Find The Exact Value Of Sin(x/2) If Sec(x) = 7/3 And 400 < X < 430

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Introduction

Understanding the relationships between trigonometric functions is essential in solving complex problems in mathematics. The problem at hand involves finding the exact value of $\sin\left(\frac{x}{2}\right)$ given that $\sec(x) = \frac{7}{3}$ and that x lies within the interval $400^\circ < x < 430^\circ$. This problem combines knowledge of reciprocal trigonometric functions, half-angle formulas, and angle measurement conversions. As we proceed, we will analyze the given data, determine the value of $\sin x$, and then use the half-angle formula to find $\sin\left(\frac{x}{2}\right)$.

Understanding the Given Data

What is $\sec(x)$?

The secant function, $\sec x$, is the reciprocal of the cosine function:

$$
\sec x = \frac{1}{\cos x}
$$

Given $\sec x = \frac{7}{3}$, it follows that:

$$
\cos x = \frac{3}{7}
$$

Since $\sec x$ is positive, $\cos x$ is positive as well.

Determining the Quadrant of x

The interval for x is $400^\circ < x < 430^\circ$. Since the standard range for angles in trigonometry is 0° to 360°, we can find the equivalent angle within this range by subtracting 360°:

$$
x_{\text{equiv}} = x - 360^\circ
$$

For x between 400° and 430°:

$$
400^\circ - 360^\circ = 40^\circ
$$

$$
430^\circ - 360^\circ = 70^\circ
$$

Thus, x is coterminal with an angle in the interval $40^\circ < x - 360^\circ < 70^\circ$. Therefore, the equivalent angle x_{ref} in the first revolution is between 40° and 70°.

Since $\cos x$ is positive ($\frac{3}{7}$) and cosine is positive in the first and fourth quadrants, but given the interval, the angle x is in the first quadrant (if considering the standard 0° to 360° range) or possibly in the second quadrant if the angle is more than 180°.

Let's verify the quadrant more precisely:

- For x between 400° and 430°, subtracting 360° gives an angle between 40° and 70°.

- Since this is between 0° and 90°, x is in the first quadrant for these coterminal angles.

However, because the original x is between 400° and 430°, and cosine is positive here, the angle is in the first or fourth quadrant. But because the angle $x - 360^\circ$ is between 40° and 70°, the original x is between 400° and 430° – in the fourth quadrant (since $360^\circ < x < 430^\circ$), where cosine is positive and sine is negative.

Summary:

- $\cos x = \frac{3}{7}$

- x is in the fourth quadrant (since x is between 400° and 430°), where $\sin x < 0$.

Calculating $\sin x$

Using Pythagoras Theorem

Given $\cos x = \frac{3}{7}$, we can find $\sin x$ using the Pythagorean identity:

$$
\sin^2 x + \cos^2 x = 1
$$

Plugging in the known value:

$$
\sin^2 x + \left(\frac{3}{7}\right)^2 = 1
$$

$$
\sin^2 x + \frac{9}{49} = 1
$$

Express 1 as $\frac{49}{49}$:

$$
\sin^2 x = \frac{49}{49} - \frac{9}{49} = \frac{40}{49}
$$

Taking the square root:

$$
\sin x = \pm \sqrt{\frac{40}{49}} = \pm \frac{\sqrt{40}}{7}

\]

Simplify \(\sqrt{40}\):

\[
\sqrt{40} = \sqrt{4 \times 10} = 2 \sqrt{10}
\]

Thus:

\[
\sin x = \pm \frac{2 \sqrt{10}}{7}
\]

Selecting the correct sign:

Since \(x\) is in the fourth quadrant (\(400^\circ\) to \(430^\circ\)), sine is negative in this quadrant:

\[
\boxed{
\sin x = - \frac{2 \sqrt{10}}{7}
}
\]

Applying the Half-Angle Formula for \(\sin\left(\frac{x}{2}\right)\)

Half-Angle Formula

The half-angle formula for sine is:

$$
\sin \left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}
$$

The sign depends on the quadrant in which $\frac{x}{2}$ lies.

Determining the Sign of $\sin \left(\frac{x}{2}\right)$

Calculating $\frac{x}{2}$

Given x is between 400° and 430°:

$$
\frac{400^\circ}{2} = 200^\circ
$$

$$
\frac{430^\circ}{2} = 215^\circ
$$

Therefore, $\frac{x}{2}$ lies between 200° and 215°.

Quadrant of $\frac{x}{2}$:

- 200° and 215° are in the third quadrant (180° to 270°), where sine is negative.

Thus:

$$
\sin \left(\frac{x}{2}\right) < 0
$$

Final sign:

$$
\sin \left(\frac{x}{2}\right) = - \sqrt{\frac{1 - \cos x}{2}}
$$

Calculating $\sin \left(\frac{x}{2}\right)$

Substitute $\cos x = \frac{3}{7}$

$$
\sin \left(\frac{x}{2}\right) = - \sqrt{\frac{1 - \frac{3}{7}}{2}}
$$

Simplify numerator:

$$
1 - \frac{3}{7} = \frac{7}{7} - \frac{3}{7} = \frac{4}{7}
$$

\]

Divide by 2:

\[
\frac{\frac{4}{7}}{2} = \frac{4}{7} \times \frac{1}{2} = \frac{4}{14} = \frac{2}{7}
\]

So:

\[
\sin \left(\frac{x}{2}\right) = - \sqrt{\frac{2}{7}}
\]

Express the radical as simplified as possible:

\[
\boxed{
\sin \left(\frac{x}{2}\right) = - \frac{\sqrt{14}}{7}
}
\]

Summary of the Solution

- Given \(\sec x = \frac{7}{3}\), \(\cos x = \frac{3}{7}\).

- Since \(x\) is in the interval \(400^\circ < x < 430^\circ\), it is in the fourth quadrant, where \(\sin x\) is negative.

- Calculated \(\sin x = - \frac{2 \sqrt{10}}{7}\).

- The half-angle formula for sine:

\[
\sin \left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}
\]

- Determined \(\frac{x}{2}\) is in the third quadrant (\(200^\circ\) to \(215^\circ\)), where sine is negative.

- Therefore, the exact value of \(\sin \left

Frequently Asked Questions

Given that sec(x) = 7/3 and 400° < x < 430°, what is the exact value of sin(x/2)?

The exact value of sin(x/2) is 4/7.

How do you find sin(x/2) when sec(x) = 7/3 and x is between 400° and 430°?

First, find cos(x) from sec(x) = 7/3, then use the half-angle formula: sin(x/2) = ±□[(1 - cos(x))/2].

What is the value of cos(x) if sec(x) = 7/3?

cos(x) = 3/7.

Since sec(x) = 7/3, is x in the first or second quadrant between 400° and 430°?

x is in the second quadrant because sec(x) is positive and x is between 400° and 430°, which corresponds to angles in the second quadrant.

How do you determine the sign of sin(x/2) in this context?

Because x is in the second quadrant and x/2 will be in the first or second quadrant, sin(x/2) is positive in this case.

What is the step-by-step process to compute sin(x/2) here?

1. Find cos(x) from sec(x). 2. Use the half-angle formula: sin(x/2) = □[(1 - cos(x))/2]. 3. Plug in the value of cos(x) and simplify.

Can you provide the numerical value of sin(x/2) given the data?

Yes, sin(x/2) = 4/7.

Why is the half-angle formula applicable in this problem?

Because we have sec(x) and need to find sin(x/2), the half-angle formulas are the appropriate tools for this half-angle calculation.

What are the key trigonometric identities used in solving this problem?

The key identities are sec(x) = 1/cos(x), and the half-angle formula: sin(x/2) = ±□[(1 - cos(x))/2].

Additional Resources

Find The Exact Value Of Sin(x/2) If Sec(x) = 7/3 And 400 < X < 430

Understanding how to find the exact value of $\sin(x/2)$ when given the secant of x and a specific interval for x is a common challenge in trigonometry. This problem combines multiple concepts, including reciprocal identities, the half-angle formula, and the importance of understanding the range of the angle x. Carefully analyzing each step involved provides a comprehensive grasp of trigonometric functions and their properties, which are fundamental to many applications in mathematics and physics.

Introduction to the Problem

The problem at hand is to determine the exact value of $\sin(x/2)$ given:

- $\sec(x) = \frac{7}{3}$,
- $400 < x < 430$.

At first glance, the problem requires us to:

1. Find $\sin(x)$ from the given $\sec(x)$,
2. Deduce the value of $\sin(x/2)$ using half-angle formulas,
3. Understand the implications of the interval for x,
4. Ensure the correct sign for $\sin(x/2)$ based on the quadrant of x.

This process involves combining identities with interval analysis to arrive at the exact value.

Understanding the Given Data

What is $\sec(x)$?

$\sec(x)$ is the reciprocal of $\cos(x)$, so:

$$
\sec(x) = \frac{1}{\cos(x)}.
$$

Given $\sec(x) = \frac{7}{3}$, this implies:

$$
\cos(x) = \frac{3}{7}.
$$

Since $\sec(x) > 0$, $\cos(x)$ is positive, indicating x is in either the first or fourth quadrant.

Interval for x: $400 < x < 430$

Angles are usually measured in degrees unless specified otherwise. Assuming degrees, the interval:

- x is between 400° and 430°.

Angles beyond 360° are coterminal with angles within the standard interval $[0°, 360°)$:

$$
x' = x - 360°,
$$

\]

where:

- When \(x = 400°\), \(x' = 40°\),
- When \(x = 430°\), \(x' = 70°\).

Thus, \(x\) lies in the interval \([400°, 430°]\), which corresponds to coterminal angles between \(40°\) and \(70°\).

Key Point: The angles \(x\) in this interval are coterminal with angles between \(40°\) and \(70°\). Since \(\cos(x)\) is positive in this range, and \(\cos(40°)\) and \(\cos(70°)\) are positive, the initial assumption about the quadrants aligns with these values.

Step-by-Step Solution

Step 1: Find \(\sin(x)\) using \(\cos(x)\)

Given:

\[
\cos(x) = \frac{3}{7}
\]

and knowing that:

\[
\sin^2(x) + \cos^2(x) = 1,

\]

we can solve for \(\sin(x)\):

\[

\sin^2(x) = 1 - \left(\frac{3}{7}\right)^2 = 1 - \frac{9}{49} = \frac{49}{49} - \frac{9}{49} = \frac{40}{49}.

\]

Therefore:

\[

\sin(x) = \pm \sqrt{\frac{40}{49}} = \pm \frac{\sqrt{40}}{7} = \pm \frac{2\sqrt{10}}{7}.

\]

Next, determine the sign of \(\sin(x)\).

Considering the interval:

- Since \(x\) coterminal with angles between \(40°\) and \(70°\),

- \(\sin(x)\) is positive in this range because sine is positive in the first quadrant (\(0°\) to \(90°\)).

Given this, we take:

\[

\sin(x) = \frac{2\sqrt{10}}{7}.

\]

Step 2: Find $\sin(x/2)$ using the half-angle formula

The half-angle formula for sine is:

$$
\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos(x)}{2}}.
$$

Determine the sign:

- Since x is between $40°$ and $70°$, $x/2$ is between $20°$ and $35°$,
- $\sin(x/2)$ is positive in this range because sine is positive in the first quadrant.

Thus, we take the positive root:

$$
\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1 - \cos(x)}{2}}.
$$

Substitute $\cos(x) = \frac{3}{7}$:

$$
\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1 - \frac{3}{7}}{2}} = \sqrt{\frac{\frac{7}{7} - \frac{3}{7}}{2}} = \sqrt{\frac{\frac{4}{7}}{2}} = \sqrt{\frac{4/7}{2}}.
$$

Simplify numerator and denominator:

$$
\sin\left(\frac{x}{2}\right) = \sqrt{\frac{4}{7} \times \frac{1}{2}} = \sqrt{\frac{4}{14}} = \sqrt{\frac{2}{7}}.
$$

Expressed in simplest radical form:

\[
\boxed{\sin\left(\frac{x}{2}\right) = \sqrt{\frac{2}{7}}}.
\]

Final Answer and Validation

The exact value of \(\sin(x/2)\), given \(\sec(x) = 7/3\) and \(400 < x < 430\), is:

\[
\boxed{\sin\left(\frac{x}{2}\right) = \sqrt{\frac{2}{7}}}.
\]

This conclusion aligns with the interval considerations and the positivity of the sine function in the relevant quadrants.

Additional Insights and Considerations

Why the Sign Matters

- The sign of the half-angle sine depends on the quadrant where \(x/2\) lies.
- Since \(x\) is between \(400°\) and \(430°\), \(x/2\) is between \(200°\) and \(215°\).

- Angles between \(180°\) and \(360°\) are in the third and fourth quadrants.
- \(\sin(x/2)\) in the third or fourth quadrants can be positive or negative depending on the specific angle.

In this case:

- \(x/2\) is between \(200°\) and \(215°\),
- \(\sin(200°)\) and \(\sin(215°)\) are negative,
- but earlier, we deduced the positive root because the original \(x\) was in the first quadrant range after coterminal adjustment.

Thus, the sign of \(\sin(x/2)\) should be negative, considering the actual angle range.

Correction:

Given the interval, the half-angle \(x/2\) falls into the third quadrant (\(180°\) to \(270°\)), where sine is negative. Therefore:

\[
\sin\left(\frac{x}{2}\right) = - \sqrt{\frac{2}{7}}.
\]

Final refined answer:

\[
\boxed{\sin\left(\frac{x}{2}\right) = - \sqrt{\frac{2}{7}}}.
\]

Summary and Key Takeaways

- The problem involves understanding reciprocal identities, interval analysis, and half-angle formulas.
- Correctly identifying the quadrant of the original angle \(x\) and the half-angle \(x/2\) is crucial for determining the sign.
- The exact value derived is \(\pm \sqrt{\frac{2}{7}}\), with the sign depending on the quadrant: negative in this case.
- The approach demonstrates the importance of breaking down complex trigonometric problems into manageable steps.

Features and Pros/Cons

Features:

- Utilizes fundamental identities and formulas in trigonometry.
- Emphasizes the importance of quadrant analysis.
- Demonstrates converting between degrees and coterminal angles.
- Provides a clear step-by-step approach to solving for exact trigonometric values.

Pros:

- Enhances understanding of half-angle formulas.
- Reinforces the significance of sign determination based on angle quadrants.
- Offers a precise, algebraic solution rather than an approximation.

Cons:

Find The Exact Value Of Sin X 2 If Sec X 7 3 And 400 Lt X Lt 430

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