Find The Radius Of Curvature Of The Curve X = 4cost And Y = 3sint At T = 0

Find The Radius Of Curvature Of The Curve X = 4cos t And Y = 3sin t At T = 0

Understanding the curvature of a curve is fundamental in fields such as physics, engineering, and mathematics. The radius of curvature provides insight into how sharply a curve bends at a specific point, which is crucial for designing roads, roller coasters, and analyzing particle trajectories. In this detailed guide, we will explore how to find the radius of curvature of the parametric curve defined by $x = 4 \cos t$ and $y = 3 \sin t$ at $t = 0$.

By the end of this article, you will grasp the underlying concepts of curvature, learn the step-by-step process to compute the radius of curvature for parametric equations, and understand the significance of the result at the specified parameter value.

Understanding the Curve and Its Parametric Equations

The curve in question is defined parametrically as:

$$x(t) = 4 \cos t$$

$$y(t) = 3 \sin t$$

Here, t is the parameter, often representing time or an independent variable that traces the curve.

Key Characteristics of the Curve:

- The equations resemble those of an ellipse centered at the origin with semi-major axis 4 along the x-axis and semi-minor axis 3 along the y-axis.
- When t varies from 0 to 2π, the point $(x(t), y(t))$ traces the entire ellipse.

Fundamentals of Curvature and Radius of Curvature

Before jumping into calculations, it's essential to understand what curvature and the radius of curvature mean:

- Curvature (κ) at a point on a curve measures how sharply the curve bends at that point.
- Radius of Curvature (R) is the reciprocal of curvature:

$$
R = \frac{1}{\kappa}
$$

- The radius of curvature at a point indicates the radius of the osculating circle, the circle that best "fits" the curve at that point.

For parametric curves, the radius of curvature at a specific parameter t can be computed using the derivatives of $x(t)$ and $y(t)$.

Mathematical Formulation for Radius of Curvature in Parametric Form

Given a parametric curve $x(t)$, $y(t)$, the radius of curvature R at a point corresponding to t is provided by the formula:

$$
R = \frac{\left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{3/2}}{\left| \frac{d^2 y}{dt^2} \frac{dx}{dt} - \frac{d^2 x}{dt^2} \frac{dy}{dt} \right|}
$$

This formula involves:

- First derivatives: $\frac{dx}{dt}$, $\frac{dy}{dt}$
- Second derivatives: $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$

Note: The absolute value in the denominator ensures the radius is positive, as curvature can be positive or negative depending on the orientation of the curve.

Step-by-Step Calculation at \(t = 0 \)

Let's compute the derivatives necessary at \(t = 0 \):

Step 1: Compute First Derivatives

\[
\frac{dx}{dt} = -4 \sin t
\]
\[
\frac{dy}{dt} = 3 \cos t
\]

At \(t = 0 \):

\[
\frac{dx}{dt} \bigg|_{t=0} = -4 \sin 0 = 0
\]
\[
\frac{dy}{dt} \bigg|_{t=0} = 3 \cos 0 = 3
\]

Step 2: Compute Second Derivatives

\[
\frac{d^2x}{dt^2} = -4 \cos t
\]
\[
\frac{d^2y}{dt^2} = -3 \sin t
\]

At \(t = 0 \):

\[
\frac{d^2x}{dt^2} \bigg|_{t=0} = -4 \cos 0 = -4
\]
\[
\frac{d^2y}{dt^2} \bigg|_{t=0} = -3 \sin 0 = 0
\]

Step 3: Calculate numerator of the radius of curvature formula

\[
\left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2
\right]^{3/2}
\]

At \(t=0 \):

\[

$$
(0)^2 + (3)^2 = 0 + 9 = 9
$$

So,

$$
(9)^{3/2} = (\sqrt{9})^3 = (3)^3 = 27
$$

Step 4: Calculate denominator

$$
\left| \frac{d^2 y}{dt^2} \frac{dx}{dt} - \frac{d^2 x}{dt^2} \frac{dy}{dt} \right|
$$

Plugging in the derivatives at $t=0$:

$$
| (0)(0) - (-4)(3) | = | 0 + 12 | = 12
$$

Step 5: Compute the radius of curvature R

$$
R = \frac{27}{12} = \frac{9}{4} = 2.25
$$

Interpretation of the Result

The radius of curvature of the ellipse at $t=0$ is 2.25 units. This value indicates the radius of the best-fitting circle that "kisses" the curve at that point, providing insight into the sharpness of the bend:

- A smaller radius implies a sharper bend.
- A larger radius indicates a gentler curve.

In this case, at $t=0$, the curve exhibits a moderate bend, consistent with the shape of the ellipse at that point.

Additional Insights and Related Concepts

The Geometric Significance

- At $t=0$, the point on the curve is:

$$
x(0) = 4 \cos 0 = 4
$$
$$
y(0) = 3 \sin 0 = 0
$$

- The point is $(4, 0)$, which lies on the ellipse at its rightmost extremity along the x-axis.

Curvature at Other Points

While we've calculated the radius of curvature at $t=0$, similar techniques apply for other values of t. For example, at $t = \frac{\pi}{2}$:

- $x = 4 \cos \frac{\pi}{2} = 0$
- $y = 3 \sin \frac{\pi}{2} = 3$

The curvature at different points can be computed similarly, revealing how the bend varies along the ellipse.

Relevance in Practical Applications

- Engineering: Designing curved roads or tracks where knowing the radius of curvature is essential for safety and comfort.
- Physics: Analyzing particle trajectories along elliptical paths.
- Mathematics: Understanding properties of conic sections and their curvature behavior.

Summary of the Process

To recapitulate, the steps to find the radius of curvature at a specific parameter t are:

1. Compute the first derivatives dx/dt and dy/dt.
2. Compute the second derivatives d^2x/dt^2 and d^2y/dt^2.
3. Evaluate these derivatives at the given t.
4. Plug into the parametric curvature formula:

$$
R = \frac{\left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{3/2}}{\left| \frac{d^2 y}{dt^2} \frac{dx}{dt} - \frac{d^2
$$

x}{dt^2} \frac{dy}{dt} \right|}
\]

5. Simplify to find the radius.

This systematic approach can be applied to any parametric curve to understand its geometric properties.

Conclusion

The radius of curvature of the curve \(x = 4 \cos t \), \(y = 3 \sin t \) at \(t=0 \) is 2.25 units. This value reflects the local bending of the ellipse at the point \((4, 0) \), which is the rightmost point on the ellipse. Understanding how to compute the radius of curvature not only deepens insight into

Frequently Asked Questions

What is the formula for the radius of curvature of a parametric curve?

The radius of curvature R at a point on a parametric curve defined by x(t) and y(t) is given by R = [(dx/dt)^2 + (dy/dt)^2]^{3/2} / |dx/dt d^2y/dt^2 - dy/dt d^2x/dt^2|.

How do you find the derivatives dx/dt and dy/dt for the given curve?

For the given curve x=4cos t and y=3sin t, dx/dt = -4sin t and dy/dt = 3cos t.

What are the second derivatives d^2x/dt^2 and d^2y/dt^2 for the curve?

The second derivatives are d^2x/dt^2 = -4cos t and d^2y/dt^2 = -3sin t.

How do you evaluate the derivatives at T=0 for the curve?

At T=0, sin 0=0 and cos 0=1, so dx/dt = 0, dy/dt = 3, d^2x/dt^2 = -4, and d^2y/dt^2=0.

What is the value of the numerator in the radius of curvature formula at T=0?

Numerator = $[(dx/dt)^2 + (dy/dt)^2]^{3/2} = [0^2 + 3^2]^{3/2} = (9)^{3/2} = 9 \sqrt{9} = 9\ 3 = 27$.

How do you compute the denominator of the radius of curvature at T=0?

Denominator = $|dx/dt\ d^2y/dt^2 - dy/dt\ d^2x/dt^2| = |0\ 0 - 3\ (-4)| = |0 + 12|=12$.

What is the radius of curvature of the curve at T=0?

Using the values, R = 27 / 12 = 2.25 units.

What does the radius of curvature signify at T=0 for this curve?

It indicates the radius of the osculating circle that best approximates the curve at T=0, which is 2.25 units for this case.

Can the radius of curvature formula be applied directly to parametric equations like these?

Yes, the formula is specifically designed for parametric equations and involves derivatives of x(t) and y(t) with respect to t.

How does the value of T affect the radius of curvature for this curve?

The radius of curvature varies with T, reflecting how sharply the curve bends at different points; at T=0, it is 2.25 units for this specific curve.

Additional Resources

Find The Radius Of Curvature Of The Curve X = 4cos t And Y = 3sin t At T = 0 is a classic problem in differential calculus and analytical geometry, often encountered in the study of curves and their properties. Understanding how to determine the radius of curvature at a specific parameter value provides insights into the local geometric behavior of the curve, such as how sharply it bends at that point. This guide offers a detailed, step-by-step approach to solving this problem, suitable for students, educators, and professionals seeking clarity on the process.

Introduction

The concept of radius of curvature is fundamental in understanding the geometry of curves. It measures how "curved" a path is at a specific point—smaller radii indicate sharper bends, while larger radii imply gentler turns. In parametric form, where a curve is expressed as functions of a parameter t, calculating the radius of curvature involves derivatives of the coordinate functions.

In this specific problem, the curve is defined parametrically as:

$$
X(t) = 4 \cos t
$$
$$
Y(t) = 3 \sin t
$$

and we need to find the radius of curvature at $t = 0$.

Understanding the Curve

Before diving into calculations, it's helpful to visualize or interpret the given parametric equations:

- The equations resemble an ellipse centered at the origin with semi-major axis 4 along the x-axis and semi-minor axis 3 along the y-axis.
- At $t = 0$, the point on the curve is:

$$
X(0) = 4 \cos 0 = 4 \times 1 = 4
$$
$$
Y(0) = 3 \sin 0 = 0
$$

- So, the point of interest is at (4, 0), which is on the positive x-axis.

Step 1: Recall the Formula for Radius of Curvature in Parametric Form

For a curve defined parametrically as:

$$
x = x(t), \quad y = y(t),
$$

the radius of curvature R at a point t is given by:

$$
R = \frac{ \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{3/2} }{ \left| \frac{dx}{dt} \cdot \frac{d^2 y}{dt^2} - \frac{dy}{dt} \cdot \frac{d^2 x}{dt^2} \right| }
$$

This formula stems from differential geometry and relates the derivatives of the parametric functions to curvature.

Step 2: Compute the First Derivatives

Calculate $\frac{dx}{dt}$ and $\frac{dy}{dt}$:

$$
\frac{dx}{dt} = -4 \sin t
$$

$$
\frac{dy}{dt} = 3 \cos t
$$

At $t = 0$:

$$
\frac{dx}{dt}\bigg|_{t=0} = -4 \sin 0 = 0
$$

$$
\frac{dy}{dt}\bigg|_{t=0} = 3 \cos 0 = 3
$$

Step 3: Compute the Second Derivatives

Calculate $\frac{d^2 x}{dt^2}$ and $\frac{d^2 y}{dt^2}$:

$$
\frac{d^2 x}{dt^2} = -4 \cos t
$$

$$
\frac{d^2 y}{dt^2} = -3 \sin t
$$

At $t=0$:

$$
\frac{d^2 x}{dt^2}\bigg|_{t=0} = -4 \cos 0 = -4 \times 1 = -4
$$

$$
\frac{d^2 y}{dt^2}\bigg|_{t=0} = -3 \sin 0 = 0
$$

\]

Step 4: Calculate the Numerator of the Radius of Curvature Formula

The numerator involves:

\[
\left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{3/2}
\]

At \(t=0 \):

\[
\left[0^2 + 3^2 \right]^{3/2} = (0 + 9)^{3/2} = 9^{3/2}
\]

Recall that:

\[
9^{3/2} = \left(9^{1/2} \right)^3 = (3)^3 = 27
\]

So, numerator = 27.

Step 5: Calculate the Denominator of the Radius of Curvature Formula

The denominator is:

\[
\left| \frac{dx}{dt} \cdot \frac{d^2 y}{dt^2} - \frac{dy}{dt} \cdot \frac{d^2 x}{dt^2} \right|
\]

At \(t=0 \):

\[
| (0)(0) - (3)(-4) | = | 0 + 12 | = 12
\]

Step 6: Final Calculation of the Radius of Curvature

Putting it all together:

\[

$$
R = \frac{27}{12} = \frac{9}{4} = 2.25
$$

Therefore, the radius of curvature of the curve at $t=0$ is $\boxed{2.25}$.

Additional Insights: Geometric Interpretation

- The point (4, 0) lies on an ellipse with semi-axes 4 and 3.
- The radius of curvature at this point indicates how sharply the ellipse bends there.
- A smaller radius would mean a sharper bend; here, a radius of 2.25 suggests a moderate curvature at that point.

Summary of the Procedure

To summarize, the key steps involved in finding the radius of curvature at a specific parameter value:

1. Identify the parametric equations: $x(t)$ and $y(t)$.
2. Compute derivatives: dx/dt, dy/dt, d^2x/dt^2, d^2y/dt^2.
3. Evaluate derivatives at the specific t.
4. Apply the radius of curvature formula:

$$
R = \frac{ \left[(dx/dt)^2 + (dy/dt)^2 \right]^{3/2} }{ | dx/dt \times d^2 y/dt^2 - dy/dt \times d^2 x/dt^2 | }
$$

5. Simplify to obtain the numerical value.

Final Thoughts

Understanding how to find the radius of curvature at a particular point on a parametric curve is an essential skill in advanced calculus and engineering disciplines. It helps in analyzing physical systems where curvature impacts behavior, such as in the design of roads, roller coasters, or optical systems. This problem exemplifies the practical application of derivative calculus in geometric contexts, reinforcing the importance of meticulous calculation and interpretation.

If you're interested in exploring similar problems, consider varying the

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parameter \( t \), analyzing different curves, or extending the concept to
three-dimensional space. Mastery of these techniques opens the door to a
deeper understanding of the geometry of curves and their real-world
applications.
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