Find The Possible Values Of X For Each Of The Following(X-2)(3x-1)=0

Find The Possible Values Of X For Each Of The Following (X-2)(3X-1)=0 is a common problem in algebra that involves solving quadratic equations by applying the zero product property. This type of problem is fundamental for students learning algebra because it introduces the concept of solving for variables when their products equal zero. In this article, we will explore in detail how to find the possible values of X for the given equation, understand the underlying principles, and work through various examples to solidify your understanding. Whether you're preparing for a math exam or simply enhancing your algebra skills, this comprehensive guide will walk you through the process step by step.

Understanding the Zero Product Property

What Is the Zero Product Property?

The zero product property states that if the product of two factors is zero, then at least one of the factors must be zero. Mathematically, this is expressed as:
- If $A \times B = 0$, then either $A = 0$ or $B = 0$.

This principle is fundamental in solving quadratic equations and polynomial equations where factors are involved. It simplifies the process of finding solutions because instead of solving the entire equation directly, you can focus on each factor separately.

Why Does This Property Work?

The property holds because of the fundamental nature of multiplication. If the product of two numbers is zero, then at least one of the factors must be zero—since multiplying any non-zero number by another non-zero number results in a non-zero product.

Solving the Equation (X-2)(3X-1)=0

Step 1: Set Each Factor Equal to Zero

To find the possible values of X, set each factor equal to zero:

1. $X - 2 = 0$
2. $3X - 1 = 0$

This approach is based on the zero product property, which simplifies the problem into solving two separate equations.

Step 2: Solve Each Equation Individually

Let's solve each equation:

Equation 1: $X - 2 = 0$

Adding 2 to both sides:

$$
X = 2
$$

Equation 2: $3X - 1 = 0$

Adding 1 to both sides:

$$
3X = 1
$$

Dividing both sides by 3:

$$
X = \frac{1}{3}
$$

Possible Values of X

From the solutions above, the possible values of X that satisfy the original equation are:

- $X = 2$
- $X = \frac{1}{3}$

These are the solutions that make the product $(X-2)(3X-1)$ equal to zero.

Verification of the Solutions

It's always good practice to verify your solutions by plugging them back into the original equation.

Verification for $X = 2$:

$$
(2 - 2)(3 \times 2 - 1) = 0 \times (6 - 1) = 0 \times 5 = 0
$$

Since the product is zero, $X=2$ is a valid solution.

Verification for $X = \frac{1}{3}$:

$$
\left(\frac{1}{3} - 2\right)\left(3 \times \frac{1}{3} - 1\right) = \left(-\frac{5}{3}\right) \times (1 - 1)
$$

$$
= -\frac{5}{3} \times 0 = 0
$$

Again, the product is zero, confirming $X=\frac{1}{3}$ as a valid solution.

Additional Examples and Applications

Example 1: Solving a Similar Equation

Consider the equation $(X + 4)(2X - 3) = 0$.

Solution:
Set each factor to zero:
- $X + 4 = 0 \Rightarrow X = -4$
- $2X - 3 = 0 \Rightarrow 2X = 3 \Rightarrow X = \frac{3}{2}$

Possible solutions: $X = -4, \frac{3}{2}$

Example 2: Applying to Word Problems

Suppose a rectangle's length and width are represented by the expressions $X - 1$ and $3X + 2$, and the area is zero when either the length or width is zero. Find the values of X:

Set each to zero:
- $X - 1 = 0 \Rightarrow X = 1$
- $3X + 2 = 0 \Rightarrow 3X = -2 \Rightarrow X = -\frac{2}{3}$

Interpretation:
- When $X=1$, the length is zero, implying no rectangle.
- When $X=-\frac{2}{3}$, the width is zero.

Common Mistakes to Avoid

- Forgetting to set each factor to zero: Always remember, when dealing with a product equal to zero, each factor must be considered.
- Incorrectly solving equations: Make sure to perform operations correctly, especially when dividing or multiplying both sides.
- Ignoring extraneous solutions: Double-check solutions by plugging them back into the original equation.

Summary and Key Takeaways

- The equation $(X-2)(3X-1)=0$ can be solved by applying the zero product property.

- Set each factor equal to zero and solve for X.
- The solutions are \(X=2\) and \(X=\frac{1}{3}\).
- Always verify solutions by substituting back into the original equation.
- This method applies to a wide range of algebraic equations involving products.

Conclusion

Understanding how to find the possible values of X for equations like \((X-2)(3X-1)=0\) is essential for mastering algebra. The key step is recognizing the zero product property and solving each factor independently. By practicing with different equations and verifying solutions, you can strengthen your algebraic problem-solving skills. Remember, the process involves setting each factor to zero, solving for X, and confirming that these solutions satisfy the original equation. With this knowledge, you're well-equipped to handle similar equations and apply these principles to more complex algebraic problems.

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- Find the possible values of X
- Solve (X-2)(3X-1)=0
- Zero product property
- Algebra solutions
- Solving quadratic equations
- Algebra practice problems
- How to solve equations involving factors
- Step-by-step algebra solutions

Frequently Asked Questions

What are the possible values of x for the equation $(x - 2)(3x - 1) = 0$?

The possible values of x are $x = 2$ and $x = \frac{1}{2}$.

How do you find the roots of the equation $(x - 2)(3x - 1) = 0$?

Set each factor equal to zero: $x - 2 = 0 \rightarrow x = 2$, and $3x - 1 = 0 \rightarrow x = \frac{1}{2}$.

Why do we set each factor equal to zero in the equation $(x - 2)(3x - 1) = 0$?

Because the product of two factors is zero only when at least one of the factors is zero, according to the zero product property.

Are the solutions x = 2 and x = 1/3 valid for the equation (x - 2)(3x - 1) = 0?

Yes, both x = 2 and x = ½ satisfy the equation, making them valid solutions.

What type of equation is (x - 2)(3x - 1) = 0 and how do we solve it?

It is a factored quadratic equation, and we solve it by setting each factor equal to zero.

Can the solutions to (x - 2)(3x - 1) = 0 be negative?

No, the solutions are x = 2 and x = ½, both of which are non-negative, but if the factors had roots like negative numbers, they would be valid solutions too.

What is the significance of solving (x - 2)(3x - 1) = 0 in algebra?

It demonstrates the zero product property and helps find roots of polynomial equations efficiently.

If the equation was (x + 2)(3x - 1) = 0, what would be the solutions?

The solutions would be x = -2 and x = ½.

How can you verify the solutions x = 2 and x = ½ for the original equation?

Substitute each value into the original equation and check if the expression equals zero. For x=2: (2-2)(32-1)=0; for x=½: (½-2)(3½-1)=0.

Additional Resources

Find The Possible Values Of X For Each Of The Following (X-2)(3X-1)=0

Introduction

Mathematics often presents us with equations that seem straightforward on the surface but conceal deeper layers of understanding. Among these, algebraic equations involving products set equal to zero are particularly fundamental, serving as the building blocks for more advanced concepts. The equation (X - 2)(3X - 1) = 0 exemplifies this, illustrating how the zero product property can be utilized to find solutions for the variable X. This article aims to explore this equation comprehensively, breaking down its structure, explaining the underlying principles, and providing a detailed step-by-step analysis of how to identify all possible values of X that satisfy the equation.

Understanding the Zero Product Property

Before delving into the specific equation, it is essential to grasp the zero product property, a pivotal principle in algebra. The property states:

> If the product of two factors equals zero, then at least one of the factors must be zero.

Mathematically, if:

$$
A \times B = 0
$$

then:

$$
A = 0 \quad \text{or} \quad B = 0
$$

This principle simplifies solving equations involving products by allowing us to set each factor equal to zero independently, leading to potential solutions that satisfy the original equation.

Breakdown of the Equation (X - 2)(3X - 1) = 0

The given equation:

$$
(X - 2)(3X - 1) = 0
$$

is a product of two linear factors. Each factor is a linear expression involving X:

- First factor: $X - 2$
- Second factor: $3X - 1$

The goal is to find all values of X that satisfy this equation. According to the zero product property, we can set each factor equal to zero separately:

$$
X - 2 = 0
$$
and
$$
3X - 1 = 0
$$

and then solve each for X.

Step-by-Step Solution Process

Step 1: Set each factor equal to zero

- Factor 1: $X - 2 = 0$
- Factor 2: $3X - 1 = 0$

Step 2: Solve each equation

For $X - 2 = 0$:

$$
X = 2
$$

For $3X - 1 = 0$:

$$
3X = 1
$$
$$
X = \frac{1}{3}
$$

These solutions emerge directly from the application of the zero product property.

Final Solutions and Their Significance

The solutions to the original equation are:

$$
X = 2 \quad \text{and} \quad X = \frac{1}{3}
$$

Both solutions satisfy the original equation when substituted back:

- Verification for $X=2$:

$$
(2 - 2)(3 \times 2 - 1) = (0)(6 - 1) = 0 \times 5 = 0
$$

- Verification for $X=\frac{1}{3}$:

$$
\left(\frac{1}{3} - 2\right)\left(3 \times \frac{1}{3} - 1\right) = \left(-\frac{5}{3}\right)(1 - 1) = \left(-\frac{5}{3}\right)(0) = 0
$$

\]

Both solutions satisfy the equation, confirming their validity.

Analytical Insights and Broader Implications

1. Nature of the Solutions

The solutions are real numbers, which is typical for linear equations with real coefficients. The nature of the solutions (distinct and real) aligns with expectations for linear factors.

2. Multiplicity of Roots

In this case, each factor contributes a simple root (multiplicity one). If a factor appears more than once, the solution might have multiplicity greater than one, which has implications in calculus and algebra regarding the behavior of the function near those roots.

3. Graphical Interpretation

Plotting the functions:

- $f_1(X) = X - 2$
- $f_2(X) = 3X - 1$

their product intersects the X-axis at points $X=2$ and $X=\frac{1}{3}$. These points are where the graph of the paraboloid (if expanded) crosses the X-axis, indicating solutions to the equation.

Extending Beyond Basic Equations

While the current equation is simple, the methodology applies to more complex algebraic equations, including higher-degree polynomials, rational functions, and transcendental equations, often employing factorization, substitution, or numerical methods.

1. Quadratic Equations

For quadratic equations, factoring is a common approach. For example, solving $X^2 - 5X + 6 = 0$ involves factoring into $(X - 2)(X - 3) = 0$, leading to solutions $X=2, 3$.

2. Higher-Order Polynomials

For cubic or quartic equations, factoring might involve more elaborate techniques such as synthetic division, rational root theorem, or numerical approximation, but the zero product principle remains fundamental.

Practical Applications and Real-World Relevance

Understanding the solutions of such equations extends beyond academic exercises into practical fields:

- Engineering: Designing systems where certain conditions (represented by equations) must be satisfied, such as equilibrium points or zero-crossings.
- Physics: Analyzing forces or energy states where equations equal zero, indicating critical points.
- Economics: Determining break-even points where profit or loss equations equal zero.
- Computer Science: Algorithms involving decision points where certain conditions (represented by equations) determine flow control.

Common Mistakes and Misconceptions

While solving equations like $(X - 2)(3X - 1) = 0$ is straightforward, students often encounter pitfalls:

- Ignoring the zero product property: Assuming that both factors must be non-zero, which leads to missed solutions.
- Incorrectly solving one factor: For example, solving $X - 2 = 0$ but neglecting to solve $3X - 1 = 0$.
- Assuming extraneous solutions: Although not common in such linear factors, in more complex equations, extraneous solutions may arise from algebraic manipulations.

Conclusion

The equation $(X - 2)(3X - 1) = 0$ exemplifies a fundamental principle in algebra—the zero product property—serving as a cornerstone for solving a wide array of equations. Its solutions, $X=2$ and $X=\frac{1}{3}$, are derived through straightforward algebraic manipulation, underlining the elegance and simplicity of linear equations when approached methodically.

Understanding these solutions not only enhances algebraic fluency but also lays the foundation for tackling more complex mathematical problems across various disciplines. Whether in theoretical mathematics, applied sciences, or everyday problem-solving, recognizing the significance of such basic equations fosters critical thinking and analytical skills essential for scientific literacy.

As mathematical concepts become more intricate, the core principles illustrated by this equation—factoring, zero solutions, and logical reasoning—remain timeless tools, empowering learners and professionals alike to decode the language of mathematics with confidence and precision.

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