Find The Smallest Positive Integer, N,such That 168n Is A Square Number.

Find The Smallest Positive Integer, N, such That 168N Is A Square Number.

Understanding the problem of finding the smallest positive integer N such that $168N$ is a perfect square is an interesting exercise in number theory. This problem involves prime factorization, properties of perfect squares, and the application of divisibility rules. In this article, we will explore the problem step-by-step, discuss relevant mathematical concepts, and provide a detailed solution approach.

Introduction to the Problem

The core question asks: What is the minimal positive integer N such that when multiplied by 168, the product becomes a perfect square?

Mathematically, this can be expressed as:

$$
\text{Find } N \in \mathbb{Z}^+ \text{ such that } 168N = k^2 \text{ for some } k \in \mathbb{Z}
$$

Understanding this problem involves analyzing the prime factorization of 168 and determining what factors N must contain to make $168N$ a perfect square.

Prime Factorization of 168

The first step is to factor 168 into its prime components:

$$
168 = 2 \times 84 = 2 \times 2 \times 42 = 2^2 \times 42
$$
$$
42 = 2 \times 21
$$
$$
21 = 3 \times 7
$$

Putting it all together:

$$
168 = 2^3 \times 3^1 \times 7^1
$$

Expressed as prime factorization:

$$
168 = 2^3 \times 3^1 \times 7^1
$$

This factorization is key to understanding what N must contain.

Conditions for a Perfect Square

A number is a perfect square if and only if all prime factors have even exponents in its prime factorization.

For the product $168N$ to be a perfect square, the combined exponents of all prime factors in $168N$ must be even:

$$
168N = 2^3 \times 3^1 \times 7^1 \times N
$$

Suppose:

$$
N = 2^a \times 3^b \times 7^c \times \text{(possibly other primes)}
$$

To minimize N, we focus on the primes involved in 168 first, and then consider whether other primes are necessary.

The combined exponents in $168N$ become:

- For 2: $3 + a$
- For 3: $1 + b$
- For 7: $1 + c$

For $168N$ to be a perfect square, each of these must be even:

$$
3 + a \equiv 0 \pmod{2}
$$
$$
1 + b \equiv 0 \pmod{2}
$$
$$
1 + c \equiv 0 \pmod{2}
$$

\]

Solving for \(a, b, c \):

- \(a \) must be odd because \(3 + a \) even \(\Rightarrow a \equiv 1 \pmod{2}\)
- \(b \) must be odd because \(1 + b \) even \(\Rightarrow b \equiv 1 \pmod{2}\)
- \(c \) must be odd because \(1 + c \) even \(\Rightarrow c \equiv 1 \pmod{2}\)

Since we are looking for the smallest positive \(N \), choose the smallest odd exponents:

\[
a = 1, \quad b = 1, \quad c = 1
\]

Therefore:

\[
N = 2^a \times 3^b \times 7^c = 2^1 \times 3^1 \times 7^1 = 2 \times 3 \times 7 = 42
\]

Verifying the Solution

Now, verify that \(N = 42 \) makes \(168N \) a perfect square:

\[
168 \times 42 = (2^3 \times 3^1 \times 7^1) \times (2^1 \times 3^1 \times 7^1) = 2^{3+1} \times 3^{1+1} \times 7^{1+1} = 2^4 \times 3^2 \times 7^2
\]

The exponents are:

- \(2^4 \), which is a perfect square.
- \(3^2 \), which is a perfect square.
- \(7^2 \), which is a perfect square.

Since all exponents are even, \(168 \times 42 \) is indeed a perfect square.

Calculate the value:

\[
168 \times 42 = 7056
\]

Check whether 7056 is a perfect square:

$$
\sqrt{7056} \approx 84
$$

Since:

$$
84^2 = 7056
$$

Indeed, 7056 is a perfect square, confirming our solution.

Conclusion and Final Answer

The smallest positive integer N such that $168N$ is a perfect square is:

$$
\boxed{42}
$$

This result is obtained by analyzing the prime factorization of 168, ensuring that all prime exponents in the product are even, and choosing the minimal odd exponents for N.

Additional Considerations

While the above method provides a straightforward solution, it also highlights several key concepts in number theory:

- **Prime Factorization:** Breaking down numbers into their prime components is fundamental for understanding divisibility and perfect squares.
- **Perfect Square Conditions:** All prime exponents in a perfect square are even; this guides how to adjust exponents via multiplication.
- **Minimal Multipliers:** To find the smallest N, always choose the minimal odd exponents necessary to make all combined exponents even.

Practical Applications and Further Study

Understanding such problems extends beyond pure mathematics into fields like cryptography, coding theory, and computational algorithms where prime factorization and number properties are critical.

Further study can include:

- Exploring similar problems with other numbers, such as finding N such that mN is a perfect cube or higher powers.
- Analyzing the role of prime factorization in modular arithmetic and number theory applications.
- Learning algorithms for prime factorization, especially for large numbers, which are vital in cryptography.

Summary

In summary, the problem of finding the smallest positive integer N such that $168N$ is a perfect square hinges on prime factorization and ensuring all prime exponents in the product are even. By carefully analyzing the exponents of 168 and adjusting them through multiplication, we find that:

$$
\boxed{N = 42}
$$

is the minimal solution, and the product $168 \times 42 = 7056$ confirms this as a perfect square. This exercise exemplifies the power of prime factorization and exponents in understanding the properties of numbers and solving classical problems in number theory.

Frequently Asked Questions

What is the problem asking for in finding the smallest positive integer N such that 168N is a perfect square?

It asks for the smallest positive integer N where multiplying it by 168 results in a perfect square number.

How do you determine if 168N is a perfect square?

By factoring 168 and N into primes, then ensuring all prime exponents in their combined factorization are even.

What is the prime factorization of 168?

$168 = 2^3 3^1 7^1$.

How can the prime factorization of N be used to find the smallest N?

By making the exponents of all primes in 168N even, N must include the prime factors raised to the minimal powers needed to achieve even exponents.

What is the minimal exponent for each prime in N to make 168N a perfect square?

For 2^3, need one more 2 to make exponent 4; for 3^1, need one more 3 to make exponent 2; for 7^1, need one more 7 to make exponent 2.

What is the smallest N based on the prime exponents identified?

$N = 2^{1} 3^{1} 7^{1} = 2 3 7 = 42$.

Why is N = 42 the smallest positive integer satisfying the condition?

Because adding the minimal necessary prime factors to make all exponents even results in N = 42, the smallest such number.

How can you verify that 168 42 is a perfect square?

Calculate 168 42 = 7056, then check if 7056 is a perfect square (which it is, since $84^2 = 7056$).

What is the general method to find the smallest N for similar problems?

Factor the initial number, identify which prime exponents are odd, and multiply by the minimal primes to make all exponents even.

Are there other methods to find N besides prime factorization?

Prime factorization is the most straightforward method; alternative approaches include using number theory algorithms or algebraic methods, but prime factorization is most common for this problem.

Additional Resources

Find The Smallest Positive Integer, N, Such That 168N Is A Square Number is a fascinating problem that combines elements of number theory, prime factorization, and algebraic reasoning. It challenges mathematicians and students alike to explore the properties of integers and their relationships with perfect squares. The quest to identify the minimal positive integer N that makes 168N a perfect square is not only an intriguing puzzle but also a gateway to understanding deeper concepts such as prime exponents, divisibility, and the structure of square numbers.

Introduction to the Problem

The problem asks us to find the smallest positive integer N such that when multiplied by 168, the product is a perfect square. Mathematically, this can be expressed as:

$$168 \times N = k^2$$

for some integer k. Our goal is to determine the minimal N that satisfies this condition.

This problem falls under the broader category of "finding the least number to make a product a perfect square," a common theme in number theory. Such problems often involve examining prime factorizations because the properties of perfect squares are directly tied to the exponents of their prime factors.

Understanding the Prime Factorization of 168

Before proceeding, it's essential to analyze the prime factorization of 168, as it forms the foundation of our calculations.

Prime Factorization of 168

Let's factor 168 into its prime components:

- Divide 168 by 2: 168 ÷ 2 = 84
- Divide 84 by 2: 84 ÷ 2 = 42
- Divide 42 by 2: 42 ÷ 2 = 21 (which is no longer divisible by 2)
- Divide 21 by 3: 21 ÷ 3 = 7
- 7 is a prime number

Thus, the prime factorization of 168 is:

$$ 168 = 2^3 \times 3^1 \times 7^1 $$

Understanding these exponents is crucial because for $168N$ to be a perfect square, the exponents of all prime factors in the product must be even.

Conditions for a Number to Be a Perfect Square

A key property of perfect squares is that, in their prime factorization, all exponents are even integers. Therefore, for:

$$ 168 \times N = 2^{3} \times 3^{1} \times 7^{1} \times N $$

to be a perfect square, the combined exponents of each prime factor in the product must be even.

Suppose N has the prime factorization:

$$ N = 2^{a} \times 3^{b} \times 7^{c} \times \text{(possibly other primes)} $$

Then,

$$ 168N = 2^{3 + a} \times 3^{1 + b} \times 7^{1 + c} \times \text{(other primes)} $$

For $168N$ to be a perfect square, the following conditions must hold:

- $3 + a$ is even
- $1 + b$ is even
- $1 + c$ is even

Since the exponents in N are non-negative integers (because N is positive), the minimal solutions for a, b, c are determined by these parity conditions.

Calculating the Minimal N

Let's analyze each prime factor's exponent separately to find the minimal values of a, b, c.

For the prime 2:

- $3 + a$ must be even
- 3 is odd, so a must be odd (since odd + odd = even)

The smallest non-negative odd integer is 1, so:

$$a = 1$$

For the prime 3:

- $1 + b$ must be even
- 1 is odd, so b must be odd

Smallest such b:

$$b = 1$$

For the prime 7:

- $1 + c$ must be even
- 1 is odd, so c must be odd

Smallest such c:

$$c = 1$$

Constructing the Minimal N

Now, with the minimal exponents identified:

$$N = 2^{a} \times 3^{b} \times 7^{c} = 2^{1} \times 3^{1} \times 7^{1}$$

Calculating this:

$$N = 2 \times 3 \times 7 = 42$$

Thus, the smallest positive integer N such that $168N$ is a perfect square is 42.

Verification of the Result

Let's verify that multiplying 168 by 42 results in a perfect square:

$$ 168 \times 42 = (2^3 \times 3^1 \times 7^1) \times (2^1 \times 3^1 \times 7^1) $$

$$ = 2^{3 + 1} \times 3^{1 + 1} \times 7^{1 + 1} $$

$$ = 2^{4} \times 3^{2} \times 7^{2} $$

Since all exponents are even:

- $2^{4} = 16$
- $3^{2} = 9$
- $7^{2} = 49$

The product:

$$ 16 \times 9 \times 49 = (16 \times 9) \times 49 = 144 \times 49 $$

Calculating:

$$ 144 \times 49 = 144 \times (50 - 1) = 144 \times 50 - 144 \times 1 = 7200 - 144 = 7056 $$

Now, check if 7056 is a perfect square:

- $84^2 = 7056$

Indeed, $84^2 = 7056$, confirming that $168 \times 42 = 84^2$ is a perfect square.

Features and Insights

Pros:

- Systematic Approach: The solution method relies on prime factorization and parity considerations, making it straightforward and reliable.
- Educational Value: Demonstrates the importance of prime exponents in understanding perfect squares.
- Minimal Solution: Finds the smallest N, which is often more meaningful in practical applications.

Cons:

- Limited to Specific Numbers: The approach is tailored to integers with known prime factorizations; more complex numbers may require advanced techniques.
- Assumption of Positivity: The analysis considers positive N; negative or zero values are not addressed.
- Prime Factor Focus: Does not handle cases involving composite or non-prime factors explicitly, which might be relevant in broader contexts.

Broader Implications and Variations

This problem exemplifies a common theme in number theory: transforming an integer into a perfect square by multiplication. Variations include:

- Finding the smallest N such that $M \times N$ is a perfect cube or higher power.
- Extending the problem to include additional prime factors or constraints.
- Analyzing similar problems in modular arithmetic or algebraic number theory.

Such problems deepen understanding of the structure of integers and their properties, with applications spanning cryptography, coding theory, and mathematical puzzles.

Conclusion

The process of finding the smallest positive integer N such that $168N$ is a perfect square underscores the elegance and utility of prime factorization in number theory. By analyzing the exponents of the prime factors of 168 and ensuring they become even upon multiplication by N, we determined that:

$$\boxed{N = 42}$$

is the minimal such number. This exercise not only reinforces core mathematical principles but also demonstrates the beauty of systematic problem-solving. Whether for academic exploration or practical problem-solving, understanding how to manipulate prime factors to achieve perfect powers is a fundamental skill that continues to offer insights into the structure of integers.

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In summary:

- Prime factorization of 168: \( 2^3 \times 3^1 \times 7^1 \)
- Conditions for perfect square: all exponents even
- Minimal \( N \): \( 2^1 \times 3^1 \times 7^1 = 42 \)
- Verified that \( 168 \times 42 = 84^2 \), a perfect square

This problem exemplifies the power of prime factorization and parity
considerations in solving number theory puzzles, providing both a clear
methodology and insights into the fundamental properties of integers.
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