Find The Verte Of The Quadratic Function Given Below F(x) = (x-4) (x+2)

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When working with quadratic functions, one of the most important features to identify is the vertex. The vertex of a parabola provides critical information about its maximum or minimum point, the axis of symmetry, and the parabola's shape. In this comprehensive guide, we'll focus on how to find the vertex of the quadratic function \(F(x) = (x-4)(x+2) \). We'll explore various methods, including expanding the function, converting it to vertex form, and using calculus, to ensure you have a thorough understanding of this process.

Understanding the Quadratic Function \(F(x) = (x-4)(x+2) \)

Before diving into the methods to find the vertex, it's essential to understand the structure of the given quadratic function.

Factored Form of the Quadratic Function

The given function is expressed in factored form:

\[

F(x) = (x - 4)(x + 2)

\]

This form reveals the roots of the parabola and makes it easier to analyze the x-intercepts.

Expanding the Function

To analyze the parabola's vertex, it's often helpful to convert the factored form into standard quadratic form:

$$
F(x) = x^2 + bx + c
$$

Let's expand:

$$
F(x) = (x - 4)(x + 2) = x \times x + x \times 2 - 4 \times x - 4 \times 2
$$

$$
F(x) = x^2 + 2x - 4x - 8
$$

$$
F(x) = x^2 - 2x - 8
$$

Now, the quadratic function is in standard form, which simplifies the process of finding the vertex.

Methods to Find the Vertex of a Quadratic Function

There are several reliable methods to find the vertex of a quadratic function. We'll explore the most common ones:

1. **Using the vertex formula (axis of symmetry method)**

2. **Completing the square**

3. **Converting to vertex form**

4. **Using calculus (derivative method)**

Each method offers unique insights and is suitable for different contexts.

Method 1: Using the Vertex Formula (Axis of Symmetry)

The vertex of a parabola given by the quadratic function $y = ax^2 + bx + c$ can be found directly using the formula:

$$
x_{v} = -\frac{b}{2a}
$$

Step-by-step process:

1. Identify coefficients:

$$
a = 1, \quad b = -2, \quad c = -8
$$

2. Calculate the x-coordinate of the vertex:

$$
x_{v} = -\frac{b}{2a} = -\frac{-2}{2 \times 1} = \frac{2}{2} = 1
$$

3. Find the y-coordinate (F(x)) at $x = 1$:

$$
F(1) = (1)^2 - 2(1) - 8 = 1 - 2 - 8 = -9
$$

4. Vertex coordinates:

$$
(x_{v}, y_{v}) = (1, -9)
$$

Summary:

The vertex of the quadratic function $F(x) = (x-4)(x+2)$ is at (1, -9).

Method 2: Completing the Square

Completing the square is a powerful technique to convert any quadratic into its vertex form:

$$
F(x) = a(x - h)^2 + k
$$

where (h, k) is the vertex.

Step-by-step process:

1. Start with the expanded form:

$$
F(x) = x^2 - 2x - 8
$$

2. Factor out the leading coefficient (if not 1):

Since $a = 1$, no need to factor.

3. Complete the square:

- Take half of the coefficient of x, which is -2:

$$
\frac{-2}{2} = -1
$$

- Square this value:

$$
(-1)^2 = 1
$$

- Rewrite the quadratic as:

$$
F(x) = (x^2 - 2x + 1) - 1 - 8
$$

- Simplify:

$$
F(x) = (x - 1)^2 - 9
$$

4. Identify the vertex:

The vertex form is:

$$

$$
F(x) = (x - 1)^2 - 9
$$

Therefore, the vertex is at:

$$
(h, k) = (1, -9)
$$

This confirms our earlier calculation.

Method 3: Converting to Vertex Form via Algebraic Manipulation

Another approach is to directly convert the factored form to vertex form:

$$
F(x) = (x - 4)(x + 2)
$$

1. Expand:

$$
F(x) = x^2 + 2x - 4x - 8 = x^2 - 2x - 8
$$

2. Complete the square:

As previously shown, this yields:

$$
F(x) = (x - 1)^2 - 9
$$

3. Identify the vertex:

$$
(1, -9)
$$

This method reinforces the previous findings.

Method 4: Using Calculus (Derivative Method)

Calculus provides a more rigorous way to find the vertex by locating the critical points where the function's slope is zero.

Step-by-step process:

1. Differentiate the function:

$$
F(x) = x^2 - 2x - 8
$$

$$
F'(x) = 2x - 2
$$

2. Set the derivative to zero to find critical points:

$$
2x - 2 = 0
$$

$$
x = 1
$$

3. Determine the y-value at $x = 1$:

$$
F(1) = 1 - 2 - 8 = -9
$$

4. Confirm it's a minimum or maximum:

The second derivative:

$$
F''(x) = 2 > 0
$$

Since the second derivative is positive, $x = 1$ corresponds to a minimum point, which is the vertex.

Result:

Vertex at (1, -9).

Graphical Interpretation of the Vertex

Understanding the geometric significance of the vertex helps in visualizing the parabola:

- The vertex represents the highest or lowest point of the parabola depending on its orientation.
- For $F(x) = (x-4)(x+2)$, since the coefficient of x^2 is positive (1), the parabola opens upward.
- The vertex at (1, -9) is the minimum point of the parabola.

Additional Insights and Applications

Knowing how to find the vertex of a quadratic function like $F(x) = (x-4)(x+2)$ has practical applications:

- Optimizing real-world problems: For example, maximizing profit or minimizing cost.
- Analyzing quadratic models: Understanding the turning point helps in predictions.
- Graphing quadratics: The vertex provides a key point for sketching the parabola accurately.

Key Points to Remember:

- The vertex form of a quadratic is $y = a(x - h)^2 + k$, with (h, k) as the vertex.
- Converting from factored or standard form to vertex form simplifies finding the vertex.
- The vertex's x-coordinate can be found using $-b/2a$, and the y-coordinate by evaluating the function at that x.
- Completing the square is a systematic way to convert any quadratic into vertex form.

- Calculus offers a rigorous approach through derivatives to locate the vertex.

Summary: Finding the Vertex of \(F(x) = (x-4)(x+2) \)

In this guide, we've demonstrated multiple methods to find the vertex:

- Using the vertex formula based on standard quadratic form yields \((1, -9) \).
- Completing the square confirms the

Frequently Asked Questions

How do you find the vertex of the quadratic function f(x) = (x - 4)(x + 2)?

First, expand the function to standard form: f(x) = x^2 - 2x - 8. Then, use the vertex formula x = -b/2a. Here, a = 1 and b = -2, so x = -(-2)/(21) = 1. Substitute x = 1 back into the function to find the y-coordinate: f(1) = (1 - 4)(1 + 2) = (-3)(3) = -9. Therefore, the vertex is at (1, -9).

What is the vertex form of the quadratic function given as f(x) = (x - 4)(x + 2)?

Expanding the original function gives f(x) = x^2 - 2x - 8. Completing the square, the vertex form is f(x) = (x - 1)^2 - 10. The vertex is at (1, -10).

Why is the vertex of the quadratic function f(x) = (x - 4)(x + 2)

important?

The vertex represents the maximum or minimum point of the parabola. For this function, which opens upward, the vertex at (1, -9) is the point of the minimum value of the quadratic, indicating the lowest point on the graph.

How does the factored form of the quadratic help in finding the vertex?

While the factored form shows the roots at x = 4 and x = -2, it doesn't directly give the vertex. However, the vertex lies midway between the roots. So, the x-coordinate of the vertex is (4 + (-2))/2 = 1, providing a quick way to find the vertex's x-coordinate before calculating the y-value.

What is the significance of the axis of symmetry in relation to the vertex for this quadratic?

The axis of symmetry passes through the vertex and divides the parabola into two mirror images. For the function, the axis of symmetry is at x = 1, which is the midpoint between the roots at x = 4 and x = -2, confirming the vertex's x-coordinate.

Additional Resources

Find The Vertex Of The Quadratic Function Given Below F(x) = (x - 4)(x + 2)

Understanding how to find the vertex of a quadratic function is a fundamental skill in algebra, essential for graphing, analyzing the behavior of the parabola, and solving various mathematical problems. The given function, F(x) = (x - 4)(x + 2), is expressed in factored form, which provides a straightforward pathway to determine its vertex. In this comprehensive guide, we will explore the process step-by-step, discussing the significance of the vertex, methods to find it, and the implications for the graph of the parabola.

Understanding the Quadratic Function

Before diving into the vertex calculation, it is crucial to understand the nature of the quadratic function provided.

Factored Form of the Function

The function:

$$F(x) = (x - 4)(x + 2)$$

is in factored form, where each factor corresponds to the roots of the quadratic. The roots are the points where the parabola intercepts the x-axis:

- $x = 4$ (since $x - 4 = 0$)
- $x = -2$ (since $x + 2 = 0$)

Factored form makes it easy to identify roots directly, but for finding the vertex, it is often helpful to convert this to standard form.

Standard Form of a Quadratic

The standard form of a quadratic function is:

$$F(x) = ax^{2} + bx + c$$

Converting the factored form into standard form allows us to use the vertex formula directly:

$$ x_{vertex} = -\frac{b}{2a} $$

which simplifies the process of finding the vertex.

Converting to Standard Form

To find the vertex, it's effective to expand the factored form into standard form.

Step-by-Step Expansion

Starting with:

$$ F(x) = (x - 4)(x + 2) $$

Apply the distributive property (FOIL):

$$
\begin{aligned}
F(x) &= x \times x + x \times 2 - 4 \times x - 4 \times 2 \\
&= x^{2} + 2x - 4x - 8 \\
&= x^{2} - 2x - 8
\end{aligned}
$$

Now, the quadratic in standard form is:

$$ F(x) = x^{2} - 2x - 8 $$

Here, $a = 1$, $b = -2$, and $c = -8$.

Finding the Vertex

With the quadratic in standard form, the vertex can be found using the vertex formula.

The Vertex Formula

The x-coordinate of the vertex is given by:

$$ x_{vertex} = -\frac{b}{2a} $$

Substituting the values:

$$
x_{vertex} = -\frac{-2}{2 \times 1} = \frac{2}{2} = 1
$$

To find the y-coordinate of the vertex, substitute $x = 1$ back into the function:

$$
F(1) = (1)^2 - 2(1) - 8 = 1 - 2 - 8 = -9
$$

Therefore, the vertex of the parabola is at:

$$ (1, -9) $$

Graphical Interpretation of the Vertex

The vertex point $(1, -9)$ signifies the parabola's minimum point because the quadratic coefficient $a = 1$ is positive, indicating the parabola opens upward.

Significance of the Vertex

- Minimum Point: Since the parabola opens upward, the vertex is the lowest point on the graph.
- Axis of Symmetry: The vertical line passing through the vertex:

$$ x = 1 $$

acts as the axis of symmetry, dividing the parabola into two mirror images.

- Range of the Function: The range starts at the y-value of the vertex and extends upward:

$$ y \geq -9 $$

Understanding the vertex allows us to sketch the parabola accurately and analyze its properties.

Features and Characteristics of the Quadratic Function

Beyond the vertex, several other features define the shape and position of the parabola.

Roots (x-intercepts)

As found earlier, the roots are at:

$$ x = 4 \quad \text{and} \quad x = -2 $$

These are the points where the parabola crosses the x-axis.

Y-intercept

To find the y-intercept, evaluate $F(0)$:

$$
F(0) = (0 - 4)(0 + 2) = (-4)(2) = -8
$$

So, the y-intercept is at $(0, -8)$.

Symmetry and Shape

- The parabola is symmetric about the line $x = 1$.
- Opens upward, indicating a minimum at the vertex.
- The distance from the vertex to the roots can inform the width of the parabola.

Pros and Cons of Different Methods to Find the Vertex

There are multiple approaches to finding the vertex of a quadratic function. Each has its advantages and limitations.

Method 1: Using the Standard Form

- Pros:
- Direct application of the vertex formula $x = -b / 2a$.
- Straightforward once the quadratic is in standard form.
- Cons:
- Requires transforming factored form into standard form if starting from factored form.
- Slightly more algebraic work.

Method 2: Completing the Square

- Pros:
- Provides insight into the vertex form and the parabola's shape.
- Useful for understanding transformations.
- Cons:
- More involved algebraically.
- Less efficient if the quadratic is already in factored form.

Method 3: Graphical Approach

- Pros:
- Visual understanding of the parabola.
- Effective for approximate solutions.
- Cons:
- Less precise unless drawn meticulously.
- Not suitable for exact calculations.

Applications and Real-World Relevance

Understanding how to find the vertex of a quadratic function like $F(x) = (x - 4)(x + 2)$ is vital in various fields:

- Physics: Analyzing projectile motion where the vertex indicates the maximum height or lowest point.
- Economics: Optimizing profit or cost functions modeled quadratically.
- Engineering: Designing parabolic reflectors and antennas, where the vertex determines focal points.
- Mathematics Education: Developing foundational skills in algebra and graphing.

Conclusion

Finding the vertex of a quadratic function such as $F(x) = (x - 4)(x + 2)$ involves understanding the function's form, converting it into standard form, and applying the vertex formula. In this case, the vertex is located at $(1, -9)$, representing the minimum point of the parabola, which opens upward.

This process not only aids in graphing but also enhances comprehension of the function's properties, symmetry, and range. Mastery of these techniques is essential for solving more complex problems involving quadratic functions and for applying these concepts across various scientific and mathematical disciplines.

Features of the process include:

- Clear conversion from factored to standard form.
- Accurate application of the vertex formula.
- Insight into the parabola's geometry and symmetry.

Pros of this approach:

- Efficiency and precision.
- Direct link between algebraic form and graphical features.

Cons:

- Requires algebraic manipulation, which might be challenging for beginners.
- Less intuitive without visual aids or graphing tools.

In summary, mastering the method to find a parabola's vertex empowers students and professionals to analyze quadratic functions effectively, leading to better problem-solving skills and deeper mathematical understanding.

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