Find The Curvature Of R(t) At The Point (7,1,1), R(t)=<7t,t2,t3>

Find The Curvature Of R(t) At The Point (7,1,1), $R(t)=<7t,t^2,t^3>$

Understanding the curvature of a space curve at a specific point is essential in differential geometry, physics, and engineering. In this article, we will explore how to find the curvature of the vector function $R(t) = <7t, t^2, t^3>$ at the point (7, 1, 1). This process involves calculating derivatives, identifying the parameter value corresponding to the point, and applying the curvature formula. Whether you're a student studying calculus or a professional applying these concepts, this comprehensive guide will walk you through each step.

Understanding the Curve $R(t) = <7t, t^2, t^3>$

Parametric Representation of Space Curves

The vector function $R(t) = <7t, t^2, t^3>$ defines a space curve parametrized by the variable t. Each component describes the position of a point in 3D space as t varies. Specifically:

- $x(t) = 7t$
- $y(t) = t^2$
- $z(t) = t^3$

This type of parametrization allows us to analyze the behavior of the curve at any point along its path by examining derivatives with respect to t.

Identifying the Point (7, 1, 1) on the Curve

To find the value of t corresponding to the point (7, 1, 1), solve for t in each coordinate:

1. From $x(t) = 7t$, set $7t = 7 \rightarrow t = 1$
2. Check $y(t) = t^2 \rightarrow t^2 = 1 \rightarrow t = \pm1$
3. Check $z(t) = t^3 \rightarrow t^3 = 1 \rightarrow t = 1$

Since all three components are consistent at $t = 1$, the point (7, 1, 1) corresponds to $t = 1$ on the curve.

Calculating Derivatives of R(t)

First Derivative R'(t)

The derivative of R(t) with respect to t gives the velocity vector:

$R'(t) = d/dt <7t, t^2, t^3> = <7, 2t, 3t^2>$

- At t = 1: $R'(1) = <7, 2(1), 3(1)^2> = <7, 2, 3>$

Second Derivative R''(t)

The second derivative indicates how the velocity vector changes along the curve:

$R''(t) = d/dt <7, 2t, 3t^2> = <0, 2, 6t>$

- At t = 1: $R''(1) = <0, 2, 6(1)> = <0, 2, 6>$

These derivatives are essential for calculating curvature, as they describe the curve's local bending.

Formula for Curvature of a Space Curve

Understanding the Curvature Formula

The curvature $\kappa(t)$ at a point on a space curve R(t) measures how sharply the curve bends at that point. The standard formula for the curvature in three dimensions is:

$\kappa(t) = |R'(t) \times R''(t)| / |R'(t)|^3$

Where:

- R'(t) is the first derivative (velocity vector)
- R''(t) is the second derivative (acceleration vector)
- × denotes the cross product
- |·| denotes the magnitude of a vector

This formula quantifies the instantaneous bending of the curve based on derivatives.

Calculating the Cross Product $R'(t) \times R''(t)$

At $t = 1$, compute $R'(1) \times R''(1)$:

$R'(1) = \langle 7, 2, 3 \rangle$

$R''(1) = \langle 0, 2, 6 \rangle$

The cross product is:

$$R'(1) \times R''(1) = \begin{vmatrix} i & j & k \\ 7 & 2 & 3 \\ 0 & 2 & 6 \end{vmatrix}$$

Calculating:

- i-component: $(2)(6) - (3)(2) = 12 - 6 = 6$
- j-component: $- [(7)(6) - (3)(0)] = - [42 - 0] = -42$
- k-component: $(7)(2) - (2)(0) = 14 - 0 = 14$

Thus,

$R'(1) \times R''(1) = \langle 6, -42, 14 \rangle$

Next, find its magnitude:

$|R'(1) \times R''(1)| = \sqrt{6^2 + (-42)^2 + 14^2} = \sqrt{36 + 1764 + 196} = \sqrt{1996} \approx 44.69$

Calculating the Magnitude $|R'(t)|$ at $t=1$

$|R'(1)| = \sqrt{7^2 + 2^2 + 3^2} = \sqrt{49 + 4 + 9} = \sqrt{62} \approx 7.87$

Cubing this magnitude:

$|R'(1)|^3 \approx (7.87)^3 \approx 487.65$

Determining the Curvature at $t=1$

Putting it all together:

$\kappa(1) = |R'(1) \times R''(1)| / |R'(1)|^3 \approx 44.69 / 487.65 \approx 0.0916$

Therefore, the curvature of the curve $R(t) = \langle 7t, t^2, t^3 \rangle$ at the point $(7, 1, 1)$ is approximately 0.0916.

Summary and Additional Insights

Key Takeaways

- The point (7, 1, 1) corresponds to t = 1 on the curve.
- Derivatives R'(t) and R''(t) are essential for curvature calculation.
- The cross product indicates the vector orthogonal to both velocity and acceleration, influencing the curvature.
- The final curvature value quantifies how sharply the curve bends at the specified point.

Applications of Curvature Calculation

- Physics: Understanding particle motion along a path.
- Engineering: Designing smooth curves and trajectories.
- Mathematics: Analyzing the geometric properties of space curves.

Additional Tips for Calculating Curvature

- Always verify the parameter t corresponding to the point of interest.
- Calculate derivatives carefully, especially when components involve different powers.
- Use the cross product to find the vector orthogonal to the velocity and acceleration vectors.
- Double-check the magnitude calculations to ensure accuracy.

Conclusion

Finding the curvature of a space curve at a specific point involves a systematic approach: identifying the parameter value, computing derivatives, applying the curvature formula, and interpreting the results. For the curve $R(t) = \langle 7t, t^2, t^3 \rangle$ at the point (7, 1, 1), the curvature is approximately 0.0916, indicating a gentle bend at that location. Mastering these steps enhances your understanding of differential geometry and its practical applications across various scientific fields.

By following the detailed steps outlined in this article, you can confidently analyze the curvature of similar space curves and deepen your grasp of geometric properties in three-dimensional space.

Frequently Asked Questions

How do you find the curvature of the vector function $R(t) = \langle 7t, t^2, t^3 \rangle$ at a specific point?

To find the curvature at a point, first determine the parameter t corresponding to that point, then compute the derivatives $R'(t)$ and $R''(t)$, and use the curvature formula $\kappa(t) = |R'(t) \times R''(t)| / |R'(t)|^3$.

What is the value of t when $R(t) = (7, 1, 1)$ for the given vector function?

Solve the equations $7t = 7$, $t^2 = 1$, and $t^3 = 1$. From $7t=7$, $t=1$; from $t^2=1$, $t=\pm1$; from $t^3=1$, $t=1$. The consistent value is $t=1$.

How do you compute the first derivative $R'(t)$ for $R(t) = \langle 7t, t^2, t^3 \rangle$?

Differentiate each component: $R'(t) = \langle 7, 2t, 3t^2 \rangle$.

What is the second derivative $R''(t)$ of $R(t) = \langle 7t, t^2, t^3 \rangle$?

Differentiate $R'(t)$: $R''(t) = \langle 0, 2, 6t \rangle$.

How do you calculate the curvature κ at t=1 for this vector function?

Evaluate $R'(1) = \langle 7, 2, 3 \rangle$ and $R''(1) = \langle 0, 2, 6 \rangle$, then compute the cross product $R'(1) \times R''(1)$, find its magnitude, and divide by $|R'(1)|^3$.

What is the cross product $R'(1) \times R''(1)$ for the given derivatives?

Calculate: $\langle 7, 2, 3 \rangle \times \langle 0, 2, 6 \rangle = \langle (2)(6) - (3)(2), (3)(0) - (7)(6), (7)(2) - (2)(0) \rangle = \langle 12 - 6, 0 - 42, 14 - 0 \rangle = \langle 6, -42, 14 \rangle$.

How do you find the magnitude $|R'(1) \times R''(1)|$?

Calculate the square root of the sum of squares: $\sqrt{6^2 + (-42)^2 + 14^2} = \sqrt{36 + 1764 + 196} = \sqrt{1996} \approx 44.66$.

What is the magnitude of $R'(1)$ and how is it used in the curvature formula?

$|R'(1)| = \sqrt{7^2 + 2^2 + 3^2} = \sqrt{49 + 4 + 9} = \sqrt{62} \approx 7.87$. The curvature is then $\kappa = |R'(1) \times R''(1)| / |R'(1)|^3 \approx 44.66 / (7.87)^3$.

Additional Resources

Find The Curvature Of R(t) At The Point (7,1,1), $R(t)=<7t, t^2, t^3>$

Understanding the curvature of a space curve at a particular point is a fundamental concept in differential geometry, providing insight into how the curve bends and twists in three-dimensional space. When given a parametric vector function such as $R(t) = <7t, t^2, t^3>$, calculating the curvature at a specific point involves several steps, including differentiating the function, evaluating derivatives at the point, and applying the curvature formula. This comprehensive analysis not only helps in grasping the geometric behavior of the curve but also enhances one's ability to interpret complex parametric equations in a spatial context.

Introduction to Curvature and Its Significance

Curvature is a measure of how sharply a curve bends at a particular point. In three-dimensional space, the curvature $\kappa(t)$ quantifies the rate of change of the unit tangent vector with respect to arc length. High curvature indicates a tight bend, while low curvature suggests a gentler curve. This concept is vital in various fields such as physics (for understanding particle trajectories), engineering (for designing smooth paths), and computer graphics (for modeling realistic motion).

Understanding the curvature at a point provides insights into the local geometric properties of the curve. For the given vector function R(t), which describes a space curve, calculating the curvature at the point (7,1,1) helps visualize how the curve behaves precisely at that location.

Analyzing the Given Vector Function R(t)

The parametric vector function provided is:

$R(t) = <7t, t^2, t^3>$

This function maps a real parameter t to points in three-dimensional space. To find the point (7,1,1) on the curve, we need to determine the value of t that satisfies R(t) = (7,1,1).

Finding t for the point (7,1,1):

- For the x-component: $7t = 7 \rightarrow t = 1$

- For the y-component: $t^2 = 1 \rightarrow t = \pm 1$
- For the z-component: $t^3 = 1 \rightarrow t = 1$

Since all components are satisfied when $t = 1$, the point (7,1,1) corresponds to $t = 1$.

This confirms that at $t=1$, the curve passes through the point of interest.

Calculating the Derivatives R'(t) and R''(t)

Calculating the derivatives of R(t) is essential for curvature computation because they provide the tangent and curvature vectors.

First derivative R'(t):

$R'(t) = d/dt \langle 7t, t^2, t^3 \rangle = \langle 7, 2t, 3t^2 \rangle$

At $t=1$:

$R'(1) = \langle 7, 2(1), 3(1)^2 \rangle = \langle 7, 2, 3 \rangle$

Second derivative R''(t):

$R''(t) = d/dt \langle 7, 2t, 3t^2 \rangle = \langle 0, 2, 6t \rangle$

At $t=1$:

$R''(1) = \langle 0, 2, 6(1) \rangle = \langle 0, 2, 6 \rangle$

Applying the Curvature Formula

The curvature $\kappa(t)$ of a space curve R(t) is given by the formula:

$\kappa(t) = |R'(t) \times R''(t)| / |R'(t)|^3$

Where:

- $R'(t) \times R''(t)$ is the cross product of R'(t) and R''(t)
- $|R'(t)|$ is the magnitude of R'(t)

Calculating $|R'(1)|$:

$|R'(1)| = \sqrt{7^2 + 2^2 + 3^2} = \sqrt{49 + 4 + 9} = \sqrt{62} \approx 7.874$

Calculating the cross product $R'(1) \times R''(1)$:

$R'(1) = \langle 7, 2, 3 \rangle$

$R''(1) = \langle 0, 2, 6 \rangle$

The cross product is:

$$\begin{vmatrix} i & j & k \\ 7 & 2 & 3 \\ 0 & 2 & 6 \end{vmatrix}$$

$= i(26 - 32) - j(76 - 30) + k(72 - 20)$

$= i(12 - 6) - j(42 - 0) + k(14 - 0)$

$= i(6) - j(42) + k(14)$

So, $R'(1) \times R''(1) = \langle 6, -42, 14 \rangle$

Magnitude of the cross product:

$|R'(1) \times R''(1)| = \sqrt{6^2 + (-42)^2 + 14^2} = \sqrt{36 + 1764 + 196} = \sqrt{1996} \approx 44.69$

Calculating the curvature $\kappa(1)$:

$\kappa(1) = |R'(1) \times R''(1)| / |R'(1)|^3$

$|R'(1)|^3 = (\sqrt{62})^3 \approx (7.874)^3 \approx 488.9$

Therefore,

$\kappa(1) \approx 44.69 / 488.9 \approx 0.0914$

Interpreting the Result

The calculated curvature at $t=1$, which corresponds to the point (7,1,1), is approximately 0.0914. This relatively small value indicates that the curve is not sharply bending at this point; instead, it exhibits gentle curvature.

Features and implications:

- Smoothness: The low curvature suggests the curve transitions smoothly around the point.
- Local geometry: The space curve behaves akin to a gently bending line at

t=1.
- Visualization: Visualizing the curve around this point would show a relatively flat or mildly curved segment.

Additional Considerations and Applications

Understanding curvature is crucial in various practical applications:

- Path planning: Ensuring smooth trajectories for robotics or autonomous vehicles.
- Structural design: Analyzing stress points where curves bend sharply.
- Animation and graphics: Creating realistic motion paths that avoid abrupt turns.

Pros of this curvature analysis:

- Provides precise measures of how the curve behaves locally.
- Helps in designing or modifying curves to achieve desired bending properties.
- Enhances understanding of the geometric nature of parametric curves.

Cons or limitations:

- Curvature calculation is local; it doesn't account for global behavior.
- Requires derivatives and cross products, which can be complex for intricate functions.
- Sensitive to parameterization; different parametrizations may produce different curvature values.

Conclusion

Calculating the curvature of the vector function $R(t) = \langle 7t, t^2, t^3 \rangle$ at the point (7,1,1) reveals that the curve exhibits gentle bending at that location, with a curvature value of approximately 0.0914. This process involves differentiating the parametric functions, evaluating derivatives at the relevant parameter, computing the cross product, and applying the curvature formula. Such an analysis provides valuable insights into the geometric structure and behavior of space curves, with broad applications in science, engineering, and computer graphics. Mastery of these techniques enhances one's ability to analyze complex curves and design smooth, efficient paths in three-dimensional space.

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