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Understanding the behavior of real-valued functions is fundamental in calculus and mathematical analysis. The given problem involves a function y, its value at a specific point, and a differential equation that governs its behavior. This comprehensive guide aims to clarify the problem, explore solution methods, and interpret the results meticulously. Whether you're a student seeking clarity or a mathematician interested in the nuances of differential equations, this article will serve as an insightful resource.

Decoding the Problem Statement

Before diving into the solution process, it's crucial to interpret the problem's components correctly. The statement appears complex and may contain typographical or notation issues, but with careful analysis, we can extract the intended meaning.

Analyzing the Given Data

The core elements provided are:

- The value of the function at $x=5$: $y(5) = 2$

- A differential equation: $y(5) = 2.16 y + 72 y + 72 y = 0$

- An expression or incomplete statement: $y=\_\_\_$

At first glance, the presentation seems inconsistent because the same notation $y(5)$ is used both as a value and as part of an equation. Likely, the original intent was:

- To specify the initial condition $y(5) = 2$

- To present a differential equation involving y: perhaps something like $y' = 2.16 y + 72 y + 72 y$

- To find the function $y(x)$ satisfying these conditions and initial values.

Given this, we can hypothesize that the differential equation is:

$$
\frac{dy}{dx} = 2.16 y + 72 y + 72 y
$$

which simplifies to:

$$
\frac{dy}{dx} = (2.16 + 72 + 72) y
$$

or

$$
\frac{dy}{dx} = 146.16 y
$$

The initial condition is:

$$
y(5) = 2
$$

and the goal is to find y at a general point, or perhaps specifically y at $x=5$, which is already given.

Mathematical Foundations: Differential Equations and Their Solutions

To solve the problem, we need to understand the nature of the differential equation involved. The most straightforward assumption is that it's a first-order linear ordinary differential equation (ODE).

First-Order Linear ODEs

A first-order linear ODE has the general form:

$$
\frac{dy}{dx} + P(x) y = Q(x)
$$

In our case, if the differential equation is:

$$
\frac{dy}{dx} = k y
$$

where k is a constant (here, 146.16), then it's a simple separable differential equation, and its solution is well-known.

Solving the Differential Equation

Given:

$$
\frac{dy}{dx} = k y
$$

The solution involves separating variables:

$$
\frac{dy}{y} = k dx
$$

Integrate both sides:

$$
\int \frac{1}{y} dy = \int k dx
$$

which yields:

$$
\ln |y| = k x + C
$$

Exponentiating both sides:

$$
|y| = e^{k x + C} = e^{C} e^{k x}
$$

Let $A = e^{C}$ (a positive constant), then:

$$
y(x) = A e^{k x}
$$

Applying the initial condition $y(5) = 2$:

$$
2 = A e^{k \times 5}
$$

which gives:

$$
A = 2 e^{-5k}
$$

Finally, the explicit solution is:

$$
\boxed{
y(x) = 2 e^{k (x - 5)}
}
$$

indicating how y depends exponentially on x.

Step-by-Step Solution Process

Having established the theoretical foundation, let's proceed with solving the specific problem.

Step 1: Simplify the Differential Equation

Assuming the differential equation is:

$$
\frac{dy}{dx} = (2.16 + 72 + 72) y = 146.16 y
$$

which simplifies to:

$$
\frac{dy}{dx} = 146.16 y
$$

This is a standard first-order linear ODE.

Step 2: Write the General Solution

From the previous derivation:

$$
y(x) = A e^{146.16 x}
$$

where A is a constant determined by initial conditions.

Step 3: Apply Initial Conditions

Given:

$$
y(5) = 2
$$

we substitute $x=5$:

$$
2 = A e^{146.16 \times 5}
$$

Calculate the exponent:

$$
146.16 \times 5 = 730.8
$$

Therefore:

$$
A = 2 e^{-730.8}
$$

which is an extremely small number, practically zero, due to the large exponential in the denominator.

Step 4: Write the Explicit Solution

The explicit solution:

$$
\boxed{
y(x) = 2 e^{146.16 (x - 5)}
}
$$

This formula defines y at any point x.

Interpreting and Applying the Solution

Understanding the behavior of this solution involves analyzing the exponential term:

- For $x > 5$, $y(x)$ grows exponentially rapidly.
- For $x < 5$, $y(x)$ diminishes exponentially.

Given the initial condition at $x=5$, the function's value at other points can be computed directly from the formula.

Practical Examples

- Find y at $x=6$:

  $$
  y(6) = 2 e^{146.16 (6 - 5)} = 2 e^{146.16} \approx \text{a very large number}
  $$

  \li>Find y at $x=4$:

  $$
  y(4) = 2 e^{146.16 (4 - 5)} = 2 e^{-146.16} \approx 0
  $$

This exponential growth or decay demonstrates the sensitivity of the solution to initial conditions and the coefficient k.

Additional Considerations and Advanced Topics

Beyond the basic solution, several factors and extensions can be considered.

1. Non-constant Coefficients

If the differential equation involved a function $P(x)$ or $Q(x)$, solutions would require integrating factor methods or numerical approaches.

2. Homogeneous vs. Non-Homogeneous Equations

The current problem appears homogeneous, with solutions involving exponential functions. Non-homogeneous equations introduce additional complexity and solutions involving particular integrals.

3. Numerical Methods

When analytical solutions are infeasible, numerical techniques such as Euler's Method or Runge-Kutta methods are employed to approximate solutions over specified intervals.

4. Stability and Sensitivity

The exponential nature indicates potential issues with stability in modeling real-world phenomena, especially with large coefficients leading to rapid growth or decay.

Conclusion: Final Expression and Summary

Based on the assumptions and calculations, the function y satisfying the differential equation and initial condition is:

$$
\boxed{
y(x) = 2 e^{146.16 (x - 5)}
}
$$

This solution encapsulates the exponential behavior dictated by the differential equation and initial data.

Key Takeaways

1. Interpreting complex or ambiguous problem statements requires careful analysis and assumptions.

2. First-order differential equations with constant coefficients have explicit solutions involving exponentials.

3. Applying initial conditions determines the specific solution from the general form.

4. Understanding the growth or decay behavior of solutions provides insights into the modeled phenomena.

5. Advanced topics such as variable coefficients and numerical methods expand the scope of differential equation analysis.

Additional Resources for Further Study

- Separable Differential Equations - Lamar University
- Linear Differential Equations - Lamar University
- Exponential Functions - Wolfram MathWorld
- Differential Equations - Khan Academy