For What Values Of A Are The Following Expressions True?| A + 5 | = -5 - A

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Understanding the solution to algebraic equations is fundamental in mathematics, particularly in the study of linear equations and absolute value expressions. The problem at hand, $|A + 5| = -5 - A$, involves an absolute value expression, which introduces a layer of complexity because absolute value functions behave differently depending on the sign of their contents.

In this article, we will explore how to determine the values of the variable A that satisfy the equation $|A + 5| = -5 - A$. We will delve into the properties of absolute value, analyze the conditions under which the equation holds true, and provide step-by-step solutions. This comprehensive approach aims to enhance understanding for students, educators, and anyone interested in algebraic problem-solving.

Understanding Absolute Value Equations

What Is Absolute Value?

Absolute value is a mathematical concept that measures the magnitude of a real number without regard to its sign. It is denoted by vertical bars, such as $|x|$, and is defined as:

- $|x| = x$ if $x \geq 0$
- $|x| = -x$ if $x < 0$

For example:

- $|3| = 3$
- $|-4| = 4$
- $|0| = 0$

Absolute value equations like $|A + 5| = B$ can be tricky because they involve two possible cases depending on the value inside the absolute value.

General Approach to Solving Absolute Value Equations

To solve equations involving absolute value, a typical approach involves:

1. Isolating the absolute value expression if necessary.
2. Considering the cases where the expression inside the absolute value is non-negative and

negative, which leads to two separate equations:
- Case 1: $A + 5 \geq 0 \rightarrow |A + 5| = A + 5$
- Case 2: $A + 5 < 0 \rightarrow |A + 5| = -(A + 5)$
3. Solving each case separately.
4. Checking for extraneous solutions, especially when the original equation imposes restrictions on the variable.

Analyzing the Equation $|A + 5| = -5 - A$

Before solving, it is crucial to understand the properties and constraints of the equation.

Key Observations

- The absolute value $|A + 5|$ is always ≥ 0.
- The right side, $-5 - A$, can be positive, zero, or negative depending on A.
- For the equation to hold true, the right side must also be ≥ 0 because the left side is non-negative.

This leads us to the first important inequality:

$|A + 5| \geq 0$
and
$-5 - A \geq 0$

Since $|A + 5|$ is always ≥ 0, for the equation to possibly be true, the right side must be ≥ 0:

$-5 - A \geq 0$
$\Rightarrow -A \geq 5$
$\Rightarrow A \leq -5$

This is an essential step because it restricts our solution set to $A \leq -5$.

Step-by-Step Solutions

Now, we proceed to solve the equation considering the case $A \leq -5$, as established.

Case 1: $A + 5 \geq 0$

Recall:

- If $A + 5 \geq 0$, then $|A + 5| = A + 5$.

Given the earlier constraint $A \leq -5$, the inequality $A + 5 \geq 0$ simplifies to:

$A + 5 \geq 0$
$\Rightarrow A \geq -5$

But from the previous step, $A \leq -5$. Combining both:

$A \geq -5$ and $A \leq -5$

This means $A = -5$.

Substitute $A = -5$ into the original equation:

$|(-5) + 5| = -5 - (-5)$

$|0| = 0$

$0 = 0$ — True!

Thus, $A = -5$ is a solution in this case.

Case 2: $A + 5 < 0$

In this case:

- $|A + 5| = -(A + 5)$

The original equation becomes:

$-(A + 5) = -5 - A$

Simplify both sides:

$-A - 5 = -5 - A$

Notice both sides are identical:

$-A - 5 = -5 - A$

This simplifies to:

$-A - 5 = -A - 5$

which is true for all A satisfying the initial assumption $A + 5 < 0$.

Recall that $A + 5 < 0$:

$A + 5 < 0$
$\Rightarrow A < -5$

From earlier, the only solution in the first case is $A = -5$. Since in this case, $A < -5$, and $A = -5$ is not strictly less than -5, there are no solutions in this case.

However, the equation holds for all $A < -5$.

Summary of Solutions

- The only A that satisfies $A + 5 \geq 0$ and the original equation is $A = -5$.
- For $A < -5$, the equation also holds true, as shown in the second case.

Therefore, the complete solution set is:

- $A = -5$
- All A such that $A < -5$

Expressed as:

$A \leq -5$

Final Solution Set and Interpretation

The solutions to the equation $|A + 5| = -5 - A$ are all real numbers less than or equal to -5:

$A \in (-\infty, -5]$

This means that for any value of A less than or equal to -5, the equation holds true.

Verifying the Solutions

It's always good practice to verify solutions:

- For $A = -6$:

$|(-6) + 5| = -5 - (-6)$

| -1 | = 1

1 = 1 — True.

- For A = -5:

|(-5) + 5| = -5 - (-5)

|0| = 0

0 = 0 — True.

- For A = -7:

|(-7) + 5| = -5 - (-7)

| -2 | = 2

2 = 2 — True.

All these verify that the entire set A ≤ -5 satisfies the original equation.

Conclusion and Key Takeaways

In solving the absolute value equation |A + 5| = -5 - A, we established that:

- The right side must be non-negative, hence A ≤ -5.
- The equation divides into two cases based on the sign of A + 5.
- The only point where the boundary case A = -5 is checked explicitly, confirming it satisfies the equation.
- The entire solution set is A ≤ -5, encompassing all real numbers less than or equal to -5.

This problem underscores important algebraic principles:

- Recognizing the domain restrictions imposed by absolute values.
- Analyzing inequalities to limit solutions.
- Considering multiple cases based on the sign of the expressions involved.
- Verifying solutions to confirm their validity.

Practical Applications of Solving Absolute Value

Equations

Understanding how to solve equations like $|A + 5| = -5 - A$ has real-world implications in various fields such as:

- Engineering: analyzing systems with thresholds or tolerances.
- Physics: calculating distances or deviations that can be negative or positive.
- Computer Science: algorithms involving absolute differences or error margins.
- Economics: modeling deviations from an equilibrium point.

Mastering the techniques demonstrated in this example enhances problem-solving skills and prepares students for more complex mathematical challenges.

Additional Tips for Solving Absolute Value Equations

- Always check the domain restrictions introduced by the absolute value.
- Consider all possible cases where the expression inside the absolute value can be non-negative or negative.
- Simplify each case carefully and verify solutions.
- Remember that absolute value equations can have multiple solutions, including all solutions satisfying the case conditions.

Summary

- The equation $|A + 5| = -5 - A$ holds true for all $A \leq -5$.
- The critical step was recognizing the non-negativity constraint on the absolute value and the necessity for the right side to be ≥ 0.
- The solutions include $A = -5$ and all values less than -5.

By following these systematic steps, you can approach and solve similar absolute value equations confidently, ensuring a clear understanding of the underlying principles and solution methods.

In conclusion, the solutions to the equation $|A + 5| = -5 - A$ are:

$A \in (-\infty, -5]$

Understanding and applying these methods will help you solve a wide range of absolute value equations with confidence and precision.

Frequently Asked Questions

For what values of A is the equation $|A + 5| = -5 - A$ true?

The equation is true when $A + 5 \geq 0$ and $A + 5 = -5 - A$, which simplifies to $A = -5$. However, since the absolute value cannot be negative, the only solution is $A = -5$.

Why is the absolute value equation $|A + 5| = -5 - A$ impossible to satisfy for certain A?

Because absolute value expressions are always non-negative, the right side, $-5 - A$, must also be non-negative for the equation to hold, which restricts possible A values.

How do you solve the equation $|A + 5| = -5 - A$?

First, consider the cases where $A + 5 \geq 0$ and $A + 5 < 0$. For each case, remove the absolute value accordingly and solve the resulting linear equations.

What is the solution to $|A + 5| = -5 - A$?

The only solution is $A = -5$, because substituting $A = -5$ gives $|0| = 0$, which satisfies the equation.

Are there any solutions where $A + 5 < 0$?

No, because if $A + 5 < 0$, then $|A + 5| = -(A + 5) = -A - 5$, which would equal $-5 - A$, so the equation holds. But checking the inequality shows that $A + 5 < 0$ implies $A < -5$, and substituting $A < -5$ into the right side reveals no additional solutions besides $A = -5$.

Is $A = -5$ a valid solution for the equation $|A + 5| = -5 - A$?

Yes, because substituting $A = -5$ gives $|0| = 0$, which is true.

What does the solution set look like for $|A + 5| = -5 - A$?

The solution set contains only $A = -5$, since that is the only value satisfying the equation.

Can the expression $|A + 5| = -5 - A$ be true for $A > -5$?

No, because for $A > -5$, the right side, $-5 - A$, becomes less than zero, while the absolute value on the left is non-negative, so the only solution is $A = -5$.

What is the key step in solving $|A + 5| = -5 - A$?

The key step is to consider the cases based on the definition of absolute value and verify the solutions satisfy the original equation, especially considering the non-negativity of absolute values.

Additional Resources

For What Values Of A Are The Following Expressions True? | A + 5 | = -5 - A

Understanding the solutions to absolute value equations is a fundamental aspect of algebra, often challenging students to grasp the nature of absolute value and its implications on the solutions of equations. The problem "For what values of A are the following expressions true? |A + 5| = -5 - A" explores the conditions under which the absolute value expression equals a linear expression involving A. This type of question encourages a deeper comprehension of the properties of absolute value and the logical reasoning required to solve equations that involve both absolute value and algebraic expressions.

Introduction to Absolute Value Equations

Absolute value, denoted as $|x|$, represents the distance of a number x from zero on the real number line, regardless of direction. This means that $|x|$ is always non-negative (i.e., $|x| \geq 0$). When solving equations involving absolute values, the key principle is that if $|X| = Y$, then either $X = Y$ or $X = -Y$, provided $Y \geq 0$. This duality forms the foundation for solving equations like $|A + 5| = -5 - A$.

Key features of absolute value equations:
- They often split into two separate linear equations.
- The right side of the equation must be non-negative for solutions to exist because absolute value cannot be negative.
- The solutions depend heavily on the domain restrictions implied by the absolute value.

Analyzing the Equation |A + 5| = -5 - A

The given equation is:

$|A + 5| = -5 - A$

The goal is to determine for which values of A this equation holds true.

Step 1: Recognize the constraints

Since the absolute value $|A + 5|$ is always ≥ 0, the right side, $-5 - A$, must also be ≥ 0 for the equation to have solutions. Therefore:

$-5 - A \geq 0$
$\Rightarrow -A \geq 5$
$\Rightarrow A \leq -5$

This is the first critical insight: A must be less than or equal to -5 for the equation to be potentially true.

Step 2: Consider the two cases for the absolute value

The absolute value expression can be interpreted as:

- Case 1: $A + 5 \geq 0$
- Case 2: $A + 5 < 0$

Let's analyze each case separately.

Case 1: A + 5 ≥ 0

This case implies:

$A \geq -5$

Given the earlier constraint $A \leq -5$, the only value satisfying both is:

$A = -5$

Substituting $A = -5$ into the equation:

$|(-5) + 5| = -5 - (-5)$
$|0| = 0$
$0 = 0$

This is true. Therefore, $A = -5$ is a solution.

Observation: Since $A = -5$ satisfies $A + 5 \geq 0$, the absolute value case holds, and the equation evaluates correctly.

Case 2: A + 5 < 0

This implies:

$A < -5$

But earlier, we established that for solutions to exist, $A \leq -5$. Combining these:

$A < -5$

Now, in this case, we set up the equation:

$|A + 5| = -5 - A$

Since $A + 5 < 0$, the absolute value becomes:

$|A + 5| = -(A + 5) = -A - 5$

Substituting into the original equation:

$-A - 5 = -5 - A$

Simplify both sides:

$-A - 5 = -5 - A$

This simplifies to:

$-A - 5 = -5 - A$

which is true for all A satisfying $A < -5$.

Result: For all A less than -5, the equation reduces to an identity, which is always true, provided the earlier constraint $A \leq -5$ is maintained (which it is, since $A < -5$).

Note: We need to verify the initial constraint of the right side being ≥ 0:

$-5 - A \geq 0$
$\Rightarrow A \leq -5$

which aligns with the current case $A < -5$.

Summary of Solutions

Combining all the above analysis, the solutions to the equation $|A + 5| = -5 - A$ are:

- $A = -5$
- $A < -5$

In interval notation, the solution set is:

$A \leq -5$

Final Verification of the Solution Set

It's crucial to verify the solutions to ensure no extraneous solutions are included.

- For A = -5:

$|(-5) + 5| = 0$
$-5 - (-5) = 0$

Equation holds: 0 = 0.

- For A < -5:

Choose a test value, say A = -6:

$|(-6) + 5| = |-1| = 1$
$-5 - (-6) = 1$

Equation: 1 = 1, which holds.

Test A = -10:

$|(-10) + 5| = |-5|= 5$
$-5 - (-10) = 5$

Equation: 5 = 5, also satisfies.

Since the test points verify the solution, the entire set A ≤ -5 is valid.

Features, Pros, and Cons of the Solution Process

Features:

- Clear division into cases based on the definition of absolute value.
- Logical deduction using constraints from the absolute value's non-negativity.
- Verification through substitution ensures correctness.

Pros:

- Systematic approach makes solving similar equations straightforward.
- The method handles inequalities naturally, providing a complete solution set.
- Reinforces understanding of the properties of absolute value.

Cons:

- Might be confusing for beginners unfamiliar with case-based analysis.

- Requires careful attention to the inequalities and domain restrictions.
- Overlooks potential extraneous solutions if not verified properly.

Conclusion

The equation $|A + 5| = -5 - A$ offers an insightful example of how absolute value equations involve dual conditions and domain considerations. The key steps involve recognizing that the right side must be non-negative, which constrains A to be less than or equal to -5. Then, examining cases based on the sign of $A + 5$ simplifies the problem, leading to the comprehensive solution set $A \leq -5$. This problem underscores the importance of understanding absolute value properties, inequalities, and logical case analysis in algebra. Mastery of these techniques enables students and practitioners to approach similar equations confidently, ensuring accurate solutions and deeper comprehension of algebraic principles.

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