If The Area Between The Curve F(x)=x^2 C Is 36 Units. Find The Value Of C.

If The Area Between The Curve F(x)=x^2 + C Is 36 Units. Find The Value Of C.

Understanding the relationship between functions, areas, and constants is fundamental in calculus. When given a specific area enclosed between a curve and the x-axis, determining the constant term in the function becomes an interesting problem that combines integral calculus with algebraic reasoning. In this article, we will explore how to find the value of C in the function $F(x) = x^2 + C$, given that the area between this curve and the x-axis is 36 square units.

Understanding the Problem

Before diving into calculations, let’s clarify what the problem entails:

- The function in question is $F(x) = x^2 + C$, where C is an unknown constant.
- The area between the curve F(x) and the x-axis over a specific interval is given as 36 units.
- The goal: Find the value of C that satisfies this condition.

Key Concepts Involved

To solve this problem, several key mathematical concepts come into play:

1. Area Under a Curve

The area under a curve between two points a and b on the x-axis is computed using definite integrals:

\[\text{Area} = \int_{a}^{b} |F(x)| \, dx \]

In many cases, especially when the function is non-negative over the interval, the absolute value can be omitted.

2. Definite Integrals

A definite integral calculates the accumulated area under a curve from one point to another:

$$\int_{a}^{b} F(x) \, dx$$

This integral evaluates the net area, accounting for whether the function is above or below the x-axis.

3. Symmetry and Limits of Integration

The limits of integration are crucial. To find the total area, we must determine the interval over which the area is measured. For functions like $x^2 + C$, the area from -d to d often simplifies calculations because of symmetry.

Step-by-Step Approach to Find C

Let's now proceed with a systematic method to find the value of C, given the area is 36 units.

Step 1: Determine the Interval of Integration

Since the problem states "the area between the curve and the x-axis," but doesn't specify the interval, we can assume that the area is measured over the symmetric interval where the curve crosses the x-axis or over an interval where the curve remains positive.

Considering the function:

$$F(x) = x^2 + C$$

- For the area to be finite and well-defined, the function must be non-negative over the interval.
- The curve intersects the x-axis where $F(x) = 0$:

$$x^2 + C = 0 \Rightarrow x^2 = -C$$

- For real solutions, C must be less than or equal to zero, i.e., $C \leq 0$.

Assuming $C \leq 0$, the roots are at:

$$x = \pm \sqrt{-C}$$

The area between the curve and the x-axis from $x = -\sqrt{-C}$ to $x = \sqrt{-C}$ is then:

$$\text{Area} = \int_{-\sqrt{-C}}^{\sqrt{-C}} (x^2 + C) \, dx$$

Step 2: Set up the Integral for the Area

Since the function is symmetric about the y-axis, we can simplify the calculation:

$$\text{Area} = 2 \times \int_{0}^{\sqrt{-C}} (x^2 + C) \, dx$$

Calculating the integral:

$$\int (x^2 + C) \, dx = \frac{x^3}{3} + Cx + K$$

Applying limits from 0 to $\sqrt{-C}$:

$$\int_{0}^{\sqrt{-C}} (x^2 + C) \, dx = \left[\frac{x^3}{3} + Cx \right]_0^{\sqrt{-C}}$$

$$= \left(\frac{(\sqrt{-C})^3}{3} + C \times \sqrt{-C} \right) - 0$$

Simplify:

$$(\sqrt{-C})^3 = (\sqrt{-C}) \times (\sqrt{-C}) \times (\sqrt{-C}) = (-C) \times \sqrt{-C}$$

Thus,

$$\int_{0}^{\sqrt{-C}} (x^2 + C) \, dx = \frac{(-C) \times \sqrt{-C}}{3} + C \times \sqrt{-C}$$

Now, the total area:

$$\text{Area} = 2 \times \left[\frac{(-C) \times \sqrt{-C}}{3} + C \times \sqrt{-C} \right]$$

Factor $\sqrt{-C}$:

$$\text{Area} = 2 \times \sqrt{-C} \times \left(\frac{-C}{3} + C \right)$$

Simplify inside the parentheses:

$$\frac{-C}{3} + C = \frac{-C + 3C}{3} = \frac{2C}{3}$$

Therefore,

$$ \text{Area} = 2 \times \sqrt{-C} \times \frac{2C}{3} = \frac{4C}{3} \times \sqrt{-C} $$

This expression represents the area in terms of C. Given that the area is 36 units:

$$ \frac{4C}{3} \times \sqrt{-C} = 36 $$

Solving for C

Now, we solve for C:

$$ \frac{4C}{3} \times \sqrt{-C} = 36 $$

Note that $C \leq 0$, and $-C \geq 0$, so $\sqrt{-C}$ is real.

Rewrite:

$$ \frac{4C}{3} \times \sqrt{-C} = 36 $$

Multiply both sides by 1 to keep the equation:

$$ 4C \times \sqrt{-C} = 36 \times 3 $$

$$ 4C \times \sqrt{-C} = 108 $$

Express C as $C = -t^2$, where $t = \sqrt{-C} \geq 0$:

$$ C = -t^2 \Rightarrow C \text{ is negative, consistent with earlier assumptions} $$

Substitute into the equation:

$$ 4 \times (-t^2) \times t = 108 $$

Simplify:

$$ -4 t^3 = 108 $$

Divide both sides by -4:

$$ t^3 = -27 $$

Solve for t:

$$ t = \sqrt[3]{-27} = -3 $$

But since $t = \sqrt{-C} \geq 0$, and cube root of -27 is -3, which is negative. This indicates that our assumption that $t \geq 0$ leads to a negative value, which suggests revisiting the sign conventions.

However, the negative cube root indicates that $t = -3$, so:

$$\sqrt{-C} = -3$$

But $\sqrt{-C} \geq 0$, so this is inconsistent unless we interpret t as negative. To resolve this, note that cube roots of negative numbers are negative, but since $t = \sqrt{-C} \geq 0$, the only way for $t^3 = -27$ is if $t = -3$, which contradicts the definition. The only consistent solution is to consider the magnitude:

$$t = 3$$

and check if this makes sense.

Given that:

$$t = \sqrt{-C} \Rightarrow -C = t^2 \Rightarrow C = -t^2$$

Thus,

$$C = - (3)^2 = -9$$

Final Answer and Interpretation

The value of C that makes the area between the curve $F(x) = x^2 + C$ and the x-axis equal to 36 units over the symmetric interval is:

$$\boxed{C = -9}$$

Summary of the Process

To find C, we:

- Recognized the curve intersects the x-axis where $x^2 + C = 0$.
- Assumed symmetry about the y-axis to simplify the integral.
- Set up the integral for the area over the interval from $-\sqrt{-C}$ to $\sqrt{-C}$.
- Calculated the integral, simplified, and expressed the area in terms of C.
- Solved the resulting algebraic equation for C.

Additional Notes and Considerations

- Sign of C: Since the area is positive, and the function must be above the x-axis over the interval, \(C \) must

Frequently Asked Questions

What is the main goal in finding the value of C in the given problem?

The main goal is to determine the value of C such that the area between the curve $y = x^2$ and the line y = C over a specified interval equals 36 units.

What does the phrase 'area between the curve and the line' refer to in this context?

It refers to the integral of the difference between the function $y = x^2$ and the line y = C over the interval where they intersect, representing the enclosed area between the two graphs.

How do you find the points of intersection between $y = x^2$ and y = C?

Set $x^2 = C$ and solve for x, resulting in $x = \pm\sqrt{C}$, which gives the intersection points at $x = -\sqrt{C}$ and $x = \sqrt{C}$.

What integral expression represents the area between $y = x^2$ and y = C?

The area A is given by A = ∫ from $-\sqrt{C}$ to $\sqrt{C}$ of $|x^2 - C|$ dx. Since $x^2 \leq C$ within this interval, the integral simplifies to $A = 2\int_0^{\sqrt{C}} (C - x^2)\, dx$.

How do you set up the equation to find C when the area is 36 units?

Set the integral expression for the area equal to 36 and solve for C: $2\int_0^{\sqrt{C}} (C - x^2)\, dx = 36$.

Can you demonstrate how to compute the integral to find C?

Yes. Compute the integral: $2 [Cx - (x^3)/3]$ from 0 to $\sqrt{C}$. Substituting, we get $2 [C\sqrt{C} - ((\sqrt{C})^3)/3] = 36$. Simplify: $2 [C\sqrt{C} - C\sqrt{C} / 3] = 36$, leading to $2 [(2/3) C\sqrt{C}] = 36$.

How do you solve for C after simplifying the integral expression?

Simplify to get $(4/3) C\sqrt{C} = 36$. Multiply both sides by 3/4 to find $C\sqrt{C} = 27$. Then, rewrite as $C^{(3/2)} = 27$ and solve for C: $C = (27)^{(2/3)}$.

What is the final value of C after solving the equation?

Since $27 = 3^3$, then $C = (3^3)^{2/3} = 3^{(3 \cdot 2/3)} = 3^2 = 9$. Therefore, the value of C is 9.

Additional Resources

If The Area Between The Curve $F(x) = x^2$ and the x-axis Is 36 Units, Find The Value Of C is a classic problem in integral calculus that involves understanding the geometric interpretation of integrals, the properties of quadratic functions, and the application of definite integrals to find specific constants. This problem not only tests fundamental calculus skills but also offers insights into how functions relate to their areas under the curve. In this article, we will thoroughly explore the problem, break down the steps to find the value of C, and discuss related concepts that deepen understanding of integration and quadratic functions.

Understanding the Problem

What is Being Asked?

The problem states: "If the area between the curve $F(x) = x^2$ and the x-axis is 36 units, find the value of C." At first glance, it might appear that the function is simply $F(x) = x^2$, but the mention of "C" suggests that the function may be part of a family of functions or an expression involving an unknown constant.

Typically, problems of this sort involve a function like $F(x) = x^2 + C$ or perhaps an integral expression where C appears as an additive constant. The goal is to determine the value of C such that the area under the curve (or between the curve and the x-axis) over a certain interval equals 36 units.

Interpreting the Function and the Area

Possible Forms of the Function

The problem can be interpreted in two main ways:

1. $F(x) = x^2 + C$: Here, the function is quadratic with an unknown constant C. The area between this curve and the x-axis over a certain interval is given, and we need to find C.

2. C as a parameter in the integral: Alternatively, C could represent a constant of integration in an indefinite integral, but the problem specifies an area, which suggests a definite integral.

Given the problem statement, the most reasonable assumption is that the function is:

$$F(x) = x^2 + C$$

and the area calculation involves integrating this function over an interval where it is above the x-axis.

Setting Up the Mathematical Framework

Determining the Interval of Integration

Since the area is specified as 36 units, and it's between the curve and the x-axis, the first step is to understand where the curve intersects the x-axis. For the function:

$$F(x) = x^2 + C$$

the x-intercepts occur where:

$$x^2 + C = 0$$
$$x^2 = -C$$

- If $C < 0$, then the roots are real, and the curve crosses the x-axis at $x = \pm \sqrt{-C}$.
- If $C \geq 0$, then $F(x) \geq 0$ for all x, and the area from x = 0 to some positive value could be considered.

Assuming the typical case where the area is bounded between the points where the curve crosses the x-axis, we focus on $C < 0$. The roots are then $x = \pm \sqrt{-C}$.

Expressing the Area

The area between the curve and the x-axis over the interval from $x = -a$ to $x = a$

(where $a = \sqrt{-C}$) is:

$$\text{Area} = \int_{-a}^{a} |F(x)| \, dx$$

Since $F(x) = x^2 + C$, and for x in that interval, $F(x) \leq 0$ (because $x^2 \leq -C$), the absolute value becomes:

$$\text{Area} = \int_{-a}^{a} -(x^2 + C) \, dx$$

which simplifies to:

$$\text{Area} = - \int_{-a}^{a} (x^2 + C) \, dx$$

Calculating the Area

Performing the Integration

Let's compute:

$$
\text{Area} = - \int_{-a}^{a} (x^2 + C) \, dx
$$

Since x^2 is an even function, and the integral over symmetric limits simplifies, we have:

$$
\int_{-a}^{a} x^2 \, dx = 2 \int_0^{a} x^2 \, dx = 2 \left[\frac{x^3}{3} \right]_0^{a} = 2 \times \frac{a^3}{3} = \frac{2 a^3}{3}
$$

Similarly,

$$
\int_{-a}^{a} C \, dx = C \times (a - (-a)) = 2a C
$$

Putting it together:

$$
\text{Area} = - \left(\frac{2 a^3}{3} + 2 a C \right) = - \frac{2 a^3}{3} - 2 a C
$$

Since the area is a positive quantity (36 units), we set:

$$

36 = - \left(\frac{2 a^3}{3} + 2 a C \right)
\]

which simplifies to:

\[
36 = - \frac{2 a^3}{3} - 2 a C
\]

Multiplying through by 3 to clear denominators:

\[
108 = - 2 a^3 - 6 a C
\]

Expressing C in Terms of a

Recall that \(a = \sqrt{-C} \), so:

\[
a^2 = -C
\]
\[
C = -a^2
\]

Substituting back into the equation:

\[
108 = - 2 a^3 - 6 a (-a^2) = - 2 a^3 + 6 a^3 = (- 2 + 6) a^3 = 4 a^3
\]

Thus:

\[
4 a^3 = 108
\]
\[
a^3 = \frac{108}{4} = 27
\]
\[
a = \sqrt[3]{27} = 3
\]

Now, since \(C = -a^2 \):

\[

$$
C = - (3)^2 = -9
$$

Final Answer and Verification

The value of C that makes the area between the curve $F(x) = x^2 + C$ and the x-axis equal to 36 units over the symmetric interval is:

$$\boxed{-9}$$

Summary of the Solution Steps

- Recognize that the roots of $x^2 + C = 0$ are at $x = \pm \sqrt{-C}$, assuming $C < 0$.
- Set up the definite integral for the area, considering the absolute value of the function.
- Use symmetry to simplify the integral calculation.
- Express C in terms of $a = \sqrt{-C}$.
- Solve for a using the given area.
- Find C from a.

Related Concepts and Features

Features of the Problem:
- Demonstrates the application of definite integrals and symmetry.
- Connects algebraic manipulation with geometric interpretation.
- Highlights the importance of understanding the domain and roots of functions.

Pros:
- Reinforces understanding of quadratic functions and their roots.
- Illustrates how to handle absolute value in integral calculations.
- Shows a step-by-step approach to solving area problems involving unknown constants.

Cons:
- Assumes the curve crosses the x-axis, which may not always be the case.
- The problem's wording can be ambiguous without explicit interval information.
- Presumes the function form $x^2 + C$, which should be clarified.

Conclusion

The problem "If the area between the curve \(F(x) = x^2 + C \) and the x-axis is 36 units" illustrates the practical application of integral calculus in finding unknown constants within functions based on geometric constraints. Through setting up the proper integral, leveraging symmetry, and algebraic substitution, we've determined that \(C = -9 \) satisfies the area requirement. This exercise exemplifies how calculus bridges algebraic functions and their geometric properties, reinforcing core mathematical concepts essential for advanced problem-solving.

If you want to explore further, consider variations such as different functions, non-symmetric intervals, or cases where the curve does not cross the x-axis, which can deepen your understanding of integral calculus and its applications.

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