Find The Linearization L(x) Of The Function At A. F(x) = X2/3, A = 64

Find The Linearization L(x) Of The Function At A. F(x) = X^{2/3}, A = 64

Introduction to Linearization

Linearization is a fundamental concept in calculus used to approximate the value of a function near a specific point. When dealing with complex or non-linear functions, calculating the exact value might be difficult or unnecessary for small intervals. Instead, we use the tangent line at a particular point to approximate the function's behavior in the vicinity of that point. This approximation is known as the linearization or linear approximation of the function.

In this article, we will determine the linearization $L(x)$ of the function $F(x) = x^{2/3}$ at the point where $A = 64$. The process involves calculating the value of the function and its derivative at A, then constructing the tangent line that best approximates the function near that point.

Understanding the Function $F(x) = x^{2/3}$

Function Characteristics

- The function $F(x) = x^{2/3}$ is a composite of basic root and power functions.
- It is defined for all $x \geq 0$ and can be extended to negative x values because the fractional exponent $2/3$ involves an even root, which is defined for negative inputs as well.
- The function is continuous and smooth (differentiable) for all $x \neq 0$.

Rewriting the Function

- For clarity, $F(x) = (x^{1/3})^2$.
- The cube root $x^{1/3}$ is well-defined for all real x, including negative values.

Step 1: Evaluating $F(A)$ at $A=64$

Calculate $F(64)$

- Since $F(x) = x^{2/3}$, substitute $x=64$:

$$
F(64) = 64^{2/3}
$$

- To compute this, recognize that:

$$
64^{2/3} = \left(64^{1/3}\right)^2
$$

- Find $64^{1/3}$:

$$
64^{1/3} = \sqrt[3]{64} = 4
$$

- Therefore,

$$
F(64) = 4^2 = 16
$$

Step 2: Find the Derivative $F'(x)$

Derivation of $F'(x)$

- The function $F(x) = x^{2/3}$ involves a fractional exponent. Recall the power rule for derivatives:

$$
\frac{d}{dx} x^{n} = n x^{n-1}
$$

- Applying this:

$$
F'(x) = \frac{2}{3} x^{(2/3) - 1} = \frac{2}{3} x^{-1/3}
$$

- Simplify the exponent:

$$
F'(x) = \frac{2}{3} x^{-1/3}
$$

- Rewrite to clarify:

$$
F'(x) = \frac{2}{3} \cdot \frac{1}{x^{1/3}} = \frac{2}{3} \cdot \frac{1}{\sqrt[3]{x}}
$$

Evaluate $F'(A)$ at $A=64$

- Substitute $x=64$:

$$
F'(64) = \frac{2}{3} \cdot \frac{1}{\sqrt[3]{64}} = \frac{2}{3} \cdot \frac{1}{4}
$$

- Since $\sqrt[3]{64} = 4$, then:

$$
F'(64) = \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}
$$

Step 3: Constructing the Linearization $L(x)$

Formula for Linearization

- The linear approximation $L(x)$ of $F(x)$ at $x = A$ is given by:

$$
L(x) = F(A) + F'(A) (x - A)
$$

- Using the values obtained:

$$
F(64) = 16
$$

$$
F'(64) = \frac{1}{6}
$$

- Substituting into the formula:

$$
L(x) = 16 + \frac{1}{6} (x - 64)
$$

Final Expression for $L(x)$

- The linearization of $F(x) = x^{2/3}$ at $A=64$ is:

$$
\boxed{
L(x) = 16 + \frac{1}{6} (x - 64)
}
$$

Applications of the Linearization

Approximate Function Values Near $A = 64$

- The linearization $L(x)$ provides a quick way to estimate $F(x)$ for values of x close to 64.

- For example, to approximate $F(65)$:

$$
L(65) = 16 + \frac{1}{6} (65 - 64) = 16 + \frac{1}{6} \times 1 = 16 + \frac{1}{6} \approx 16.1667
$$

- The actual $F(65)$ can be computed as:

$$
F(65) = 65^{2/3} = \left(65^{1/3}\right)^2
$$

- $65^{1/3}$ is approximately 4.02, so:

$$
F(65) \approx (4.02)^2 \approx 16.16
$$

- The linear approximation 16.1667 is very close, demonstrating the effectiveness of linearization.

Understanding the Scope and Limitations

- The linear approximation works best for x values near 64.
- As x moves further from 64, the approximation becomes less accurate.
- For more precise estimates over larger intervals, higher-order approximations or exact calculations are necessary.

Summary of the Process

- Identify the function and the point of interest: $F(x) = x^{2/3}$, $A=64$.
- Calculate $F(A)$: $F(64) = 16$.
- Compute the derivative $F'(x) = \frac{2}{3} x^{-1/3}$.
- Evaluate the derivative at $A=64$: $F'(64) = \frac{1}{6}$.
- Construct the linearization formula: $L(x) = F(64) + F'(64) (x - 64)$.

- Write the final linearization: $\boxed{L(x) = 16 + \frac{1}{6} (x - 64)}$.

Conclusion

The process of finding the linearization of a function at a specific point is a powerful method for approximation in calculus. For the function $F(x) = x^{2/3}$ at $A=64$, the linearization $L(x)$ is:

$$
L(x) = 16 + \frac{1}{6} (x - 64)
$$

This linear approximation allows us to estimate the function's value near $x=64$ efficiently and with reasonable accuracy. Such techniques are essential in various applications, including physics, engineering, and economics, where exact calculations may be complex or unnecessary for initial estimates. Understanding how to derive and apply linearizations not only enhances problem-solving skills but also deepens one's grasp of the fundamental concepts of calculus.

Frequently Asked Questions

What is the general process to find the linearization L(x) of a function at a point A?

To find the linearization $L(x)$ of a function at a point A, you evaluate the function and its derivative at A, then use the formula $L(x) = f(A) + f'(A)(x - A)$.

Given the function $f(x) = x^{2/3}$ and A = 64, how do you compute f(64)?

Since $f(64) = 64^{2/3}$, you calculate $64^{1/3} = 4$, then square it: $4^2 = 16$. So, $f(64) = 16$.

How do you find the derivative $f'(x)$ of $f(x) = x^{2/3}$?

Using the power rule, $f'(x) = (2/3) x^{(2/3) - 1} = (2/3) x^{-1/3}$.

What is the value of f'(64) for the given function?

f'(64) = (2/3) 64^{-1/3}. Since 64^{1/3} = 4, then 64^{-1/3} = 1/4. Therefore, f'(64) = (2/3) (1/4) = 2/12 = 1/6.

What is the linearization L(x) of f(x) = x^{2/3} at A = 64?

Using the formula, L(x) = f(64) + f'(64)(x - 64) = 16 + (1/6)(x - 64).

Why is linearization useful for approximating function values near A?

Because it uses the tangent line at A to provide a close approximation of the function's value for x close to A, simplifying calculations.

How can you use the linearization L(x) to estimate f(65)?

Plug x = 65 into L(x): L(65) = 16 + (1/6)(65 - 64) = 16 + (1/6)(1) = 16 + 1/6 ≈ 16.1667.

What are some limitations of using linearization for approximation?

Linearization provides accurate estimates only near the point A; as x moves further away, the approximation may become less accurate due to the curvature of the function.

Additional Resources

Find The Linearization L(x) Of The Function At A. F(x) = x^{2/3}, A = 64

Introduction

In the realm of calculus, linearization serves as a fundamental technique for approximating complex functions near a specific point. It allows us to replace a potentially complicated function with a linear one that closely mimics its behavior in a small neighborhood. This process is not only pivotal for theoretical analysis but also has practical applications across engineering, physics, and computer science.

In this comprehensive investigation, we focus on deriving the linearization \(L(x)\) of the function \(f(x) = x^{2/3}\) at the point \(A=64\). This function presents an interesting case due to its fractional exponent, which introduces nuanced considerations in differentiation and approximation near the point of expansion.

Background and Significance of Linearization

Linearization, often called the tangent line approximation, is rooted in the fundamental theorem of calculus and differential calculus principles. For a function $f(x)$ differentiable at a point a, the linearization $L(x)$ provides an approximation of $f(x)$ near a:

$$
L(x) = f(a) + f'(a)(x - a)
$$

This approximation effectively replaces the curve of $f(x)$ with its tangent line at a, facilitating easier calculations and insights into the function's local behavior. The accuracy of this approximation improves as x approaches a.

The importance of linearization extends to:

- Estimating function values where direct computation is complex
- Analyzing rates of change in real-world phenomena
- Solving differential equations approximately
- Understanding local behavior of functions in advanced calculus and analysis

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The Function $f(x) = x^{2/3}$: An Analytical Overview

The function under scrutiny, $f(x) = x^{2/3}$, is a fractional power function. Its properties and behavior near specific points reveal intriguing characteristics:

- Domain and Range: Defined for all $x \geq 0$ and $x \leq 0$, with the output always non-negative for positive x. At $x=0$, $f(0)=0$.
- Differentiability: For $x \neq 0$, the function is smooth and differentiable. At $x=0$, the derivative exists but exhibits a cusp, which warrants careful analysis when linearizing at points close to zero.
- Behavior at $x=64$: Since $A=64$, a perfect cube, evaluating $f(64)$ and its derivative is straightforward, facilitating a precise linear approximation at this point.

Step-by-Step Derivation of the Linearization $L(x)$ at $A=64$

1. Calculating $f(64)$

Given:

$$
f(x) = x^{2/3}

\]

At \(x=64\):

\[
f(64) = 64^{2/3}
\]

Recall that \(64 = 4^3\). Therefore:

\[
f(64) = (4^3)^{2/3} = 4^{3 \times \frac{2}{3}} = 4^{2} = 16
\]

Result:

\[
f(64) = 16
\]

---

2. Computing the Derivative \(f'(x)\)

Using the power rule:

\[
f'(x) = \frac{d}{dx} x^{2/3} = \frac{2}{3} x^{(2/3) - 1} = \frac{2}{3} x^{-1/3}
\]

This derivative exists for all \(x \neq 0\). At \(x=64\):

\[
f'(64) = \frac{2}{3} \times 64^{-1/3}
\]

Calculate \(64^{-1/3}\):

\[
64^{1/3} = \sqrt[3]{64} = 4
\]

Thus:

$$
64^{-1/3} = \frac{1}{4}
$$

Substituting back:

$$
f'(64) = \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}
$$

Result:

$$
f'(64) = \frac{1}{6}
$$

---

3. Formulating the Linearization $L(x)$

The linear approximation of $f(x)$ near $x=64$ is:

$$
L(x) = f(64) + f'(64)(x - 64)
$$

Substituting the computed values:

$$
L(x) = 16 + \frac{1}{6}(x - 64)
$$

This expression provides a straightforward, linear estimate of $f(x)$ for x close to 64.

Interpretation and Applications of the Linearization

Practical Usage

The derived linearization:

$$

$$
L(x) = 16 + \frac{1}{6}(x - 64)
$$

can be employed to estimate $f(x)$ for values of x near 64. For instance, if one needs to approximate $f(65)$:

$$
L(65) = 16 + \frac{1}{6}(65 - 64) = 16 + \frac{1}{6} \times 1 = 16 + \frac{1}{6} \approx 16.1667
$$

The actual value:

$$
f(65) = 65^{2/3}
$$

is approximately 16.160, indicating the linear approximation's accuracy.

Significance in Analysis and Engineering

- Error estimation: The difference between $f(x)$ and $L(x)$ quantifies the approximation error.
- Sensitivity analysis: The slope $\frac{1}{6}$ indicates how rapidly $f(x)$ changes near $x=64$.
- Design and control systems: Approximations like these simplify complex models, making real-time calculations feasible.

Deeper Insights: Behavior Near the Point of Expansion

While the linearization provides a good approximation near $x=64$, the behavior of $f(x) = x^{2/3}$ can vary significantly further away:

- For $x < 64$: The function decreases in a smooth, concave manner.
- For $x > 64$: The function increases, maintaining smoothness due to the differentiability for $x > 0$.

The derivative's form, involving $x^{-1/3}$, indicates a non-linear rate of change:

- As $x \to 0^+$, $f'(x) \to \infty$, indicating a steep slope near zero.
- At larger x, the slope diminishes, approaching zero as $x \to \infty$.

This characteristic emphasizes the importance of local linear approximations, which provide accurate estimates only within a limited vicinity of the expansion point.

Conclusion

The process of finding the linearization $L(x)$ of the function $f(x) = x^{2/3}$ at $A=64$ underscores the essential principles of differential calculus. Through systematic calculation of the function's value and derivative at the point of interest, we derived:

$$
L(x) = 16 + \frac{1}{6}(x - 64)
$$

This linear approximation offers a practical and insightful tool for estimating the function's behavior near $x=64$. Its applications span scientific computation, engineering design, and mathematical analysis, exemplifying the profound utility of linearization in understanding complex functions.

The exploration of fractional power functions like $x^{2/3}$ also enriches our appreciation of calculus's depth, revealing how local linear models adapt to nuanced behaviors, especially near non-smooth points or at the edges of a function's domain. As such, this investigation not only provides a concrete example of linearization but also highlights its broader significance within the mathematical sciences.

References

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Final Remarks

Understanding how to find the linearization of a function like $x^{2/3}$ at a specific point enhances both theoretical knowledge and practical problem-solving skills. It bridges the gap between abstract calculus concepts and tangible applications, affirming the importance of precise computation and interpretation in mathematical analysis.

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