Find D2y/dx2 If 4x2 + 7y2 = 10 Provided Your Answer Below :d2y/dx2 =

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Understanding how to find the second derivative $\frac{d^2 y}{dx^2}$ from implicit equations is a fundamental skill in calculus, especially when dealing with curves defined implicitly rather than explicitly. The equation $4x^2 + 7y^2 = 10$ is an example of an implicit relation describing an ellipse. In this article, we will explore step-by-step methods to differentiate such an equation, find the first derivative $\frac{dy}{dx}$, and subsequently compute the second derivative $\frac{d^2 y}{dx^2}$. We will also discuss the significance of second derivatives in analyzing the curvature and concavity of the curve.

Understanding Implicit Differentiation

Implicit differentiation allows us to find derivatives of functions where y is not explicitly expressed as a function of x. Instead, the relation involves both x and y in a combined form.

Key points:

- When differentiating terms involving y, treat y as a function of x, i.e., $y = y(x)$.
- Use the chain rule for derivatives involving y, which introduces $\frac{dy}{dx}$.

Given Equation and Its Geometric Significance

The given relation:

$$
4x^2 + 7y^2 = 10
$$

represents an ellipse centered at the origin with semi-axes determined by the coefficients:

- Semi-major and semi-minor axes depend on the constants in the equation.

- The ellipse is symmetric about both axes.

Understanding the geometric shape helps in visualizing the behavior of the derivatives and the curvature.

Step-by-Step Calculation of $\frac{dy}{dx}$

To find $\frac{dy}{dx}$, differentiate both sides of the equation with respect to x:

$$
\frac{d}{dx}(4x^2 + 7y^2) = \frac{d}{dx}(10)
$$

Applying the derivatives:

$$
8x + 14y \frac{dy}{dx} = 0
$$

Rearranged to solve for $\frac{dy}{dx}$:

$$
14y \frac{dy}{dx} = -8x
$$

$$
\frac{dy}{dx} = -\frac{8x}{14y} = -\frac{4x}{7y}
$$

This is the first derivative, expressing the slope of the tangent line at any point (x, y) on the ellipse.

Calculating the Second Derivative $\frac{d^2 y}{dx^2}$

The second derivative measures the curvature or concavity of the curve at a point. To find it, differentiate $\frac{dy}{dx}$ with respect to x:

$$
\frac{dy}{dx} = -\frac{4x}{7y}
$$

Let's denote:

$$
M = -\frac{4x}{7y}
$$

Differentiate M with respect to x:

$$
\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(-\frac{4x}{7y} \right)
$$

Apply the quotient rule or treat as a product:

$$
\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(-\frac{4}{7} \times \frac{x}{y} \right)
$$

Since $-\frac{4}{7}$ is a constant:

$$
\frac{d^2 y}{dx^2} = -\frac{4}{7} \times \frac{d}{dx} \left(\frac{x}{y} \right)
$$

Now, differentiate $\frac{x}{y}$:

$$
\frac{d}{dx} \left(\frac{x}{y} \right) = \frac{y \times 1 - x \times \frac{dy}{dx}}{y^2}
$$

Recall that $\frac{dy}{dx} = -\frac{4x}{7y}$, so substitute:

$$
\frac{d}{dx} \left(\frac{x}{y} \right) = \frac{y - x \times \left(-\frac{4x}{7y} \right)}{y^2}
$$

Simplify numerator:

$$
y + \frac{4x^2}{7y}
$$

Express numerator over a common denominator $7y$:

$$
\frac{7y^2 + 4x^2}{7y}
$$

Now, the entire derivative:

$$
\frac{d}{dx} \left(\frac{x}{y} \right) = \frac{\frac{7y^2 + 4x^2}{7y}}{y^2} = \frac{7y^2 + 4x^2}{7y \times y^2} = \frac{7y^2 + 4x^2}{7 y^3}
$$

Finally, multiply by $-\frac{4}{7}$:

$$
\frac{d^2 y}{dx^2} = -\frac{4}{7} \times \frac{7 y^2 + 4x^2}{7 y^3} = -\frac{4(7 y^2 + 4x^2)}{7 \times 7 y^3} = -\frac{4(7 y^2 + 4x^2)}{49 y^3}
$$

Simplify numerator:

$$
\boxed{
\frac{d^2 y}{dx^2} = -\frac{4(7 y^2 + 4x^2)}{49 y^3}
}
$$

This is the second derivative expressed in terms of x and y.

Expressing $\frac{d^2 y}{dx^2}$ Solely in Terms of x and Known Quantities

Since $4x^2 + 7y^2 = 10$, we can express $7 y^2$ as:

$$
7 y^2 = 10 - 4x^2
$$

Substitute into the second derivative:

$$
\frac{d^2 y}{dx^2} = -\frac{4((10 - 4x^2) + 4x^2)}{49 y^3}
$$

Simplify numerator:

$$
(10 - 4x^2) + 4x^2 = 10
$$

Thus:

\[
\frac{d^2 y}{dx^2} = -\frac{4 \times 10}{49 y^3} = -\frac{40}{49 y^3}
\]

Now, \(y\) can be expressed from the original equation:

\[
7 y^2 = 10 - 4x^2 \Rightarrow y^2 = \frac{10 - 4x^2}{7}
\]

Therefore:

\[
y = \pm \sqrt{\frac{10 - 4x^2}{7}}
\]

And:

\[
y^3 = y \times y^2 = y \times \frac{10 - 4x^2}{7}
\]

Substitute \(y\):

\[
y^3 = \pm \sqrt{\frac{10 - 4x^2}{7}} \times \frac{10 - 4x^2}{7}
\]

Expressed explicitly:

\[
y^3 = \pm \frac{(10 - 4x^2)^{3/2}}{7 \sqrt{7}}
\]

Finally, the second derivative becomes:

\[
\frac{d^2 y}{dx^2} = -\frac{40}{49} \times \frac{7 \sqrt{7}}{(10 -
4x^2)^{3/2}} = -\frac{40 \times 7 \sqrt{7}}{49 (10 - 4x^2)^{3/2}}
\]

Simplify numerator:

\[
\frac{280 \sqrt{7}}{49 (10 - 4x^2)^{3/2}} = \frac{280 \sqrt{7}}{49 (10 -
4x^2)^{3/2}}
\]

Divide numerator and denominator by 7:

\[

\frac{40 \sqrt{7}}{7 (10 - 4x^2)^{3/2}}
\]

Final Expression for \(\frac{d^2 y}{dx^2}\)

The second derivative of \(y\) with respect to \(x\) for the ellipse \(4x^2 + 7y^2 = 10\) is:

\[
\boxed{
\frac{d^2 y}{dx^2} = -\

Frequently Asked Questions

How do I find the second derivative d^2y/dx^2 for the equation $4x^2 + 7y^2 = 10$?

First, differentiate implicitly to find dy/dx, then differentiate dy/dx to find d^2y/dx^2. For $4x^2 + 7y^2 = 10$, differentiate both sides to get 8x + 14y(dy/dx) = 0, solve for dy/dx, then differentiate again to find d^2y/dx^2.

What is the first step in differentiating $4x^2 + 7y^2 = 10$ implicitly?

Differentiate both sides with respect to x: 8x + 14y(dy/dx) = 0.

How do I express dy/dx from the implicit differentiation of $4x^2 + 7y^2 = 10$?

Rearranged as dy/dx = - (8x) / (14y) = - (4x) / (7y).

After finding dy/dx, what is the next step to compute d^2y/dx^2?

Differentiate dy/dx = - (4x) / (7y) with respect to x, applying the quotient rule or implicit differentiation.

How do I differentiate dy/dx = - (4x) / (7y) to find d^2y/dx^2?

Use the quotient rule: d/dx [u/v] = (v du/dx - u dv/dx) / v^2, where u = -4x and $v = 7y$.

What is the derivative of $u = -4x$ with respect to x?

$du/dx = -4$.

What is the derivative of $v = 7y$ with respect to x?

$dv/dx = 7\, dy/dx = 7\, (-4x / 7y) = -4x / y$.

What is the final expression for d^2y/dx^2 for the given equation?

$d^2y/dx^2 = [\ (7y)(-4) - (-4x)(-4x / y)\ ] / (7y)^2 = [\ -28y - (16x^2 / y)\ ] / 49 y^2$.

Can you give the simplified form of d^2y/dx^2 based on the previous calculations?

Yes, $d^2y/dx^2 = [\ -28 y - (16 x^2 / y)\ ] / (49 y^2)$.

Additional Resources

Mathematics: An In-Depth Exploration of Finding $\frac{d^2 y}{dx^2}$ for the Equation $4x^2 + 7y^2 = 10$

Introduction

In the realm of calculus, the process of differentiating implicitly holds a pivotal role, especially when dealing with equations where y is not explicitly expressed as a function of x. The problem at hand—finding the second derivative $\frac{d^2 y}{dx^2}$ of the implicit relation:

$$
4x^2 + 7y^2 = 10
$$

—serves as an excellent example to demonstrate the power of implicit differentiation, chain rule application, and algebraic manipulation. This article endeavors to guide you through a comprehensive, step-by-step derivation, highlighting key concepts, common pitfalls, and best practices. Whether you're a student brushing up on calculus techniques or an educator seeking clarity on implicit differentiation, this detailed exploration aims to deepen your understanding.

Understanding the Problem

Before jumping into derivatives, it's essential to understand the structure and nature of the given equation:

$$
4x^2 + 7y^2 = 10
$$

This is a conic section—specifically an ellipse—whose shape depends on the coefficients of x^2 and y^2. Our goal is to find the second derivative $\frac{d^2 y}{dx^2}$, which provides insights into the curvature and concavity of the ellipse's graph.

Key points to consider:

- The equation relates x and y implicitly; y is not isolated.
- To find derivatives, implicit differentiation must be employed.
- The first derivative $\frac{dy}{dx}$ is a prerequisite for the second derivative.

Step 1: Differentiating Implicitly to Find $\frac{dy}{dx}$

Applying the Chain Rule

Differentiation of both sides with respect to x:

$$
\frac{d}{dx}(4x^2) + \frac{d}{dx}(7y^2) = \frac{d}{dx}(10)
$$

Calculating each derivative:

- $\frac{d}{dx}(4x^2) = 8x$
- $\frac{d}{dx}(7y^2) = 7 \times 2y \times \frac{dy}{dx} = 14 y \frac{dy}{dx}$
- $\frac{d}{dx}(10) = 0$

Assembling the first derivative:

$$
8x + 14 y \frac{dy}{dx} = 0
$$

Solving for $\frac{dy}{dx}$:

$$
14 y \frac{dy}{dx} = -8x
$$

$$
\frac{dy}{dx} = -\frac{8x}{14 y} = -\frac{4x}{7 y}
$$

Result:

$$
\boxed{
\frac{dy}{dx} = -\frac{4x}{7 y}
}
$$

This expression captures how y changes with respect to x, considering the implicit relationship.

Step 2: Differentiating $\frac{dy}{dx}$ to Obtain $\frac{d^2 y}{dx^2}$

The Chain Rule and Quotient Rule

To find the second derivative, differentiate $\frac{dy}{dx}$ with respect to x:

$$
\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(-\frac{4x}{7 y} \right)
$$

It's often easier to extract the constant factor:

$$
\frac{d^2 y}{dx^2} = -\frac{1}{7} \frac{d}{dx} \left(\frac{4x}{y} \right)
$$

Now, focus on differentiating:

$$
\frac{d}{dx} \left(\frac{4x}{y} \right)
$$

which is a quotient involving two functions of x: numerator $4x$ and denominator y.

Step 3: Applying the Quotient Rule

Recall, the quotient rule states:

\[
\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u

\frac{dv}{dx}}{v^2}
\]

Identify:

- $u = 4x$
- $v = y$

Calculate derivatives:

- $\frac{du}{dx} = 4$
- $\frac{dv}{dx} = \frac{dy}{dx} = -\frac{4x}{7 y}$ (from earlier)

Now, apply the quotient rule:

$$
\frac{d}{dx} \left(\frac{4x}{y} \right) = \frac{ y \times 4 - 4x \times \left(-\frac{4x}{7 y} \right) }{ y^2 }
$$

Simplify numerator:

$$
4 y + \frac{16 x^2}{7 y}
$$

Expressed over a common denominator $7 y$:

$$
\frac{ 4 y \times 7 y + 16 x^2 }{ 7 y } = \frac{28 y^2 + 16 x^2}{7 y}
$$

Note: Since numerator and denominator are explicit, write the entire derivative as:

$$
\frac{28 y^2 + 16 x^2}{7 y}
$$

Now, recall the earlier factor:

$$
\frac{d^2 y}{dx^2} = -\frac{1}{7} \times \frac{28 y^2 + 16 x^2}{7 y}
$$

Combine constants:

$$
\frac{d^2 y}{dx^2} = - \frac{28 y^2 + 16 x^2}{49 y}
$$

Step 4: Final Expression for $\frac{d^2 y}{dx^2}$

Simplify numerator and denominator:

$$
\boxed{
\frac{d^2 y}{dx^2} = - \frac{28 y^2 + 16 x^2}{49 y}
}
$$

This is the second derivative expressed in terms of x and y. However, to make this expression more elegant and meaningful, it's often beneficial to relate x and y via the original equation.

Step 5: Expressing $\frac{d^2 y}{dx^2}$ in Terms of x and y

Recall, from the original equation:

$$
4x^2 + 7 y^2 = 10
$$

which can be rearranged as:

$$
7 y^2 = 10 - 4 x^2
$$

or:

$$
y^2 = \frac{10 - 4 x^2}{7}
$$

Similarly, y^2 can be substituted into the second derivative for a more explicit formula if needed.

Additional Insights and Practical Considerations

1. Significance of $\frac{d^2 y}{dx^2}$:

- The second derivative indicates the curvature of the curve defined by the original equation.
- Positive $\frac{d^2 y}{dx^2}$ indicates concave upward regions; negative indicates concave downward.

2. Application Scope:

- Such derivatives are essential in physics, engineering, and computer graphics for understanding the shape and motion along curves.
- The implicit derivatives can also help determine points of inflection and curvature extrema.

3. Common Pitfalls:

- Forgetting the chain rule when differentiating y.
- Confusing the derivatives of numerator and denominator in the quotient rule.
- Not simplifying the algebraic expressions thoroughly, leading to confusing or overly complicated formulas.

Summary Table

Step	Action	Result
1	Differentiate implicitly to find $\frac{dy}{dx}$	$\boxed{\frac{dy}{dx} = -\frac{4x}{7 y}}$
2	Differentiate $\frac{dy}{dx}$ to find $\frac{d^2 y}{dx^2}$	$\frac{d^2 y}{dx^2} = - \frac{28 y^2 + 16 x^2}{49 y}$
3	Simplify and relate x, y via original equation	\text{Expressed in terms of original variables}

Conclusion

The derivation of $\frac{d^2 y}{dx^2}$ for the relation $4x^2 + 7 y^2 = 10$ exemplifies the elegance and complexity of implicit differentiation. By methodically applying the chain rule, quotient rule, and algebraic simplification, we arrive at a meaningful expression that captures the curvature behavior of an ellipse.

Final Answer:

\[
\boxed{
\frac

Find D2y Dx2 If 4x2 7y2 10 Provided Your Answer Below D2y Dx2

Find D2y Dx2 If 4x2 7y2 10 Provided Your Answer Below D2y Dx2

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