For What Value Of A Will The Expressions 11(a+2) And 55-22a Be Equal?

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When dealing with algebraic expressions, a common problem is to find the value of a variable that makes two expressions equal. In this case, we are asked to determine the value of a for which the expressions 11(a+2) and 55 - 22a are equal. Understanding how to approach this problem involves applying fundamental algebraic principles, primarily the properties of equality and the distributive property. This article explores the process step by step, providing a comprehensive explanation to ensure clarity and understanding.

Understanding the Problem

Given Expressions

The two expressions provided are:

- Expression 1: 11(a + 2)
- Expression 2: 55 - 22a

The goal is to find the value of a such that:

11(a + 2) = 55 - 22a

This is a simple linear equation in one variable. Solving it involves isolating a on one side of the equation to find its specific value.

Step-by-Step Solution Process

Step 1: Write the Equation

Start by setting the two expressions equal:

$$
11(a + 2) = 55 - 22a
$$

This equation equates the two expressions, and solving it will give the value of a.

Step 2: Expand Both Sides

Apply the distributive property to expand 11(a + 2):

\[
11 \times a + 11 \times 2 = 55 - 22a
\]
\[
11a + 22 = 55 - 22a
\]

Now, the equation is simplified to:

\[
11a + 22 = 55 - 22a
\]

Step 3: Collect Like Terms

The goal is to gather all terms containing a on one side and constants on the other. Let's add 22a to both sides to move all a terms to the left:

\[
11a + 22 + 22a = 55 - 22a + 22a
\]
\[
(11a + 22a) + 22 = 55
\]
\[
33a + 22 = 55
\]

Next, subtract 22 from both sides to isolate the term with a:

\[
33a + 22 - 22 = 55 - 22
\]
\[
33a = 33
\]

Step 4: Solve for a

Divide both sides of the equation by 33:

$$
a = \frac{33}{33} = 1
$$

Thus, the value of a is 1.

Verification of the Solution

Substitute a = 1 back into the original expressions:

- For 11(a + 2):

$$
11(1 + 2) = 11 \times 3 = 33
$$

- For 55 - 22a:

$$
55 - 22 \times 1 = 55 - 22 = 33
$$

Both expressions equal 33 when a = 1, confirming that the solution is correct.

Understanding the Significance of the Solution

Why is finding such a value important?

Determining the value of a that satisfies the equality helps in understanding relationships between different algebraic expressions. This type of problem appears frequently in various contexts, such as:

- Solving for parameters in equations
- Finding intersection points in coordinate geometry
- Analyzing functions and their behaviors

Implications in real-world problems

Suppose these expressions represent some quantities, such as costs, distances, or other measurable entities, and the goal is to find under what conditions they are equal. The process of solving such equations provides insights into the relationships between these quantities.

Additional Concepts and Variations

Handling More Complex Equations

The problem can be extended to more complex equations involving quadratic terms, fractions, or absolute values. The fundamental approach remains:

- Expand and simplify expressions
- Collect like terms
- Isolate the variable
- Verify the solution

Common Mistakes to Avoid

- Forgetting to distribute correctly when expanding brackets
- Making errors while moving terms across the equal sign
- Dividing by zero when the coefficient of the variable is zero (which indicates the original expressions are not equal for any a)

Special Cases

- If, after simplifying, the variable cancels out and the statement reduces to a true or false statement, it indicates infinitely many solutions or no solution, respectively.
- For example, if the simplified equation becomes $0 = 0$, then a can be any value, implying infinite solutions.

Summary

- The key to solving the problem is setting the two expressions equal and applying algebraic operations systematically.
- The process involves expanding, collecting like terms, and isolating the variable.
- The solution $a = 1$ ensures both expressions are equal, verified by substitution.
- Understanding such problems enhances algebraic manipulation skills and prepares learners for more advanced mathematical challenges.

Conclusion

The question of finding the value of a that makes 11(a+2) and 55 - 22a equal is an excellent example of applying basic algebraic principles in practice. By carefully expanding, simplifying, and solving the linear equation, we discover that a = 1 satisfies the condition. Such problems are fundamental in algebra and serve as building blocks for understanding more complex mathematical concepts. Mastering these techniques is essential for students and anyone interested in mathematical problem-solving, providing a strong foundation for future learning in mathematics.

Frequently Asked Questions

For what value of a are the expressions 11(a+2) and 55 - 22a equal?

They are equal when a = 0.

How do you find the value of a that makes 11(a+2) equal to 55 - 22a?

Set 11(a+2) equal to 55 - 22a and solve for a: 11a + 22 = 55 - 22a, then combine like terms and solve to find a = 0.

What steps are involved in solving for a in the equation 11(a+2) = 55 - 22a?

First expand the left side to 11a + 22, then set it equal to 55 - 22a, add 22a to both sides, subtract 22 from both sides, and finally divide to find a = 0.

Is the value of a that makes the expressions equal unique?

Yes, the value a = 0 is unique because the equation simplifies to a single solution.

Can the expressions 11(a+2) and 55 - 22a be equal for any other value of a?

No, the only value where they are equal is a = 0, as shown by solving the equation.

Additional Resources

For What Value Of A Will The Expressions 11(a+2) And 55-22a Be Equal?

Mathematics often presents intriguing problems that challenge our understanding of algebraic relationships and logical reasoning. One such problem asks us to determine the value of the parameter \(a\) for which two algebraic expressions are equal: \(11(a+2)\) and \(55 - 22a\). At first glance, this query appears straightforward, but a comprehensive analysis reveals deeper insights into algebraic manipulation, equation solving, and the importance of understanding parameters within an expression. This article aims to explore this problem in depth, providing clarity through detailed explanations, step-by-step solutions, and contextual understanding, all while maintaining a journalistic tone that emphasizes analytical thinking.

Understanding the Problem: Breaking Down the Expressions

Before diving into calculations, it is crucial to understand what the problem entails. The core question is:

"For what value of \(a\) do the expressions \(11(a+2)\) and \(55 - 22a\) become equal?"

In algebraic terms, this translates to:

\[11(a+2) = 55 - 22a \]

Understanding this setup involves recognizing the components:

- Expression 1: \(11(a+2)\) — a linear expression involving \(a\), scaled by 11.
- Expression 2: \(55 - 22a\) — another linear expression involving \(a\), with a different structure.

The goal is to find the value of \(a\) that satisfies the equation where these two expressions are equal.

Step-by-Step Solution: Solving the Equation

The process of solving this problem is a classic illustration of algebraic techniques — particularly, the method of isolating the variable \(a\). Here are the detailed steps:

Step 1: Write the equation

As per the problem statement:

\[11(a+2) = 55 - 22a \]

Step 2: Expand both sides

Apply the distributive property to the left side:

$$ 11 \times a + 11 \times 2 = 55 - 22a $$

which simplifies to:

$$ 11a + 22 = 55 - 22a $$

Step 3: Gather like terms

To solve for a, bring all terms involving a to one side and constants to the other:

$$ 11a + 22a = 55 - 22 $$

Adding $11a$ and $22a$:

$$ (11a + 22a) = 33a $$

Subtracting 22 from 55:

$$ 55 - 22 = 33 $$

So, the equation becomes:

$$ 33a = 33 $$

Step 4: Solve for a

Divide both sides by 33:

$$ a = \frac{33}{33} = 1 $$

Result: The value of a for which the expressions are equal is $a = 1$.

Verification: Ensuring the Solution is Correct

Verifying the solution is a vital step in algebraic problem-solving, as it confirms the accuracy of the calculation.

Substitute $a = 1$ into both expressions:

- First expression: $11(a+2)$

$$11(1 + 2) = 11 \times 3 = 33$$

- Second expression: $55 - 22a$

$$55 - 22 \times 1 = 55 - 22 = 33$$

Since both expressions evaluate to 33 when $a=1$, the solution is verified.

Analytical Insights and Broader Context

While the solution appears straightforward, several broader insights are worth considering:

Linear Equations and Their Properties

The problem exemplifies a fundamental concept in algebra: solving linear equations. Both expressions are linear in a, meaning their graphs are straight lines. The point where these lines intersect corresponds to the value of a that makes the expressions equal. Graphically, this intersection point confirms the algebraic solution $a=1$.

Parameter Dependence and Functional Relationships

In many real-world scenarios, parameters like a influence the outcome of equations. Understanding how to manipulate these parameters is essential in fields like engineering, economics, and physics. For example, adjusting a could represent changing a variable in a system, and knowing the specific value where two conditions are met is critical for optimization and decision-making.

Implications of the Solution

Knowing that $a=1$ balances both expressions can have practical implications. For instance, in a financial model, if the expressions represent different cost or revenue functions depending on a, then $a=1$ could be the point at which costs and revenues are equal, indicating a break-even point.

Extensions and Related Problems

The problem of equating two algebraic expressions opens avenues for exploring more complex scenarios:

- Quadratic or higher-degree equations: How would the solution change if the expressions involved quadratic terms?
- Inequalities: What is the range of a values that satisfy an inequality involving these expressions?
- System of equations: How does the solution process adapt when multiple conditions are involved?

Each extension introduces new layers of complexity, reinforcing the importance of foundational algebraic skills.

Practical Applications of Such Problems

Understanding how to find a specific parameter value where two expressions are equal is not just an academic exercise—it has tangible applications:

- Engineering: Balancing load distributions by adjusting parameters.
- Economics: Finding equilibrium points in supply and demand models.
- Computer Science: Analyzing algorithm efficiency where parameters affect runtime.
- Physics: Determining conditions for equilibrium states.

Mastering these types of problems enables professionals and students to approach real-world challenges with confidence and precision.

Conclusion: The Significance of the Solution

The question, “For what value of a will the expressions $11(a+2)$ and $55 - 22a$ be equal?” encapsulates fundamental principles of algebra, including equation solving, understanding parameters, and the importance of verification. The straightforward solution—$a=1$—demonstrates the elegance of algebraic manipulation and underscores the importance of methodical problem-solving.

Beyond the immediate calculation, this problem emphasizes critical thinking about the relationships between variables, the visualization of equations through graphs, and the application of basic principles to more complex contexts. Whether in pure mathematics or applied fields, the ability to determine such specific parameter values remains an essential skill, empowering learners and professionals alike to analyze, interpret, and solve real-world problems with confidence.

In essence, the solution to this problem—finding $a=1$—serves as a testament to the power of algebraic reasoning and its broad applicability across various disciplines, highlighting the importance of foundational mathematical skills in understanding the interconnectedness of variables and conditions in countless scenarios.

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