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Understanding derivatives is fundamental to mastering calculus, a branch of mathematics that deals with change and motion. Derivatives help us analyze how a function behaves at any given point, giving insights into rates of change, slopes of curves, and optimization problems. In this article, we will explore the process of finding the derivative of the function G(t) = 5t + 4t algebraically, and then evaluate it at a specific point, T = -8. The goal is to provide a comprehensive, step-by-step explanation suitable for students and enthusiasts looking to deepen their understanding of derivatives.

The Significance of Derivatives in Mathematics

Before diving into the calculation, it’s important to understand why derivatives matter. Derivatives quantify how a function changes as its input changes. For example:

- In physics, derivatives represent velocity as the rate of change of position.
- In economics, they can indicate marginal cost or revenue.
- In engineering, derivatives help analyze system stability and response.

Mathematically, the derivative of a function g(t) is denoted as g'(t) or $\frac{dg}{dt}$ and provides the slope of the tangent line to the graph of g(t) at any point t.

Analyzing the Function G(t) = 5t + 4t

The function under consideration is:

$$ G(t) = 5t + 4t $$

At first glance, this looks like a simple linear function, but it’s important to understand its structure and how to differentiate it correctly.

Simplification of G(t)

Notice that the function involves two terms with similar variables:

$$ G(t) = 5t + 4t $$

These can be combined since they are like terms:

$$ G(t) = (5 + 4) t = 9t $$

Simplifying the function makes the differentiation process straightforward, but for educational purposes, we will also demonstrate how to differentiate each term separately before simplifying.

Step-by-Step Algebraic Differentiation

1. Differentiating Each Term Separately

Using the basic rules of differentiation, specifically the power rule:

$$ \frac{d}{dt} [a t^n] = a n t^{n-1} $$

for a constant a and power n.

Since both terms are linear (power of 1), their derivatives are:

- Derivative of $5t$:

$$ \frac{d}{dt} [5t] = 5 \times 1 \times t^{1-1} = 5 \times 1 \times t^{0} = 5 \times 1 = 5 $$

- Derivative of $4t$:

$$ \frac{d}{dt} [4t] = 4 \times 1 \times t^{0} = 4 \times 1 = 4 $$

2. Summing the Derivatives

Adding the derivatives of the individual terms:

$$ G'(t) = 5 + 4 = 9 $$

Thus, the derivative of the original function $G(t) = 5t + 4t$ is:

$$ G'(t) = 9 $$

3. Confirming the Simplification

Alternatively, by simplifying $G(t)$ first:

$$ G(t) = 9t $$

Differentiating:

$$ G'(t) = \frac{d}{dt} [9t] = 9 \times 1 = 9 $$

which matches our previous result.

Evaluating the Derivative at T = -8

Now that we know $G'(t) = 9$ for all values of t, including t = -8, we can evaluate the derivative at that specific point:

$$ G'(-8) = 9 $$

However, the initial statement indicates that $G'(-8) = 4$. This suggests that there might be an inconsistency or that the original function was misrepresented. To clarify this, let’s analyze the possible reasons.

Possible Discrepancy Explanation

- If the original function was G(t) = 5t + 4t, then its derivative is 9, and at $t = -8$, $G'(-8) = 9$.
- If the derivative is given as G'(-8) = 4, then perhaps the function is different or the derivative is evaluated differently.

Assuming the function is correctly given as $G(t) = 5t + 4t$, then the derivative is constantly 9, not 4. Alternatively, if the function was intended to be:

$$ G(t) = 5t + 4 $$

then the derivative would be:

$$ G'(t) = 5 $$

which is still not 4. Or, if the function was:

$$ G(t) = 4t $$

then $G'(-8) = 4$, matching the given derivative value.

Clarifying the Function and Derivative

Given the initial statement, it’s likely that the function is:

$$ G(t) = 5t + 4 $$

and the derivative is:

$$ G'(t) = 5 $$

which remains constant at 5, so at $t = -8$, the derivative is still 5, not 4.

Alternatively, perhaps the function is:

$$ G(t) = 4t $$

and the derivative is:

$$ G'(t) = 4 $$

which matches the initial derivative value at any point, including \(t =

-8\).

Final Clarification

For the purpose of this article, assuming the original function is:

$$ G(t) = 9t $$

then:

$$ G'(t) = 9 $$

and:

$$ G'(-8) = 9 $$

which contradicts the initial statement $G'(-8) = 4$. The discrepancy suggests that either the function or the derivative value is misprinted or misunderstood.

Conclusion:
Based on the algebraic differentiation process, the derivative of $G(t) = 5t + 4t$ is 9, and it remains constant across all t-values, including t = -8.

Importance of Correct Function Specification

When working with derivatives, it’s crucial to:

- Clearly identify the function involved.
- Simplify the function where possible.
- Apply differentiation rules meticulously.
- Confirm the derivative value at specific points.

Misinterpretation of the function can lead to incorrect conclusions about the derivative value at a given point.

Applications of Derivatives in Real-World Contexts

Understanding how to find derivatives algebraically has numerous applications:

Physics: Velocity and Acceleration

- Derivatives of position functions give velocity.
- Derivatives of velocity functions give acceleration.

Economics: Marginal Analysis

- Derivatives of cost or revenue functions help determine marginal costs or revenues.

Engineering: System Response

- Derivatives analyze the rate at which systems respond to inputs or changes.

Data Science and Machine Learning

- Derivatives are essential in gradient descent algorithms for optimizing models.

Practice Problems to Reinforce Learning

To solidify your understanding, try solving these problems:

1. Find the derivative of \(H(t) = 3t^2 + 2t\).
2. Evaluate the derivative \(H'(t)\) at \(t = 5\).
3. If \(F(t) = 7t - 3\), what is \(F'(t)\) and what is its value at \(t = -8\)?
4. Simplify and differentiate \(J(t) = 4t + 2t^2\).

Solutions:

1. \(H'(t) = 6t + 2\)
2. \(H'(5) = 6(5) + 2 = 30 + 2 = 32\)
3. \(F'(t) = 7\), so at \(t = -8\), \(F'(-8) = 7\)
4. Simplify: \(J(t) = 4t + 2t^2\), derivative: \(J'(t) = 4 + 4t\)

Conclusion

Mastering the process of finding derivatives algebraically is essential for anyone delving into calculus. It involves understanding the basic differentiation rules, simplifying functions when possible, and carefully evaluating derivatives at specific points. In the case of the function \(G(t) = 5t + 4t\), the derivative simplifies to a constant \(9\), which remains the same regardless of the value of \(t\), including \(t = -8\).

Always ensure the function and derivative values are correctly specified and consistent. Doing so not only enhances accuracy but also deepens your understanding of how functions behave and change.

By practicing these techniques and understanding their applications, you can develop a solid foundation in calculus that will serve you across various scientific and engineering disciplines.

Frequently Asked Questions

How do you find the derivative of G(t) = 5t + 4t

algebraically?

Combine like terms to get G(t) = 9t, then differentiate to find G'(t) = 9.

What is the derivative of G(t) = 5t + 4t at t = -8?

Since G(t) simplifies to 9t, G'(-8) = 9, not 4. The given G'(-8)=4 suggests a different interpretation or typo.

Why does the derivative of G(t) = 5t + 4t equal 9?

Because the derivative of 5t is 5, the derivative of 4t is 4, and adding them gives 9.

How do you evaluate the derivative at a specific point t = -8?

Substitute t = -8 into the derivative G'(t) = 9 to get G'(-8) = 9.

Is G'(-8) = 4 consistent with G(t) = 5t + 4t?

No, because the derivative G'(t) = 9, so G'(-8) should be 9, not 4. There may be a mistake or different function involved.

Can the derivative be 4 at t = -8 for G(t) = 5t + 4t?

No, because the derivative is constant and equals 9 everywhere, including at t = -8.

What is the correct derivative of G(t) = 5t + 4t, and its value at t = -8?

The derivative is G'(t) = 9, and G'(-8) = 9.

Additional Resources

Find The Derivative Of G(t) = 5t + 4t At T = -8 Algebraically is a fundamental problem in calculus that exemplifies the process of differentiation and the application of derivatives to evaluate the rate of change at specific points. Understanding how to find derivatives algebraically not only deepens comprehension of calculus concepts but also enhances problem-solving skills essential for advanced mathematics, physics, and engineering. This article provides a detailed, step-by-step approach to deriving the derivative of the given function and evaluating it at T = -8, along with insights into the significance and applications of derivatives.

Understanding the Function G(t) = 5t + 4t

Before diving into differentiation, it is crucial to analyze the function itself. The given function is:

$$ G(t) = 5t + 4t $$

At first glance, this appears to be a linear function composed of two terms involving the variable t. However, recognizing that these terms are similar allows for simplification, which streamlines the differentiation process.

Simplifying the Function

The function can be simplified by combining like terms:

$$ G(t) = (5 + 4) t = 9t $$

This simplification reveals that $G(t)$ is essentially a linear function with a slope of 9.

Features of the simplified function:

- Linear nature: The graph of $G(t)$ is a straight line.
- Constant slope: The coefficient of t indicates the rate of change.
- Easy to differentiate: The derivative of a linear function is simply its slope.

Algebraic Derivation of the Derivative G'(t)

Having simplified the function, the next step involves finding its derivative algebraically.

Applying Basic Differentiation Rules

The differentiation rule for a linear function $G(t) = at$ where a is a constant is straightforward:

$$ \frac{d}{dt} (a t) = a $$

Applying this to $G(t) = 9t$:

$$G'(t) = \frac{d}{dt} (9t) = 9$$

This indicates that the derivative of $G(t)$ with respect to t is a constant 9, meaning the rate of change of $G(t)$ is constant at all points.

Implication of the Derivative

- The constant derivative confirms the linearity of the function.
- The slope of the original function is 9, consistent with the derivative.
- The derivative provides information about the instantaneous rate of change at any point t.

Evaluating the Derivative at T = -8

With the derivative $G'(t) = 9$, evaluating at $T = -8$ becomes a simple process.

Direct Substitution

Since the derivative is constant, evaluating at any T yields the same result:

$$G'(-8) = 9$$

However, the problem states that $G'(-8) = 4$, which suggests a discrepancy. This could be due to an initial misinterpretation or typo in the problem statement. Given the algebraic derivation, the derivative is definitively 9, but if the problem claims $G'(-8) = 4$, perhaps the original function was different or more complex.

Possible explanations:

- The function might have been $G(t) = 4t$, whose derivative is 4.
- There may have been a typo or misprint in the problem statement.
- The problem might intend for the student to verify the derivative algebraically and then compare with the provided derivative value.

Assuming the function is indeed $G(t) = 5t + 4t$, the correct derivative is 9, and at $t = -8$, the derivative remains 9.

Confirming the Calculation

Given the algebraic steps:

- Simplify $G(t) = 5t + 4t = 9t$.
- Derivative: $G'(t) = 9$.
- At $t = -8$, $G'(-8) = 9$.

The provided value $G'(-8) = 4$ appears inconsistent unless the function was different. For the purpose of this analysis, the algebraic derivative remains 9.

Significance of Derivatives in Mathematics and Applied Fields

Understanding how to find derivatives algebraically and evaluate them at specific points is crucial across various disciplines.

Applications of Derivatives

- Physics: To determine velocity and acceleration.
- Economics: To find marginal cost and revenue.
- Engineering: For analyzing system stability.
- Biology: To model rates of change in populations.

Pros and Cons of Algebraic Differentiation

Pros:

- Straightforward for simple functions.
- Provides exact derivatives.
- Enhances understanding of function behavior.

Cons:

- Can be complex for complicated functions.
- Prone to algebraic errors if not careful.
- Less efficient for higher-order derivatives or non-elementary functions.

Features and Best Practices in Differentiation

To ensure accurate and efficient differentiation, consider the following features and best practices:

- Familiarity with differentiation rules: Power rule, product rule, quotient rule, chain rule.
- Simplification of functions before differentiation: Reduces errors and complexity.
- Verification: Always verify results, especially when discrepancies arise.
- Understanding the context: Recognize whether the derivative should be constant or variable.

Conclusion

The process of finding the derivative of $G(t) = 5t + 4t$ algebraically highlights fundamental calculus principles. Simplifying the function to $9t$ makes differentiation straightforward, with the derivative being a constant 9. While the initial problem suggests evaluating $G'(-8)$ as 4, algebraically, it is 9, emphasizing the importance of precise function definitions. Derivatives serve as powerful tools for understanding the behavior of functions, measuring rates of change, and applying mathematical insights across various fields. Mastery of algebraic differentiation not only solidifies foundational calculus skills but also prepares students for tackling more complex mathematical challenges with confidence.

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