Find The Average Value Of F(x) = 4(x + 1) / X^2 Over The Interval [2:41

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Understanding how to find the average value of a function over a specific interval is a fundamental concept in calculus. In this article, we will explore the process of determining the average value of the function $f(x) = \frac{4(x + 1)}{x^2}$ over the interval $[2, 41]$. We will break down the steps involved, explain relevant concepts, and provide illustrative examples to ensure clarity. Whether you're a student preparing for exams or a math enthusiast seeking to deepen your understanding, this comprehensive guide will walk you through each stage of the calculation, emphasizing best practices for accuracy and efficiency.

Understanding the Concept of the Average Value of a Function

Before diving into the calculations, it’s vital to grasp what the average value of a function over an interval signifies.

Definition of Average Value of a Function

The average value f_{avg} of a continuous function $f(x)$ on the interval $[a, b]$ is given by the formula:

$$
f_{avg} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
$$

This formula essentially computes the mean height of the function over the specified interval, offering insight into its overall behavior.

Why Is the Average Value Important?

- Practical Applications: Engineers and scientists often need to find the average value of a quantity over time or space.
- Mathematical Analysis: It helps in understanding the overall trend of a function.
- Problem Solving: It serves as a basis for solving real-world problems

involving averages and means.

Analyzing the Function $f(x) = \frac{4(x + 1)}{x^2}$

Let's analyze the given function to understand its properties, which will aid in integration and interpretation.

Function Breakdown

The function is:

$$
f(x) = \frac{4(x + 1)}{x^2}
$$

which can be rewritten as:

$$
f(x) = 4 \cdot \frac{x + 1}{x^2}
$$

This form allows us to split the function into more manageable parts.

Simplifying the Function

Express $f(x)$ as a sum of simpler fractions:

$$
f(x) = 4 \left(\frac{x}{x^2} + \frac{1}{x^2} \right) = 4 \left(\frac{1}{x} + \frac{1}{x^2} \right)
$$

Therefore:

$$
f(x) = \frac{4}{x} + \frac{4}{x^2}
$$

This form makes the integration process straightforward, as we now have basic power functions.

Calculating the Integral of $f(x)$

The core step in finding the average value involves calculating the definite integral of $f(x)$ over the interval $[2, 41]$.

Integral of $f(x)$

Given:

$$
f(x) = \frac{4}{x} + \frac{4}{x^2}
$$

We need to compute:

$$
\int_{2}^{41} f(x) \, dx = \int_{2}^{41} \left(\frac{4}{x} + \frac{4}{x^2} \right) dx
$$

This integral can be separated into two parts:

$$
\int_{2}^{41} \frac{4}{x} \, dx + \int_{2}^{41} \frac{4}{x^2} \, dx
$$

Step-by-Step Integration

1. Integrate $\frac{4}{x}$:

$$
\int \frac{4}{x} \, dx = 4 \int \frac{1}{x} \, dx = 4 \ln|x| + C
$$

2. Integrate $\frac{4}{x^2}$:

Recall that:

$$
\int x^{-n} \, dx = \frac{x^{-n+1}}{-n+1} + C,\quad n \neq 1
$$

Here, $n=2$, so:

$$
\int x^{-2} \, dx = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = -\frac{1}{x} + C
$$

Multiplying by 4:

$$
\int \frac{4}{x^2} \, dx = 4 \left(- \frac{1}{x} \right) = - \frac{4}{x} + C
$$

Evaluating the Definite Integral

Putting it all together:

$$
\int_{2}^{41} f(x) \, dx = \left[4 \ln|x| - \frac{4}{x} \right]_{x=2}^{x=41}
$$

Calculating at the bounds:

$$
= \left(4 \ln 41 - \frac{4}{41} \right) - \left(4 \ln 2 - \frac{4}{2} \right)
$$

Simplify each term:

- $4 \ln 41$ remains as is.
- $\frac{4}{41}$ remains as is.
- $4 \ln 2$ remains as is.
- $\frac{4}{2} = 2$.

Thus:

$$
\int_{2}^{41} f(x) \, dx = \left(4 \ln 41 - \frac{4}{41} \right) - \left(4 \ln 2 - 2 \right)
$$

Rearranged:

$$
= 4 \ln 41 - \frac{4}{41} - 4 \ln 2 + 2
$$

Calculating the Average Value

Recall the formula for the average value:

$$
f_{avg} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
$$

where $a=2$, $b=41$. The interval length:

$$
b - a = 41 - 2 = 39
$$

Therefore:

$$
f_{avg} = \frac{1}{39} \left(4 \ln 41 - \frac{4}{41} - 4 \ln 2 + 2 \right)
$$

Simplify numerator:

$$
f_{avg} = \frac{1}{39} \left(4 \ln 41 - 4 \ln 2 - \frac{4}{41} + 2 \right)
$$

Factor where possible:

$$
f_{avg} = \frac{1}{39} \left(4 (\ln 41 - \ln 2) - \frac{4}{41} + 2 \right)
$$

Recognize that $\ln 41 - \ln 2 = \ln \left(\frac{41}{2} \right)$:

$$
f_{avg} = \frac{1}{39} \left(4 \ln \left(\frac{41}{2} \right) - \frac{4}{41} + 2 \right)
$$

Final Expression and Numerical Approximation

The exact average value of the function over $[2, 41]$ is:

$$
\boxed{
f_{avg} = \frac{1}{39} \left(4 \ln \left(\frac{41}{2} \right) - \frac{4}{41} + 2 \right)
}
$$

For practical purposes, let's approximate this value numerically.

Numerical Calculation

- $\ln \left(\frac{41}{2} \right) = \ln(20.5) \approx 3.021$
- $4 \times 3.021 \approx 12.084$
- $\frac{4}{41} \approx 0.09756$
- 2 remains as is.

Plugging in:

$$
f_{avg} \approx \frac{1}{39} \left(12.084 - 0.09756 + 2 \right) = \frac{1}{39} (13.98644)
$$

Finally:

$$
f_{avg} \approx \frac{13.98644}{39} \approx 0.3594
$$

Thus, the average value of $f(x)$ over $[2, 41]$ is approximately 0.3594.

Summary of Key Steps

- Step 1: Recognize the formula for the average value of a function.
- Step 2: Simplify the function $f(x)$ into manageable parts.
- Step 3: Compute the

Frequently Asked Questions

What is the function given for finding the average

value?

The function is $F(x) = 4(x + 1) / x^2$.

Over which interval do we need to find the average value of F(x)?

The interval is from $x = 2$ to $x = 41$.

What is the formula to compute the average value of a function over an interval?

The average value is given by $(1 / (b - a)) \int$ from a to b of $F(x)\,dx$.

How do you set up the integral to find the average value of F(x) over [2, 41]?

You set up $(1 / (41 - 2)) \int$ from 2 to 41 of $(4(x + 1) / x^2)\,dx$.

What is the simplified form of the integrand $4(x + 1) / x^2$?

The integrand simplifies to $4(1 + 1/x)$.

How do you evaluate the integral $\int$ from 2 to 41 of $4(1 + 1/x)\,dx$?

You split it into $4\int 1\,dx + 4\int (1/x)\,dx$, which evaluates to $4[x] + 4[\ln|x|]$, evaluated from 2 to 41.

What is the value of the integral after evaluation?

The integral evaluates to $4(41 - 2) + 4(\ln 41 - \ln 2)$.

How do you compute the average value after evaluating the integral?

Divide the result of the integral by $(41 - 2)$, which is 39.

What is the final approximate value of the average of F(x) over [2, 41]?

The approximate average value is $(4\ 39 + 4\ (\ln 41 - \ln 2)) / 39$, which simplifies to approximately $4 + (4 / 39) \ln(41/2)$.

Additional Resources

Find The Average Value Of $F(x) = 4(x + 1) / x^2$ Over The Interval [2, 41]

Understanding how to find the average value of a function over a specified interval is a fundamental concept in calculus, offering insights into the overall behavior of the function. In this comprehensive review, we will delve into the process of calculating the average value of the function $F(x) = \frac{4(x + 1)}{x^2}$ over the interval $[2, 41]$. This exploration will cover the theoretical foundations, step-by-step calculations, interpretations of results, and practical applications, providing a clear and detailed understanding suitable for students, educators, or anyone interested in calculus.

Introduction to the Concept of Average Value of a Function

Before we proceed with the specific function, it’s essential to grasp what the average value of a function signifies in calculus. The average value of a continuous function $f(x)$ on a closed interval $[a, b]$ is given by the formula:

$$
f_{avg} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
$$

This formula essentially computes the mean of the function's outputs over the interval, providing a single representative value that reflects the overall “height” of the function across that interval.

Key points:
- The integral computes the total accumulated value of the function over $[a, b]$.
- Dividing by $(b - a)$ gives the average height of the function over that interval.
- This concept is used in various applications, such as physics (average velocity), economics (average cost), and engineering.

Understanding the Function $F(x) = \frac{4(x + 1)}{x^2}$

The function under consideration is:

$$
F(x) = \frac{4(x + 1)}{x^2}
$$

which can be rewritten for clarity as:

$$
F(x) = 4 \cdot \frac{x + 1}{x^2}
$$

This rational function combines polynomial expressions in the numerator and denominator, and its behavior varies across the interval based on the values of x.

Features of $F(x)$:

- Domain: Since the denominator is x^2, the function is undefined at $x=0$, but our interval $[2, 41]$ does not include zero, so the function is continuous in the interval.
- Behavior: As x increases, the function's value tends to decrease because the denominator grows faster than the numerator.
- End behavior: For large x, $F(x)$ approaches zero, indicating the function diminishes as we move towards larger x.

Calculating the Integral $\int_{2}^{41} F(x) \, dx$

The core step in finding the average value involves calculating the definite integral of $F(x)$ over $[2, 41]$.

Step 1: Simplify the integrand

Express $F(x)$ in a more manageable form:

$$
F(x) = 4 \cdot \frac{x + 1}{x^2} = 4 \left(\frac{x}{x^2} + \frac{1}{x^2} \right) = 4 \left(\frac{1}{x} + x^{-2} \right)
$$

This simplifies the integral to:

$$
\int_{2}^{41} F(x) \, dx = 4 \int_{2}^{41} \left(\frac{1}{x} + x^{-2} \right) dx
$$

\]

Step 2: Break into separate integrals

\[
4 \left(\int_{2}^{41} \frac{1}{x} dx + \int_{2}^{41} x^{-2} dx \right)
\]

Step 3: Calculate each integral

- Integral of \(1/x\):

\[
\int \frac{1}{x} dx = \ln|x| + C
\]

- Integral of \(x^{-2}\):

\[
\int x^{-2} dx = \int x^{-2} dx = -x^{-1} + C
\]

Step 4: Evaluate definite integrals

\[
4 \left[\left(\ln|x| \right)_{2}^{41} + \left(-x^{-1} \right)_{2}^{41}
\right]
\]

which expands to:

\[
4 \left[\left(\ln 41 - \ln 2 \right) + \left(-\frac{1}{41} + \frac{1}{2}
\right) \right]
\]

Step 5: Simplify the expression

\[
4 \left(\ln \frac{41}{2} + \left(-\frac{1}{41} + \frac{1}{2} \right)
\right)
\]

Calculating the Numerical Values

Let's compute each part:

- $\ln \frac{41}{2}$:

$$
\ln 20.5 \approx 3.0204
$$

- $-\frac{1}{41} \approx -0.0244$

- $\frac{1}{2} = 0.5$

Adding $-\frac{1}{41} + \frac{1}{2}$:

$$
-0.0244 + 0.5 = 0.4756
$$

Now, summing with $\ln \frac{41}{2}$:

$$
3.0204 + 0.4756 = 3.496
$$

Finally, multiply by 4:

$$
4 \times 3.496 = 13.984
$$

Thus, the value of the integral is approximately 13.984.

Calculating the Average Value

Recall the formula:

$$
f_{avg} = \frac{1}{b - a} \int_{a}^{b} F(x) \, dx
$$

Here, $a=2$, $b=41$:

$$
f_{avg} = \frac{1}{41 - 2} \times 13.984 = \frac{1}{39} \times 13.984
$$

Performing the division:

$$

f_{avg} \approx 0.3591
\]

Therefore, the average value of \(F(x)\) over \([2, 41]\) is approximately 0.3591.

Interpretation of the Result

The computed average value of approximately 0.3591 indicates that, over the interval from 2 to 41, the function \(F(x)\) hovers around this value on average. Given the nature of the function, which diminishes as \(x\) increases, this average reflects the overall trend rather than the specific behavior at individual points.

Significance:

- Since \(F(x)\) tends to decrease over the interval, the average value leans closer to the function's value at larger \(x\), but the integral's weighting over the entire interval balances the larger values near \(x=2\) with the smaller values near \(x=41\).
- This average can be useful in applications where the function models a real-world quantity that varies with \(x\), such as average speed, cost, or resource consumption over a period or range.

Practical Applications and Contexts

Understanding how to compute and interpret the average value of a function like \(F(x)\) has numerous applications:

- Physics: Calculating average velocity or acceleration over a given time interval.
- Economics: Determining average cost or profit over a specified period.
- Engineering: Assessing average load, stress, or resource utilization.
- Environmental Science: Computing average pollutant levels over a region or time.

In each case, the average value provides a meaningful summary metric that simplifies complex variable behaviors into a single, interpretable number.

Pros and Cons of the Methodology

Pros:
- Provides a clear summary: Summarizes complex function behavior over an interval into a single value.
- Applicable to diverse fields: Useful across science, engineering, economics, etc.
- Mathematically rigorous: Based on the fundamental theorem of calculus, ensuring accuracy for continuous functions.

Cons:
- Assumes continuity: Cannot be directly applied if the function is discontinuous over the interval.
- Averages out local variations: May obscure important local behavior or fluctuations within the interval.
- Requires integration skills: Demands understanding of integral calculus and potential symbolic manipulation.

Conclusion

Calculating the average value of $F(x) = \frac{4(x + 1)}{x^2}$ over $[2, 41]$ illustrates the power and utility of integral calculus. Through methodical simplification, integration, and evaluation, we determined that the average value is approximately 0.3591. This process not only reinforces key calculus concepts but also demonstrates practical applications in analyzing real-world functions. Whether for academic purposes or practical modeling, mastering such calculations

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