Davis, S. (2012). Calculus. Wiley.

This detailed examination underscores the importance of understanding domains in multivariable functions and provides a thorough approach to analyzing and visualizing such functions for academic, instructional, and practical purposes.

[Find And Sketch The Domain Of The Function F X Y Ln 16 X2]

16y2

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