Find The Slope Of The Line Tangent To The Graph Of Y = 10x/x-3 At X = -2.

Find The Slope Of The Line Tangent To The Graph Of Y = 10x / (x - 3) At x = -2.

Understanding how to determine the slope of a tangent line to a curve at a specific point is a fundamental concept in calculus. The slope of the tangent line provides insight into the instantaneous rate of change of the function at that point. In this article, we will explore the process of finding this slope for the function \(y = \frac{10x}{x - 3} \) at \(x = -2 \). We will walk through the necessary steps, including differentiation techniques, simplifying expressions, and calculating the derivative's value at the given point.

Understanding the Function and Its Significance

Overview of the Function \(y = \frac{10x}{x - 3} \)

The function \(y = \frac{10x}{x - 3} \) is a rational function, which means it is expressed as the ratio of two polynomials. Its behavior is characterized by potential asymptotes, discontinuities, and specific points where the slope can be evaluated. The function is undefined at \(x = 3 \), which causes a vertical asymptote, but our point of interest \(x = -2 \) is well within the domain of the function.

Why Find the Slope at a Specific Point?

The slope of the tangent line at a point \(x = a \) is the derivative \(y' \) evaluated at \(x = a \). This slope indicates how rapidly the function is increasing or decreasing at that precise point. For example, if the derivative is positive, the function is increasing; if negative, decreasing. Understanding this slope is crucial in many applications, from physics to economics, where the rate of change impacts decision-making and analysis.

Step 1: Differentiating the Function $y = \frac{10x}{x - 3}$

Choosing a Differentiation Method

Since the function is a ratio of two functions, the quotient rule is the most appropriate method for differentiation. The quotient rule states:

$$
\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right] = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{[v(x)]^2}
$$

where $u(x)$ is the numerator and $v(x)$ is the denominator.

Identifying $u(x)$ and $v(x)$

- $u(x) = 10x$

- $v(x) = x - 3$

Calculating the Derivatives of $u(x)$ and $v(x)$

- $u'(x) = 10$

- $v'(x) = 1$

Applying the Quotient Rule

Using the quotient rule:

$$
y' = \frac{(x - 3) \times 10 - 10x \times 1}{(x - 3)^2}
$$

Simplify numerator:

$$
y' = \frac{10(x - 3) - 10x}{(x - 3)^2}
$$

Step 2: Simplifying the Derivative Expression

Expanding and Combining Like Terms

Calculate the numerator:

$$
10x - 30 - 10x
$$

Notice that $10x$ and $-10x$ cancel out:

$$
(10x - 10x) - 30 = 0 - 30 = -30
$$

Therefore, the derivative simplifies to:

$$
y' = \frac{-30}{(x - 3)^2}
$$

This is a key result because it allows us to evaluate the slope directly at the given x-value.

Step 3: Evaluating the Derivative at $x = -2$

Substituting $x = -2$ into the Derivative

Plug in $x = -2$:

$$
y'(-2) = \frac{-30}{(-2 - 3)^2}
$$

Calculate the denominator:

$$
-2 - 3 = -5
$$

Square it:

$$
(-5)^2 = 25
$$

Now, compute the slope:

$$
y'(-2) = \frac{-30}{25} = -\frac{6}{5}
$$

Result:

The slope of the tangent line to the graph of $y = \frac{10x}{x - 3}$ at $x = -2$ is $-\frac{6}{5}$.

Interpreting the Result

Significance of the Slope

The negative value indicates that at $x = -2$, the function is decreasing—meaning the graph is slanting downward as it passes through that point. The magnitude $\frac{6}{5}$ shows the rate of change; for each unit increase in x, y decreases approximately 1.2 units.

Implications in Applied Contexts

- In physics, if y represented position and x time, the slope would represent velocity. A negative slope indicates motion in the reverse direction.
- In economics, it could represent decreasing returns or diminishing marginal utility at that point.
- In calculus, this process exemplifies how derivatives enable the analysis of the behavior of functions at specific points.

Summary and Key Takeaways

- Identifying the correct differentiation rule (quotient rule) is crucial when dealing with rational functions.
- Simplifying the derivative expression makes evaluation straightforward.
- Evaluating the derivative at a specific point involves substituting the x-value directly into the simplified derivative expression.
- The resulting slope provides immediate insights into the function's behavior at that point.

Final Answer:

The slope of the tangent line to the graph of $y = \frac{10x}{x - 3}$ at $x = -2$ is $-\frac{6}{5}$.

Additional Considerations

Verifying the Point on the Graph

While the primary goal was to find the slope, confirming the exact point on the graph can deepen understanding. To do this, substitute $x = -2$ into the original function:

$$
y = \frac{10 \times (-2)}{-2 - 3} = \frac{-20}{-5} = 4
$$

So, the point of tangency is $(-2, 4)$.

Graphical Interpretation

Plotting the function and the tangent line at $(-2, 4)$ with the calculated slope will visually demonstrate the tangent's orientation. The tangent line equation can be written as:

$$
y - 4 = -\frac{6}{5}(x + 2)
$$

which can be used for further visualization or analysis.

Conclusion

Calculating the slope of the tangent line to a rational function involves understanding the quotient rule, simplifying derivatives, and carefully evaluating at the specific point. For the function $y = \frac{10x}{x - 3}$, the derivative simplifies elegantly to $y' = \frac{-30}{(x - 3)^2}$. Evaluating this at $x = -2$ yields a slope of $-\frac{6}{5}$, indicating a decreasing function at that point with a moderate rate of change. Mastery of these techniques is essential in calculus, providing invaluable tools for analyzing the behavior of complex functions across various disciplines.

Frequently Asked Questions

How do you find the slope of the tangent line to the graph of y = 10x / (x - 3) at x = -2?

To find the slope, first differentiate y = 10x / (x - 3) using the quotient rule, then substitute x = -2 into the derivative.

What is the derivative of y = 10x / (x - 3)?

Using the quotient rule, the derivative is y' = (10(x - 3) - 10x) / (x - 3)^2 = -30 / (x - 3)^2.

What is the slope of the tangent line at x = -2 for the given function?

Substituting x = -2 into the derivative, y' = -30 / (-2 - 3)^2 = -30 / (-5)^2 = -30 / 25 = -6/5.

Why do we use the quotient rule to differentiate y = 10x / (x - 3)?

Because y is a ratio of two functions, and the quotient rule provides the correct derivative for such functions.

What is the equation of the tangent line at x = -2 on the graph of y = 10x / (x - 3)?

The slope is -6/5, and the point at x = -2 gives y = 10(-2) / (-2 - 3) = -20 / -5 = 4. So, the tangent line is y - 4 = (-6/5)(x + 2).

Additional Resources

Find The Slope Of The Line Tangent To The Graph Of Y = 10x / (x - 3) At x = -2

Introduction

Understanding how to find the slope of a tangent line to a curve is a fundamental concept in calculus, providing insights into the behavior of functions at specific points. The problem at hand involves a rational function:

$$ y = \frac{10x}{x - 3} $$

and requires us to determine the slope of the tangent line when $x = -2$. This task involves several key steps, including differentiating the function to find its derivative (which represents the slope of the tangent line at any point), evaluating that derivative at the specified x-value, and interpreting the result.

In this detailed exploration, we will:

- Break down the function and its properties
- Derive the derivative using appropriate calculus rules
- Calculate the slope at the specified point
- Discuss the significance of the tangent slope
- Explore potential nuances and related concepts

Understanding the Function $y = \frac{10x}{x - 3}$

Before delving into the derivative, it's essential to analyze the function's basic properties:

1. Nature of the Function

- Type: Rational function (ratio of two polynomials)
- Numerator: \(10x \), a linear polynomial
- Denominator: \(x - 3 \), also linear

This combination leads to a function with potential asymptotes and interesting behavior near certain points.

2. Domain

- The function is defined for all \(x \) except where the denominator equals zero
- Vertical asymptote at: \(x = 3 \)

Thus, the domain:

\[\boxed{ \text{All real } x \neq 3 } \]

Since \(x = -2 \neq 3 \), the point of interest falls within the domain.

3. Behavior at \(x = -2 \)

- The function is continuous at \(x = -2 \), enabling us to find the tangent slope there.

Calculating the Derivative \(\frac{dy}{dx} \)

The core step in finding the tangent slope is differentiating \(y \) with respect to \(x \). Because \(y \) is a quotient, the quotient rule is the most straightforward approach.

1. Recall the Quotient Rule

Given a function:

\[y = \frac{u(x)}{v(x)} \]

the derivative is:

\[y' = \frac{u'v - uv'}{v^2} \]

where:

- \(u(x) = 10x \)
- \(v(x) = x - 3 \)

2. Compute \(u' \) and \(v' \)

- \(u' = \frac{d}{dx} (10x) = 10 \)
- \(v' = \frac{d}{dx} (x - 3) = 1 \)

3. Apply the Quotient Rule

\[
\begin{aligned}
\frac{dy}{dx} &= \frac{(10)(x - 3) - (10x)(1)}{(x - 3)^2} \\
&= \frac{10x - 30 - 10x}{(x - 3)^2}
\end{aligned}
\]

4. Simplify the Derivative

Observe that:

\[10x - 10x = 0 \]

so the numerator simplifies to:

\[-30 \]

Therefore:

\[
\boxed{
\frac{dy}{dx} = \frac{-30}{(x - 3)^2}
}
\]

This expression gives the slope of the tangent line at any point \(x \neq 3 \).

Evaluating the Derivative at $x = -2$

With the general derivative in hand, substitute $x = -2$:

$$
\frac{dy}{dx} \bigg|_{x = -2} = \frac{-30}{(-2 - 3)^2}
$$

Calculate the denominator:

$$
-2 - 3 = -5
$$
$$
(-5)^2 = 25
$$

Thus:

$$
\frac{dy}{dx} \bigg|_{x = -2} = \frac{-30}{25} = -\frac{6}{5}
$$

Final Answer:

$$
\boxed{
\text{The slope of the tangent line at } x = -2 \text{ is } -\frac{6}{5}
}
$$

Interpreting the Result

The negative value indicates that at $x = -2$, the graph of the function is decreasing. The magnitude $\frac{6}{5}$ suggests a moderate slope, and the negative sign confirms the downward trend at that point.

This slope is crucial for understanding the local behavior of the function:

- Rate of change: The function decreases at a rate of approximately 1.2 units vertically for every 1 unit horizontally to the right.
- Tangent line: The linear approximation at the point $x = -2$ can be constructed using the point-slope form of a line, which is valuable for graphing or analyzing the function's local behavior.

Additional Insights and Related Concepts

While the primary goal was to find the tangent slope at a specific point, exploring related ideas enhances comprehension:

1. Connection to the Derivative Function

- The derivative $y' = \frac{-30}{(x - 3)^2}$ is valid for all $x \neq 3$.
- It shows the slope is always negative (since numerator and denominator are both positive or negative, but denominator squared makes it positive).
- As $x \to 3$, the derivative tends to negative infinity, reflecting the vertical asymptote.

2. Behavior Near Asymptotes and Critical Points

- The function's slope approaches zero as $x \to \pm \infty$, indicating a flattening trend.
- Near $x = 3$, the derivative becomes unbounded, illustrating the steepness of the graph.

3. Geometric Interpretation

- The tangent line at $x = -2$ has the equation:

$$
y = y(-2) + m(x + 2)
$$

where:

- $y(-2)$ is the function value at $x = -2$:

$$
y(-2) = \frac{10 \times (-2)}{-2 - 3} = \frac{-20}{-5} = 4
$$

- $m = -\frac{6}{5}$

Thus, the tangent line:

$$
y = 4 - \frac{6}{5}(x + 2)
$$

This line approximates the function near $x = -2$.

4. Practical Applications

- Tangent slopes are used in optimization problems, where understanding the rate of change helps identify maxima, minima, or points of inflection.
- In physics, the slope of a position-time graph corresponds to velocity.
- In economics, the derivative indicates marginal changes.

Conclusion

Finding the slope of the tangent line to a curve at a specific point involves a systematic process rooted in calculus principles. For the function:

$$ y = \frac{10x}{x - 3} $$

the derivative computed via the quotient rule reveals:

$$ y' = \frac{-30}{(x - 3)^2} $$

which, when evaluated at $x = -2$, yields:

$$ \boxed{ -\frac{6}{5} } $$

This result encapsulates the local behavior of the function at that point, demonstrating a decreasing trend with a moderate slope. Such analysis forms the foundation for more advanced topics like optimization, curve sketching, and understanding real-world phenomena modeled by rational functions. Recognizing the derivative's role and carefully interpreting its value at specific points are vital skills for students and practitioners working with calculus concepts.

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