# Find The Following Limit Using L'Hpital's Rule: Lim X0 + (tanx)^tanx

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When encountering limits involving indeterminate forms such as 0^0, 1^∞, or ∞^0, standard algebraic techniques often fall short. In such cases, L'Hôpital's Rule becomes an invaluable tool for evaluating these complex limits. The problem at hand—finding the limit of $x \to 0^+$ for the expression $( \tan x )^{\tan x}$—is a classic example where direct substitution results in an indeterminate form. This article aims to guide you through the detailed process of solving this limit using L'Hôpital's Rule, supplemented with essential mathematical insights and step-by-step calculations.

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## Understanding the Limit and Its Indeterminate Nature

Before jumping into the solution, it’s crucial to analyze the behavior of the function as $x \to 0^+$. The limit in question is:

$$
\lim_{x \to 0^+} (\tan x)^{\tan x}
$$

At first glance, substituting $x = 0$ directly:

- $\tan 0 = 0$
- $(\tan 0)^{\tan 0} = 0^{0}$

The expression $0^{0}$ is an indeterminate form, which means that a straightforward substitution does not give a meaningful answer. Instead, it indicates that the limit may tend to a finite value, zero, or infinity, and requires further analysis.

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## Transforming the Expression for Easier Evaluation

To evaluate limits involving exponents like $a^{b}$, a common method is to take the natural logarithm. This transformation simplifies the problem because it allows us to handle the exponent separately.

Define:

\[

L = \lim_{x \to 0^+} (\tan x)^{\tan x}
\]

Then take the natural logarithm:

\[
\ln L = \lim_{x \to 0^+} \tan x \cdot \ln(\tan x)
\]

If we can find \( \ln L \), then the original limit \( L \) can be obtained by exponentiation:

\[
L = e^{\ln L}
\]

Thus, our problem reduces to evaluating:

\[
\lim_{x \to 0^+} \tan x \cdot \ln(\tan x)
\]

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## Analyzing the Transformed Limit

As \( x \to 0^+ \):

- \( \tan x \to 0^+ \)
- \( \ln(\tan x) \to -\infty \)

Therefore, the expression:

\[
\lim_{x \to 0^+} \tan x \cdot \ln(\tan x)
\]

becomes a \( 0 \times (-\infty) \) form, which is indeterminate. To evaluate this limit, we need to manipulate it into a form suitable for applying L'Hôpital's Rule.

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## Rewriting the Limit for L'Hôpital's Rule

Express the limit as a quotient:

\[

\lim_{x \to 0^+} \tan x \cdot \ln(\tan x) = \lim_{x \to 0^+} \frac{\ln(\tan x)}{\cot x}
\]

since:

\[
\frac{\ln(\tan x)}{\cot x} = \frac{\ln(\tan x)}{\frac{\cos x}{\sin x}}
\]

Alternatively, since \( \cot x = \frac{1}{\tan x} \), the limit becomes:

\[
\lim_{x \to 0^+} \frac{\ln(\tan x)}{\frac{1}{\tan x}}
\]

which simplifies to:

\[
\lim_{x \to 0^+} \tan x \cdot \ln(\tan x)
\]

But this is the original form. To apply L'Hôpital's Rule, we need to ensure the limit is in a \( 0/0 \) or \( \infty/\infty \) form. Let's consider:

\[
\lim_{x \to 0^+} \frac{\ln(\tan x)}{\frac{1}{\tan x}}
\]

As \( x \to 0^+ \):

- \( \tan x \to 0^+ \)
- \( \ln(\tan x) \to -\infty \)
- \( \frac{1}{\tan x} \to +\infty \)

So, the numerator approaches \( -\infty \), and the denominator approaches \( +\infty \). The limit is of the form \( -\infty / +\infty \), which simplifies to an indeterminate form suitable for L'Hôpital's Rule.

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# Applying L'Hôpital's Rule

Now, we can differentiate numerator and denominator separately with respect to \( x \):

- Numerator: \( \ln(\tan x) \)

- Denominator: \( \frac{1}{\tan x} \)

Calculations:

1. Derivative of numerator:

$$
\frac{d}{dx} \left[ \ln(\tan x) \right] = \frac{1}{\tan x} \cdot \sec^2 x
$$

2. Derivative of denominator:

$$
\frac{d}{dx} \left[ \frac{1}{\tan x} \right] = - \frac{1}{\tan^2 x} \cdot \sec^2 x
$$

Applying L'Hôpital's Rule:

$$
\lim_{x \to 0^+} \frac{\ln(\tan x)}{\frac{1}{\tan x}} = \lim_{x \to 0^+} \frac{\frac{\sec^2 x}{\tan x}}{ - \frac{\sec^2 x}{\tan^2 x} } = \lim_{x \to 0^+} \frac{\frac{\sec^2 x}{\tan x}}{ - \frac{\sec^2 x}{\tan^2 x} }
$$

Simplify numerator and denominator:

$$
= \lim_{x \to 0^+} \frac{\sec^2 x / \tan x}{ - \sec^2 x / \tan^2 x } = \lim_{x \to 0^+} \frac{1 / \tan x}{ - 1 / \tan^2 x } = \lim_{x \to 0^+} \frac{1 / \tan x}{ - 1 / \tan^2 x }
$$

Further simplifying:

$$
= \lim_{x \to 0^+} \left( \frac{1 / \tan x}{ - 1 / \tan^2 x } \right) = \lim_{x \to 0^+} \left( \frac{\tan^2 x}{ - \tan x } \right) = \lim_{x \to 0^+} \left( - \tan x \right)
$$

Now, as $x \to 0^+$:

$$
- \tan x \to 0
$$

Thus, the limit of the logarithmic expression is 0:

$$
\lim_{x \to 0^+} \tan x \ln(\tan x) = 0
$$

---

## Determining the Final Limit

Recall that:

$$
\ln L = \lim_{x \to 0^+} \tan x \ln(\tan x) = 0
$$

Therefore:

$$
L = e^{0} = 1
$$

This means:

$$
\boxed{
\lim_{x \to 0^+} (\tan x)^{\tan x} = 1
}
$$

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## Summary of the Solution Steps

To summarize, the process involved the following key steps:

1. Recognize the indeterminate form $0^0$ and transform the limit by taking the natural logarithm.
2. Express the transformed limit in a form suitable for applying L'Hôpital's Rule, such as a quotient that results in $0/0$ or $\infty/\infty$.
3. Apply L'Hôpital's Rule by differentiating numerator and denominator separately.
4. Simplify the resulting expression and evaluate the limit as $x \to 0^+$.
5. Exponentiate the result to find the original limit.

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# Additional Tips for Applying L'Hôpital's Rule Effectively

While the above example demonstrates a straightforward application, here are some general tips:

- Always check the form of the limit before applying L'Hôpital's Rule.
- If the limit remains indeterminate after the first application, consider applying L'Hôpital's Rule multiple times.
- Transform complex expressions using logarithms or other algebraic manipulations to simplify the limit evaluation.
- Be cautious with domain restrictions, especially when dealing

## Frequently Asked Questions

### How do I apply L'Hôpital's Rule to evaluate the limit of (tan x)^tan x as x approaches 0+?

First, rewrite the limit as an exponential: $\lim_{x\to 0^+} (\tan x)^{\tan x} = \lim_{x\to 0^+} e^{\tan x \cdot \ln(\tan x)}$. Then analyze the exponent $\lim_{x\to 0^+} \tan x \cdot \ln(\tan x)$. Since both $\tan x \to 0$ and $\ln(\tan x) \to -\infty$, the product results in an indeterminate form $0 \cdot (-\infty)$. To handle this, rewrite as $\ln(\tan x) / (1 / \tan x)$ and apply L'Hôpital's Rule to this new form.

### What is the key step in solving lim x→0+ (tan x)^tan x using L'Hôpital's Rule?

The key step is to take the natural logarithm of the original expression to convert the power into a product, simplify the limit of the exponent, and then apply L'Hôpital's Rule to that exponent's limit, which often results in an indeterminate form suitable for differentiation.

### Why do we rewrite (tan x)^tan x as e^{tan x · ln(tan x)} when finding the limit?

Rewriting it as an exponential allows us to handle the limit more easily by focusing on the exponent, which simplifies the analysis of the indeterminate form, especially when both the base and exponent approach zero or infinity.

## What is the limit of tan x as x approaches 0+?

The limit of tan x as x approaches 0+ is 0.

## What is the limit of ln(tan x) as x approaches 0+?

As x approaches 0+, tan x approaches 0+, so ln(tan x) approaches -∞.

## How do you evaluate the limit of tan x · ln(tan x) as x approaches 0+?

Express the limit as a quotient, such as ln(tan x) / (1 / tan x), and then apply L'Hôpital's Rule to evaluate the resulting indeterminate form.

## What is the final value of the original limit lim x→0+ (tan x)^tan x?

The limit is 1, because after analysis, the exponent tends to 0, and e^0 = 1.

## Can L'Hôpital's Rule be directly applied to (tan x)^tan x without logarithmic transformation?

No, because (tan x)^tan x is not directly in a form suitable for L'Hôpital's Rule. We need to take the logarithm first to convert the problem into a form involving indeterminate ratios.

## What does the indeterminate form 0 · (−∞) imply in this limit problem?

It indicates that the limit of the exponent is an indeterminate form, which requires further analysis—usually by rewriting as a quotient and applying L'Hôpital's Rule—to find the actual limit.

## Is it necessary to use L'Hôpital's Rule multiple times in

### this problem?

Typically, applying L'Hôpital's Rule once to the transformed limit suffices. Additional applications might be needed if the resulting expressions remain indeterminate, but often a single application is enough after proper rewriting.

## Additional Resources

Find The Following Limit Using L'Hôpital's Rule: Lim x → 0+ (tan x)^{tan x}

Understanding the behavior of functions as variables approach specific points is a foundational aspect of calculus. Among the many tools mathematicians employ, L'Hôpital's Rule stands out as a powerful technique for evaluating indeterminate forms—limits that initially appear undefined or ambiguous. In this article, we delve into a classic yet intriguing problem: finding the limit of the function \( ( \tan x )^{\tan x} \) as \( x \) approaches zero from the right, i.e., \( x \to 0^+ \). This exploration will not only demonstrate the practical application of L'Hôpital's Rule but also deepen our understanding of how exponential and trigonometric functions interact near critical points.

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The Challenge of Limits Involving Indeterminate Forms

When approaching limits in calculus, one encounters expressions that seem to defy straightforward evaluation. Such expressions often take on the forms \( 0/0 \) or \( \infty/\infty \), known as indeterminate forms. Recognizing these situations is crucial because they signal the need for more advanced techniques, such as L'Hôpital's Rule.

The specific limit under review involves a function raised to a power that itself depends on the variable:

\[
\lim_{x \to 0^+} (\tan x)^{\tan x}
\]

At first glance, as \( x \to 0^+ \):

- \( \tan x \to 0^+ \)
- Therefore, the base \( \tan x \to 0^+ \)
- The exponent \( \tan x \to 0^+ \)

This results in a classic indeterminate form of \( 0^0 \), which is undefined in a strict algebraic sense. Nonetheless, in calculus, such forms can be evaluated by transforming the expression into a more manageable form, often involving logarithms.

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## Transforming the Expression: From Power to Exponential

The key to evaluating the limit lies in rewriting the original expression into a form that simplifies analysis. Recall that for any positive $a$ and $b$:

$$
a^b = e^{b \ln a}
$$

Applying this to our function:

$$
(\tan x)^{\tan x} = e^{\tan x \cdot \ln (\tan x)}
$$

Thus, our limit becomes:

$$
\lim_{x \to 0^+} e^{\tan x \cdot \ln (\tan x)}
$$

Since the exponential function is continuous, the limit of the entire expression equals the exponential of the limit of the exponent, provided that the latter exists:

$$
\lim_{x \to 0^+} (\tan x)^{\tan x} = e^{\lim_{x \to 0^+} \left( \tan x \cdot \ln (\tan x) \right)}
$$

Hence, the problem reduces to finding:

$$
L = \lim_{x \to 0^+} \left( \tan x \cdot \ln (\tan x) \right)
$$

---

## Evaluating the Inner Limit: Handling $\tan x \cdot \ln (\tan x)$

As $x \to 0^+$:

- $\tan x \to 0^+$
- $\ln (\tan x) \to -\infty$

Therefore, the product $\tan x \cdot \ln (\tan x)$ takes the form:

$$
0 \times (-\infty) \to 0 \times -\infty
$$

which is an indeterminate form of type $0 \times -\infty$. To analyze it further, we can rewrite the product as a quotient to enable the use of L'Hôpital's Rule:

$$
\tan x \cdot \ln (\tan x) = \frac{\ln (\tan x)}{1 / \tan x}
$$

Now, as $x \to 0^+$:

- $\ln (\tan x) \to -\infty$
- $1 / \tan x \to +\infty$

So, the limit transforms into:

$$
L = \lim_{x \to 0^+} \frac{\ln (\tan x)}{1 / \tan x}
$$

This is of the indeterminate form $\frac{-\infty}{\infty}$, suitable for applying L'Hôpital's Rule.

---

Applying L'Hôpital's Rule: Differentiating Numerator and Denominator

L'Hôpital's Rule states that if the limit $\lim_{x \to a} \frac{f(x)}{g(x)}$ yields an indeterminate form $\frac{0}{0}$ or $\frac{\pm \infty}{\pm \infty}$, then:

$$
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}
$$

assuming this new limit exists.

Calculating derivatives:

- Numerator:

$$
f(x) = \ln (\tan x), \quad f'(x) = \frac{1}{\tan x} \cdot \sec^2 x
$$

- Denominator:

$$
g(x) = \frac{1}{\tan x}, \quad g'(x) = - \frac{\sec^2 x}{\tan^2 x}
$$

Thus, applying L'Hôpital's Rule:

$$
L = \lim_{x \to 0^+} \frac{f'(x)}{g'(x)} = \lim_{x \to 0^+} \frac{\frac{\sec^2 x}{\tan x}}{-\frac{\sec^2 x}{\tan^2 x}}
$$

Simplify numerator and denominator:

\[
L = \lim_{x \to 0^+} \frac{\sec^2 x / \tan x}{- \sec^2 x / \tan^2 x} = \lim_{x \to 0^+}
\frac{\sec^2 x}{\tan x} \times \frac{\tan^2 x}{- \sec^2 x}
\]

Canceling common terms:

\[
L = \lim_{x \to 0^+} \left( \frac{\cancel{\sec^2 x}}{\tan x} \times \frac{\tan^2 x}{-
\cancel{\sec^2 x}} \right) = \lim_{x \to 0^+} \frac{\tan^2 x}{- \tan x}
\]

Simplify:

\[
L = - \lim_{x \to 0^+} \tan x
\]

As \( x \to 0^+ \), \( \tan x \to 0^+ \). Therefore:

\[
L = -0 = 0
\]

---

Final Calculation: Recovering the Original Limit

Recall that:

\[
\lim_{x \to 0^+} (\tan x)^{\tan x} = e^{L}
\]

and we've just determined:

\[
L = 0
\]

Therefore:

\[
\boxed{
\lim_{x \to 0^+} (\tan x)^{\tan x} = e^{0} = 1
}
\]

---

Broader Insights and Applications

This problem exemplifies how transforming complex limits involving powers and logarithms can simplify the evaluation process. The key steps involved:

- Recognizing the indeterminate form $0^0$
- Applying logarithmic transformation to convert the power into a product
- Rewriting the product as a quotient suitable for L'Hôpital's Rule
- Carefully differentiating numerator and denominator
- Simplifying derivatives to find the inner limit

Such techniques are essential in calculus, especially when dealing with limits involving exponential, logarithmic, and trigonometric functions. They are widely applicable in fields like physics, engineering, and economics, where understanding the behavior of functions near critical points is vital.

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Summary

In conclusion, the limit:

$$
\boxed{
\lim_{x \to 0^+} (\tan x)^{\tan x} = 1
}
$$

demonstrates the elegance and utility of calculus tools. By transforming the original expression using logarithms and then applying L'Hôpital's Rule, we navigated through indeterminate forms to arrive at a definitive answer. Such problems reinforce the importance of strategic transformations and the power of derivatives in analyzing the subtle behavior of functions near points of interest.

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Final Remarks

Mastering these techniques enhances problem-solving skills and deepens mathematical intuition. Whether dealing with limits involving exponential powers, roots, or complex combinations, the principles illustrated here serve as a robust framework for tackling similar challenges across mathematical disciplines.

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