Find The Inverse Laplace Transform Of The Function H(s) = As + B . (s)2 +2

Find The Inverse Laplace Transform Of The Function H(s) = As + B / (s)^2 + 2

The process of finding the inverse Laplace transform of a given function is a fundamental skill in engineering, physics, and mathematics, especially when dealing with differential equations and systems analysis. Today, we will focus on determining the inverse Laplace transform of the function:

$$ H(s) = \frac{As + B}{s^2 + 2} $$

Understanding how to approach this problem involves recognizing the structure of the function, decomposing it into simpler parts, and applying known Laplace transform pairs.

Understanding the Function H(s) = (As + B) / (s^2 + 2)

Before diving into the solution, it is essential to analyze the components of the function. The denominator, $s^2 + 2$, resembles the standard form associated with sinusoidal functions, particularly those involving sine or cosine.

The numerator, $As + B$, is a linear combination of s and a constant, which suggests that the inverse transform may involve derivatives or shifts of basic functions.

Key Concepts for Finding the Inverse Laplace Transform

1. Recognizing Standard Laplace Transform Pairs

The inverse Laplace transform relies heavily on known pairs, including:

- $\mathcal{L}^{-1}\left\{\frac{1}{s^2 + a^2}\right\} = \frac{\sin at}{a}$
- $\mathcal{L}^{-1}\left\{\frac{s}{s^2 + a^2}\right\} = \cos at$

Given the denominator $s^2 + 2$, note that:

$$a^2 = 2 \implies a = \sqrt{2}$$

Thus, the basic inverse transforms involve $\sin(\sqrt{2} t)$ and $\cos(\sqrt{2} t)$.

2. Decomposition of the Numerator

Since numerator $As + B$ includes both s and a constant term, it is advantageous to split $H(s)$ into two parts:

$$
H(s) = \frac{As}{s^2 + 2} + \frac{B}{s^2 + 2}
$$

This separation simplifies the process, allowing us to find inverse transforms for each part individually.

Step-by-Step Solution

Let's proceed step by step to find the inverse Laplace transform of $H(s)$.

Step 1: Express H(s) as a sum of simpler fractions

Rewrite:

$$
H(s) = \frac{As}{s^2 + 2} + \frac{B}{s^2 + 2}
$$

This allows us to handle each term separately.

Step 2: Recall inverse transforms for each term

Using standard Laplace pairs:

- For $\frac{s}{s^2 + a^2}$:

$$
\mathcal{L}^{-1}\left\{\frac{s}{s^2 + a^2}\right\} = \cos at
$$

- For $\frac{1}{s^2 + a^2}$:

$$
\mathcal{L}^{-1}\left\{\frac{1}{s^2 + a^2}\right\} = \frac{\sin at}{a}
$$

\]

Applying these with \(a = \sqrt{2} \):

- \(\mathcal{L}^{-1}\left\{\frac{s}{s^2 + 2}\right\} = \cos (\sqrt{2} t) \)

- \(\mathcal{L}^{-1}\left\{\frac{1}{s^2 + 2}\right\} = \frac{\sin (\sqrt{2} t)}{\sqrt{2}} \)

Step 3: Find inverse Laplace transforms of each term

- For \(\frac{As}{s^2 + 2} \):

\[
\mathcal{L}^{-1}\left\{\frac{As}{s^2 + 2}\right\} = A \cos (\sqrt{2} t)
\]

- For \(\frac{B}{s^2 + 2} \):

\[
\mathcal{L}^{-1}\left\{\frac{B}{s^2 + 2}\right\} = B \times \frac{\sin (\sqrt{2} t)}{\sqrt{2}}
\]

Step 4: Combine the results to obtain the inverse transform

Hence, the inverse Laplace transform of \(H(s) \) is:

\[
h(t) = A \cos (\sqrt{2} t) + \frac{B}{\sqrt{2}} \sin (\sqrt{2} t)
\]

This result provides a complete time-domain representation corresponding to the original Laplace function.

Special Cases and Additional Considerations

1. When A or B are Zero

- If $A = 0$, the inverse simplifies to:

$$
h(t) = \frac{B}{\sqrt{2}} \sin (\sqrt{2} t)
$$

- If $B = 0$, then:

$$
h(t) = A \cos (\sqrt{2} t)
$$

2. Impact of Coefficient Values

The coefficients A and B determine the amplitude and phase of the sinusoidal response. Adjusting these parameters alters the system's response in frequency domain and time domain accordingly.

Applications of the Inverse Laplace Transform in Engineering and Physics

The ability to find the inverse Laplace transform of functions like $H(s) = \frac{As + B}{s^2 + 2}$ is crucial in various fields:

- **Control Systems:** Analyzing system responses to inputs
- **Electrical Engineering:** Solving circuit differential equations
- **Mechanical Systems:** Determining displacement or velocity over time
- **Signal Processing:** Reconstructing signals from their Laplace domain representations

Understanding how to manipulate and invert Laplace transforms enables engineers and scientists to predict system behavior accurately and design more efficient systems.

Summary

In summary, finding the inverse Laplace transform of $H(s) = \frac{As + B}{s^2 + 2}$ involves:

- Recognizing the standard forms associated with sinusoidal functions
- Decomposing the numerator into parts corresponding to known inverse transforms

- Applying standard pairs to find the inverse in the time domain
- Combining the results to obtain the final expression: $h(t) = A \cos (\sqrt{2} t) + \frac{B}{\sqrt{2}} \sin (\sqrt{2} t)$

Mastering these techniques enhances problem-solving skills in differential equations and system analysis, making it an essential part of engineering curricula and applied sciences.

Final Thoughts

Whether you are analyzing electrical circuits, mechanical vibrations, or control systems, understanding how to find the inverse Laplace transform of functions like $H(s) = \frac{As + B}{s^2 + 2}$ is invaluable. Proper decomposition, recognition of standard forms, and application of transform pairs streamline the process, allowing for quick and accurate solutions that translate complex frequency domain functions into meaningful time-domain responses.

By mastering these methods, you can confidently tackle a wide range of problems involving differential equations and system dynamics, ultimately enhancing your analytical capabilities in engineering and scientific disciplines.

Frequently Asked Questions

What is the inverse Laplace transform of the function $H(s) = (A s + B) / (s^2 + 2)$?

The inverse Laplace transform is $h(t) = A \cos(\sqrt{2} t) + (B / \sqrt{2}) \sin(\sqrt{2} t)$.

How do you approach finding the inverse Laplace transform of a rational function like $H(s) = (A s + B) / (s^2 + 2)$?

You decompose the function into terms matching known Laplace pairs, often splitting into parts involving s and constants, then apply inverse transforms of cosine and sine functions.

What standard Laplace transform pairs are useful in finding the inverse of $H(s) = (A s + B) / (s^2 + 2)$?

The pairs are $L^{-1}\{s / (s^2 + a^2)\} = \cos(a t)$ and $L^{-1}\{1 / (s^2 + a^2)\} = (1 / a) \sin(a t)$.

Can the inverse Laplace transform of $H(s) = (A s + B) / (s^2 + 2)$ be expressed in terms of basic trigonometric functions?

Yes, it can be expressed as $h(t) = A \cos(\sqrt{2}\, t) + (B / \sqrt{2}) \sin(\sqrt{2}\, t)$.

If B equals zero, what does the inverse Laplace transform of $H(s) = (A s + B) / (s^2 + 2)$ simplify to?

It simplifies to $h(t) = A \cos(\sqrt{2}\, t)$.

Are there any special conditions or parameters to consider when finding the inverse Laplace of $H(s) = (A s + B) / (s^2 + 2)$?

Yes, ensuring that A and B are constants, and recognizing that the denominator $s^2 + 2$ corresponds to a damping factor with a frequency $\sqrt{2}$, which influences the form of the inverse transform.

What is the significance of the constants A and B in the inverse Laplace transform of H(s)?

Constants A and B determine the amplitudes of the cosine and sine components in the time domain solution, shaping the overall response.

Additional Resources

Find the Inverse Laplace Transform of the Function H(s) = (A s + B) / (s^2 + 2)

Introduction

In the realm of engineering, mathematics, and applied sciences, the Laplace transform is a powerful tool used to analyze systems, particularly in control theory, signal processing, and differential equations. It simplifies complex differential equations into algebraic equations, making them easier to manipulate and solve. The process of finding the inverse Laplace transform is equally vital as it allows us to revert back from the s-domain (complex frequency domain) to the time domain, providing meaningful physical interpretations.

One common challenge encountered by students and professionals is determining the inverse Laplace transform of rational functions, especially when they involve polynomial expressions in the numerator and denominator. This article focuses on a specific function:

$$
H(s) = \frac{A s + B}{s^2 + 2}
$$

where A and B are constants. The goal is to derive a comprehensive, detailed, and analytical approach to find its inverse Laplace transform, $h(t)$. Through systematic exploration, we will dissect the problem, leverage known Laplace transform pairs, and develop a step-by-step method to solve similar functions efficiently.

Overview of the Laplace Transform and Its Inverse

What is the Laplace Transform?

The Laplace transform of a time-domain function $f(t)$, denoted as $F(s)$, is defined as:

$$
F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-s t} f(t) dt
$$

This integral transforms the function from the time domain into the complex frequency domain, where algebraic techniques can be employed to solve differential equations.

Importance of the Inverse Laplace Transform

While the Laplace transform simplifies the initial problem, the ultimate goal is to interpret the solution in the original time domain. The inverse Laplace transform, denoted as $\mathcal{L}^{-1}\{F(s)\}$, accomplishes this task, providing the response or behavior $f(t)$ corresponding to the frequency domain representation.

Decomposition of $H(s) = \frac{A s + B}{s^2 + 2}$

Recognizing the Structure

The function $H(s)$ is a rational function with a quadratic denominator. The numerator is a linear polynomial, $A s + B$, which suggests that the inverse transform might involve basic functions like sines and cosines, possibly multiplied by constants.

The denominator, $s^2 + 2$, resembles the Laplace transform of sinusoidal functions, specifically:

$$
\mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2 + \omega^2}
$$

$$
\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2 + \omega^2}
$$

In our case, $\omega^2 = 2$, so $\omega = \sqrt{2}$.

Step-by-Step Approach to Find the Inverse Laplace Transform

1. Express the Function in Terms of Known Transform Pairs

The key is to decompose $H(s)$ into parts that correspond to standard Laplace transform pairs.

$$
H(s) = \frac{A s + B}{s^2 + 2}
$$

This can be written as:

$$
H(s) = A \frac{s}{s^2 + 2} + B \frac{1}{s^2 + 2}
$$

which suggests a linear combination of two standard inverse transforms:

- $\mathcal{L}^{-1}\left\{\frac{s}{s^2 + \omega^2}\right\} = \cos(\omega t)$
- $\mathcal{L}^{-1}\left\{\frac{\omega}{s^2 + \omega^2}\right\} = \sin(\omega t)$

Since the second term involves $\frac{1}{s^2 + 2}$, we can relate it to $\sin(\sqrt{2} t)$ via the known pair:

$$
\mathcal{L}^{-1}\left\{\frac{1}{s^2 + \omega^2}\right\} = \frac{1}{\omega} \sin(\omega t)
$$

---

2. Rewrite $H(s)$ Using Standard Pairs

Express $H(s)$ explicitly:

$$
H(s) = A \frac{s}{s^2 + 2} + B \frac{1}{s^2 + 2}
$$

Recall the inverse transforms:

$$
\mathcal{L}^{-1}\left\{\frac{s}{s^2 + \omega^2}\right\} = \cos(\omega t)
$$

$$
\mathcal{L}^{-1}\left\{\frac{1}{s^2 + \omega^2}\right\} = \frac{1}{\omega} \sin(\omega t)
$$

where $\omega = \sqrt{2}$.

Thus,

\[

h(t) = A \cos(\sqrt{2} t) + B \times \frac{1}{\sqrt{2}} \sin(\sqrt{2} t)

\]

Analytical Derivation of the Inverse Laplace Transform

3. Formal Expression of the Result

The inverse Laplace transform of \(H(s)\) can now be written explicitly as:

\[

\boxed{

h(t) = A \cos(\sqrt{2} t) + \frac{B}{\sqrt{2}} \sin(\sqrt{2} t)

}

\]

This formula reveals that the original time-domain function is a linear combination of sinusoidal functions with angular frequency \(\sqrt{2}\), scaled by the constants \(A\) and \(B\).

Additional Insights and Considerations

a. Significance of Constants \(A\) and \(B\)

- The constant \(A\) influences the amplitude of the cosine component.
- The constant \(B\) modulates the sine component, scaled by \(\frac{1}{\sqrt{2}}\).

These constants could originate from initial conditions or specific system parameters in applications like electrical circuits or mechanical oscillators.

b. Impact of Parameter Variations

Changing A and B alters the system's response:

- For $A = 1$, $B=0$, the response is purely cosine.
- For $A=0$, $B=1$, the response is purely sine scaled by $\frac{1}{\sqrt{2}}$.
- For general values, the response is a phase-shifted sinusoid, which can be expressed as a single sinusoid with amplitude and phase shift.

Generalization and Broader Implications

1. Extending to Other Functions

The approach demonstrated here is applicable to a wide class of rational functions with quadratic denominators, especially when the numerator is linear or polynomial. The key steps include:

- Decomposition into known transform pairs.
- Recognizing standard forms.
- Expressing the inverse in terms of sine and cosine functions.

2. Connection to Differential Equations

The inverse Laplace transform derived corresponds to the solution of a second-order homogeneous differential equation:

$$
\frac{d^2 h(t)}{dt^2} + \omega^2 h(t) = 0
$$

with solutions involving sinusoidal functions, consistent with the form obtained.

Practical Applications

a. Control Systems

In control engineering, such inverse transforms are essential for analyzing system responses to sinusoidal inputs or initial disturbances, especially in the context of natural frequency and damping.

b. Signal Processing

Understanding how to invert Laplace transforms aids in designing filters and analyzing signals in the frequency domain.

c. Mechanical and Electrical Systems

Oscillatory systems like springs, pendulums, and RLC circuits often produce responses characterized by functions like $h(t)$.

Conclusion

The process of finding the inverse Laplace transform of $H(s) = \frac{A s + B}{s^2 + 2}$ illustrates the power of pattern recognition, standard transform pairs, and algebraic manipulation. By decomposing the function into familiar components, we determine that:

\[
\boxed{

h(t) = A \cos(\sqrt{2} t) + \frac{B}{\sqrt{2}} \sin(\sqrt{2} t)

}

\]

This result encapsulates the behavior of systems characterized by such transfer functions, providing insights into their oscillatory nature, response stability, and the influence of system parameters.

Understanding these techniques enhances one's ability to analyze complex systems across various scientific and engineering disciplines, underscoring the importance of the Laplace transform as a foundational mathematical tool.

References and Further Reading

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