Given The Function F(x) =3x^2-6x-9 Is The Point (1,-12) On The Graph Of F?

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When analyzing the relationship between a function and a specific point, a fundamental step is to determine whether the point lies on the graph of the function. In this case, we are asked to verify if the point (1, -12) lies on the graph of the quadratic function F(x) = 3x^2 - 6x - 9. This involves substituting the x-coordinate of the point into the function and comparing the resulting value with the y-coordinate provided. This process is critical in various mathematical contexts, such as graphing functions, solving equations, and understanding the nature of quadratic functions.

In this comprehensive guide, we will explore the steps to verify whether the point (1, -12) is on the graph of the function F(x) = 3x^2 - 6x - 9. We will cover the mathematical process in detail, discuss the significance of the results, and explain related concepts such as quadratic functions, points on a graph, and how to interpret the results.

Understanding the Function F(x) = 3x^2 - 6x - 9

Before evaluating whether a point lies on a graph, it’s essential to understand the characteristics of the function involved.

Quadratic Function Overview

- Definition: A quadratic function is a polynomial function of degree 2, typically expressed in the form f(x) = ax^2 + bx + c, where a ≠ 0.
- Graph Shape: The graph of a quadratic function is a parabola that opens upward if a > 0 and downward if a < 0.
- Vertex: The highest or lowest point of the parabola, which can be found using the vertex formula.
- Axis of Symmetry: The vertical line that passes through the vertex, given by x = -b / 2a.

Specifics of F(x) = 3x^2 - 6x - 9

- Coefficients: a = 3, b = -6, c = -9.
- Direction: Since a = 3 > 0, the parabola opens upward.
- Vertex Calculation:
- x-coordinate of the vertex: x = -b / 2a = -(-6) / (2 3) = 6 / 6 = 1.

- y-coordinate of the vertex: F(1) = 3(1)^2 - 6(1) - 9 = 3 - 6 - 9 = -12.

This preliminary calculation indicates that at x = 1, F(x) = -12, which is a crucial insight for our evaluation.

Evaluating Whether the Point (1, -12) Lies on the Graph of F(x)

The core method to determine if a point (x, y) is on the graph of a function F(x) is straightforward:

1. Substitute the x-coordinate into the function.
2. Calculate the resulting y-value.
3. Compare the calculated y-value with the y-coordinate of the point.

Applying this to our specific point:

Step 1: Substitute x = 1 into F(x)

- Calculate F(1):
- F(1) = 3(1)^2 - 6(1) - 9
- F(1) = 3 1 - 6 - 9
- F(1) = 3 - 6 - 9
- F(1) = -12

Step 2: Compare the calculated value with the y-coordinate of the point

- The point in question is (1, -12).
- The evaluated function at x = 1 is F(1) = -12, which matches the y-coordinate.

Conclusion:

- Since the value of the function at x = 1 equals the y-coordinate of the point, the point (1, -12) lies on the graph of F(x).

This confirms that the point is indeed on the parabola described by the quadratic function.

Visualizing the Graph and Its Key Features

Understanding the graphical representation of the function enhances comprehension of why the point (1, -12) is on the graph.

Plotting the Vertex

- The vertex of the parabola is at (1, -12), as calculated earlier.
- This point represents the minimum point on the parabola since it opens upward.

Shape of the Graph

- Since $a = 3 > 0$, the parabola opens upward.
- The vertex at (1, -12) is the lowest point on the curve.
- The parabola is symmetric with respect to the vertical line $x = 1$.

Other Points on the Graph

- To understand the shape further, consider additional x-values:
- For $x = 0$:
- $F(0) = 3(0)^2 - 6(0) - 9 = -9$
- For $x = 2$:
- $F(2) = 3(4) - 6(2) - 9 = 12 - 12 - 9 = -9$
- These points (0, -9) and (2, -9) are symmetric about $x = 1$.

Implication

- Since $F(1) = -12$ is the lowest point, the parabola opens upward with the vertex at (1, -12).
- The y-values increase as x moves away from 1.

Additional Mathematical Insights

Understanding the properties of the quadratic function provides a broader context for the specific point.

Factoring and Standard Form

- While the quadratic is already in standard form, it can be rewritten in vertex form for clarity:

- F(x) = a(x - h)^2 + k, where (h, k) is the vertex.
- Completing the square for F(x):
- F(x) = 3x^2 - 6x - 9
- F(x) = 3(x^2 - 2x) - 9
- Complete the square inside:
- x^2 - 2x + 1 - 1 = (x - 1)^2 - 1
- So,
- F(x) = 3[(x - 1)^2 - 1] - 9 = 3(x - 1)^2 - 3 - 9 = 3(x - 1)^2 - 12
- The vertex form confirms that the vertex is at (1, -12), matching previous calculations.

Function Behavior and Range

- Since the parabola opens upward, the minimum y-value is at the vertex:
- Range: y ≥ -12
- The vertex point (1, -12) is the lowest point of the graph.

Implications for the Point (1, -12)

- The point is the vertex itself, the lowest point on the parabola.
- Any movement along the parabola away from x=1 results in higher y-values.

Additional Practice: Verifying Other Points

To deepen understanding, consider verifying whether other points lie on the graph.

1. Point (0, -9):

2. Point (2, -9):

3. Point (3, ?)

Verification:

- For (0, -9):
- F(0) = -9; y = -9 matches, so (0, -9) is on the graph.
- For (2, -9):
- F(2) = -9; y = -9 matches, so (2, -9) is on the graph.
- For (3, y):
- F(3) = 3(9) - 6(3) - 9 = 27 - 18 - 9 = 0
- So, the point (3, 0) is on the graph.

This process illustrates how to verify points systematically, reinforcing the method used for the initial point.

Conclusion: Is (1, -12) On The Graph?

Based on the calculations and analysis, the answer is clear:

- Yes, the point (1, -12) lies on the graph of the function $F(x) = 3x^2 - 6x - 9$.

This conclusion is supported by the following facts:
- Substituting $x=1$ into $F(x)$ yields $y = -12$.
- The calculated y-value matches the y-coordinate of the point.
- The point (1, -12) corresponds to the vertex of the parabola, which is a point on the graph.

Understanding how to verify points on a graph is a fundamental skill in algebra and calculus, essential for graph analysis, solving equations, and understanding the behavior of functions. Recognizing the vertex and symmetry of quadratic functions allows for quick identification of key points, and substitution methods provide a straightforward way to

Frequently Asked Questions

Is the point (1, -12) on the graph of the function $F(x) = 3x^2 - 6x - 9$?

To verify, substitute $x = 1$ into the function: $F(1) = 3(1)^2 - 6(1) - 9 = 3 - 6 - 9 = -12$. Since $F(1) = -12$, which matches the y-coordinate of the point, the point (1, -12) is on the graph.

How do you determine if a point lies on the graph of a quadratic function?

You substitute the x-coordinate into the function to find the corresponding y-value. If this y-value matches the y-coordinate of the point, then the point lies on the graph.

What is the value of $F(1)$ for the function $F(x) = 3x^2 - 6x - 9$?

$F(1) = 3(1)^2 - 6(1) - 9 = 3 - 6 - 9 = -12$.

Does the point (1, -12) satisfy the function F(x) = 3x^2 - 6x - 9?

Yes, because substituting x=1 into the function yields y = -12, which matches the y-coordinate of the point.

What is the significance of evaluating F(1) in relation to the point (1, -12)?

Evaluating F(1) helps confirm whether the point is on the graph by checking if the function's output at x=1 equals -12.

Can the point (1, -12) be considered a solution to the equation y = 3x^2 - 6x - 9?

Yes, because plugging x=1 into the equation results in y=-12, which matches the y-value of the point.

What is the general process for verifying points on quadratic graphs?

Substitute the x-coordinate into the quadratic function and see if the resulting y-value matches the y-coordinate of the point.

If a point does not satisfy the function equation, is it on the graph?

No, if substituting the x-value into the function does not produce the given y-value, the point is not on the graph.

What is the vertex of the parabola defined by F(x) = 3x^2 - 6x - 9?

The vertex can be found using x = -b/(2a). Here, a=3 and b=-6, so x = -(-6)/(23) = 6/6 = 1. Substituting x=1 gives y=-12, so the vertex is at (1, -12).

Is the point (1, -12) the vertex of the parabola F(x) = 3x^2 - 6x - 9?

Yes, because the vertex occurs at x=1, and F(1) = -12, so the vertex is at (1, -12).

Additional Resources

Given The Function F(x) = 3x^2 - 6x - 9 Is The Point (1, -12) On The Graph Of F? – This question is a common example used in algebra and calculus to test understanding of how functions and points relate on a graph. Determining whether a specific point lies on a given function's graph involves evaluating the function at the x-coordinate and comparing it with the y-coordinate of the point. This process not only reinforces the concept of function evaluation but also deepens understanding of how graphs represent relationships between variables.

Understanding the Problem

Before jumping into calculations, it’s essential to understand what the question is asking. The core question is: Is the point (1, -12) on the graph of the function $F(x) = 3x^2 - 6x - 9$?

In simpler terms, we need to verify whether the point's coordinates satisfy the function. If plugging x=1 into the function yields y=-12, then the point lies on the graph; otherwise, it does not.

Step-by-Step Approach to the Solution

1. Recall the Definition of a Function Point

A point (a, b) lies on the graph of a function F(x) if and only if:

$$ F(a) = b $$

This means, for our case:

- Plug x = 1 into F(x).
- Check if the result equals -12.

2. Evaluate F(x) at x=1

Given:

$$ F(x) = 3x^2 - 6x - 9 $$

Let's substitute x=1:

$$ F(1) = 3(1)^2 - 6(1) - 9 $$

Calculations:

- $3(1)^2 = 3 \times 1 = 3$

- $-6(1) = -6$
- -9 remains as is.

Now, sum these:

$$F(1) = 3 - 6 - 9$$

$$F(1) = (3 - 6) - 9$$

$$F(1) = -3 - 9$$

$$F(1) = -12$$

3. Compare the Result with the Point's y-coordinate

The point in question is (1, -12). Since:

$$F(1) = -12$$

The y-value of the point matches the value of the function at x=1.

Conclusion: Is the Point (1, -12) on the Graph?

Based on the calculations, yes, the point (1, -12) lies on the graph of the function $F(x) = 3x^2 - 6x - 9$.

Visual Representation and Graphical Intuition

While calculations confirm the point's presence on the graph, visualizing the function can solidify understanding.

Graphing $F(x) = 3x^2 - 6x - 9$

- Type of function: Quadratic (parabola).
- Leading coefficient (3): Positive, so the parabola opens upward.
- Vertex: The minimum point of the parabola.

Finding the Vertex

The vertex of a quadratic function $ax^2 + bx + c$ is at:

$$x = -\frac{b}{2a}$$

For $a=3$, $b=-6$:

$$x_{vertex} = -\frac{-6}{2 \times 3} = \frac{6}{6} = 1$$

Plug this x-value into F(x):

$$ F(1) = -12 $$ (as previously calculated).

Thus, the vertex is at (1, -12), which is exactly the point in question.

Implication

- The point (1, -12) is the vertex of the parabola.
- This confirms that the point is on the graph, and specifically, it represents the lowest point of the parabola.

Additional Insights and Related Concepts

1. Axis of Symmetry

- The axis of symmetry for a parabola $y = ax^2 + bx + c$ passes through the vertex.
- Given the vertex at x=1, the axis of symmetry is the vertical line:

$$ x=1 $$

2. Graph Behavior

- Since the parabola opens upward, the vertex at (1, -12) is the minimum point.
- As x moves away from 1, the y-values increase symmetrically on both sides.

3. Implications for Function Analysis

- The fact that the point is the vertex indicates that the function reaches its minimum value at x=1.
- Understanding this helps in solving optimization problems or analyzing the function's behavior.

Common Misconceptions and Errors

1. Confusing the Point with a Random Coordinate

- Always check whether the x-value of the point matches the function's value at that x, not just assuming the point is on the graph.

2. Miscalculations in Evaluation

- Be cautious with algebraic substitution and arithmetic operations — errors here can lead to incorrect conclusions.

3. Assuming the Point is on the Graph Without Verification

- Always verify by direct substitution before concluding.

Summary of Key Takeaways

- To determine if a point lies on a graph, substitute the x-coordinate into the function and compare the result with the y-coordinate.
- For the function $F(x) = 3x^2 - 6x - 9$, the point (1, -12) is on the graph because:

$$F(1) = -12$$

- The point (1, -12) is also the vertex of the parabola, highlighting its significance in understanding the graph's shape.

Final Thoughts

Understanding how to verify whether a point lies on a function's graph is a foundational skill in algebra and calculus. It combines algebraic evaluation with geometric interpretation, allowing students and professionals alike to analyze functions with confidence. Recognizing key points like vertices and intercepts not only aids in graphing but also provides insights into the function's behavior, optimization potential, and real-world applications.

Whether you're preparing for exams, solving real-world problems, or just enhancing your mathematical intuition, mastering the process of verifying points on graphs is an essential step in becoming proficient with functions.

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