

In Triangle ABC, AC=13, BC=84, And AB=85. Find The Measure Of Angle C

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Understanding the properties of triangles and how to calculate their angles is fundamental in geometry. When given specific side lengths, such as in triangle ABC with sides AC=13, BC=84, and AB=85, it is possible to determine the measure of an unknown angle using various mathematical tools, primarily the Law of Cosines. This article provides a comprehensive guide to solving for angle C in this triangle, including explanations of relevant concepts, step-by-step calculations, and practical applications.

Overview of Triangle Properties and Notation

Before delving into calculations, it's important to understand the basic notation and properties related to triangles.

Side and Angle Notation

In triangle ABC:

- Side AB is opposite angle C, so it is often denoted as c .
- Side AC is opposite angle B, often denoted as b .
- Side BC is opposite angle A, often denoted as a .

Given data:

- AC = 13 (side b)
- BC = 84 (side a)
- AB = 85 (side c)

Correspondences:

Side	Length	Opposite Angle
a	84	A
b	13	B
c	85	C

The goal is to determine the measure of angle C, which is opposite side c .

Understanding the Law of Cosines

The Law of Cosines is a powerful tool for calculating an angle when the lengths of all three sides are known. It is expressed as:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Rearranged to solve for $\cos C$:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Once $\cos C$ is calculated, the measure of angle C can be obtained by taking the inverse cosine (arccosine).

Applicability of the Law of Cosines

This law is particularly useful when:

- All three side lengths are known.
- You need to find an angle opposite a known side.

In our case, since side c (AB=85), and sides a and b are known, we can apply the Law of Cosines directly to find angle C.

Step-by-Step Calculation of Angle C

Let's now proceed to the calculation.

Step 1: Write down the known side lengths

- $a = 84$
- $b = 13$
- $c = 85$

Step 2: Plug into the Law of Cosines formula

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{(84)^2 + (13)^2 - (85)^2}{2 \times 84 \times 13}$$

\]

Step 3: Calculate the numerator

\[

$$84^2 = 7056$$

\]

\[

$$13^2 = 169$$

\]

\[

$$85^2 = 7225$$

\]

So,

\[

$$\text{Numerator} = 7056 + 169 - 7225 = (7225 - 7225) = 0$$

\]

Step 4: Calculate the denominator

\[

$$2 \times 84 \times 13 = 2 \times 1092 = 2184$$

\]

Step 5: Compute $\cos C$

\[

$$\cos C = \frac{0}{2184} = 0$$

\]

Since the numerator is zero, $\cos C = 0$.

Step 6: Find the measure of angle C

\[

$$C = \arccos(0)$$

\]

The cosine of 0 corresponds to a 90-degree angle. Therefore,

\[

$$C = 90^\circ$$

\\

Conclusion: The measure of angle C in triangle ABC is 90 degrees.

Verifying the Result and Geometric Implications

Having deduced that angle C is 90° , it indicates that triangle ABC is a right triangle with the right angle at vertex C.

Verifications:

- Pythagorean Theorem check:

Since side c (AB) is 85, and sides a and b are 84 and 13 respectively, check whether:

\\

$$a^2 + b^2 = c^2$$

\\

\\

$$84^2 + 13^2 = 7225$$

\\

\\

$$7056 + 169 = 7225$$

\\

\\

$$7225 = 7225$$

\\

The equality holds, confirming that the triangle is right-angled at C.

Geometric Significance:

- Triangle ABC is a right triangle with hypotenuse AB=85.
- Sides adjacent to the right angle are AC=13 and BC=84.
- The sides satisfy the Pythagorean theorem, which is consistent with our calculation for $\cos C=0$.

Practical Applications of the Calculation

Understanding how to determine angles in triangles has numerous applications across various fields:

- Engineering and Construction: Ensuring structures are built with precise angles.
- Navigation and GPS: Calculating angles between routes.
- Physics: Analyzing forces and vector components.
- Computer Graphics: Rendering scenes with accurate angles and perspectives.
- Mathematics Education: Teaching triangle properties and trigonometry.

Additional Methods for Confirming the Result

While the Law of Cosines provided a straightforward solution, other methods can be employed to verify the result.

Using the Pythagorean Theorem

As established, the sides satisfy $(a^2 + b^2 = c^2)$, confirming the right-angled nature at C.

Using Coordinate Geometry

Assign coordinate points:

- Place point C at the origin: $(C(0,0))$.
- Since $AC = 13$, place A at $(A(13,0))$.
- To satisfy $BC=84$, point B could be at $(B(0,84))$.

Calculate the distance AB:

$$AB = \sqrt{(13 - 0)^2 + (0 - 84)^2} = \sqrt{169 + 7056} = \sqrt{7225} = 85$$

This confirms the initial data and supports the conclusion that the angle at C is 90° .

Summary and Final Remarks

This detailed exploration reveals that in triangle ABC with sides $AC=13$, $BC=84$, and $AB=85$, the measure of angle C is exactly 90 degrees. The key steps involved applying the Law of Cosines and verifying the Pythagorean theorem, both confirming the right-angled nature of the triangle.

Key Takeaways:

- The Law of Cosines is an essential tool for solving triangles with known side lengths and unknown angles.
- When the sides satisfy the Pythagorean theorem, the triangle is right-angled.
- Multiple methods, including coordinate geometry, can be used to verify the calculations.

- Recognizing special triangles simplifies problem-solving in geometry.

This example demonstrates the importance of understanding fundamental geometric principles and their applications in real-world and academic contexts. Whether for academic purposes, engineering design, or computational geometry, mastering these techniques enhances problem-solving capabilities and deepens comprehension of geometric relationships.

Frequently Asked Questions

In triangle ABC with sides AC=13, BC=84, and AB=85, how can we find the measure of angle C?

We can use the Law of Cosines: $\cos C = (AC^2 + BC^2 - AB^2) / (2 AC BC)$. Plugging in the values: $\cos C = (13^2 + 84^2 - 85^2) / (2 \cdot 13 \cdot 84)$. Calculating: $\cos C = (169 + 7056 - 7225) / (2 \cdot 13 \cdot 84) = 0 / 2184 = 0$. Therefore, angle C = $\arccos(0) = 90^\circ$.

Is triangle ABC a right triangle based on the given side lengths?

Yes, since the side lengths satisfy the Pythagorean theorem: $13^2 + 84^2 = 169 + 7056 = 7225$, which equals $85^2 = 7225$. Thus, triangle ABC is a right triangle with a right angle at C.

What is the measure of angle C in triangle ABC with sides AC=13, BC=84, AB=85?

The measure of angle C is 90 degrees because the sides satisfy the Pythagorean theorem, confirming a right angle at C.

Can we determine the angles of triangle ABC using the given side lengths?

Yes. Using the Law of Cosines, we find that angle C is 90° , and then use the Law of Sines or Law of Cosines to find angles A and B if needed.

What formula is used to find angle C in a triangle when sides are known?

The Law of Cosines is used: $\cos C = (AC^2 + BC^2 - AB^2) / (2 AC BC)$.

Why does the calculation show that angle C is 90° in this triangle?

Because the side lengths satisfy the Pythagorean theorem, indicating a right triangle with a 90° angle at C.

If side AC is 13, BC is 84, and AB is 85, what is the significance of these lengths?

These lengths form a Pythagorean triple, confirming that triangle ABC is a right triangle with the right angle at C, and the hypotenuse AB.

Additional Resources

Triangle Geometry Analysis: Determining the Measure of Angle C in Triangle ABC

Understanding the internal angles of a triangle is fundamental to the study of geometry, with applications spanning architecture, engineering, and computer graphics. In this detailed exploration, we focus on a specific triangle, Triangle ABC, characterized by side lengths $AC = 13$, $BC = 84$, and $AB = 85$. Our goal is to meticulously determine the measure of Angle C, providing a comprehensive breakdown of the underlying mathematical principles involved.

Introduction to Triangle ABC and Its Known Sides

Before delving into calculations, let's establish a clear understanding of our given data:

- Side AC = 13 units
- Side BC = 84 units
- Side AB = 85 units

In triangle notation, typically:

- Point A is connected to points B and C
- Side a is opposite to angle A
- Side b is opposite to angle B
- Side c is opposite to angle C

Assigning these:

- $(AB = c = 85)$
- $(AC = b = 13)$
- $(BC = a = 84)$

Our target: Find $(\angle C)$, which is the angle at vertex C, opposite side c (which is AB).

Approach and Strategies for Calculation

To find $(\angle C)$, various methods can be employed. The most straightforward approach in this context is the Law of Cosines, which relates the lengths of the sides of a triangle to the cosine of one of its angles. Given the side lengths, the Law of Cosines states:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Rearranged to solve for $(\cos C)$:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Once $(\cos C)$ is determined, the measure of $(\angle C)$ can be found via the inverse cosine function.

Key considerations:

- Confirm the validity of side lengths forming a triangle (triangle inequality).
- Carefully substitute the known values.
- Use precise calculations to determine $(\angle C)$.

Verifying Triangle Validity

Before proceeding, ensure that the side lengths satisfy the triangle inequality theorem, which states that:

- The sum of any two sides must be greater than the third.

Check:

1. $(AC + BC = 13 + 84 = 97 > AB = 85)$ $\square$
2. $(AC + AB = 13 + 85 = 98 > BC = 84)$ $\square$
3. $(BC + AB = 84 + 85 = 169 > AC = 13)$ $\square$

All inequalities hold, confirming that these side lengths can indeed form a valid triangle.

Applying the Law of Cosines to Find $(\angle C)$

Given:

$$a = 84 \quad (BC)$$

$$b = 13 \quad (AC)$$

$$c = 85 \quad (AB)$$

Recall the formula:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Substituting the known values:

$$\cos C = \frac{(84)^2 + (13)^2 - (85)^2}{2 \times 84 \times 13}$$

Calculations step-by-step:

1. $(84)^2 = 7056$
2. $(13)^2 = 169$
3. $(85)^2 = 7225$

Now, numerator:

$$7056 + 169 - 7225 = (7056 + 169) - 7225 = 7225 - 7225 = 0$$

Denominator:

$$2 \times 84 \times 13 = 2 \times 1092 = 2184$$

Thus:

$$\cos C = \frac{0}{2184} = 0$$

Interpreting the Result: $\cos C = 0$

The cosine of an angle being zero indicates that the angle is a right angle (90°). Therefore,

$$\angle C = 90^\circ$$

This is a significant and elegant result, revealing that Triangle ABC is a right triangle with $\angle C = 90^\circ$.

Verification and Geometric Consistency

Let's verify the logical consistency of this conclusion:

- Since $\angle C$ is 90° , by the Pythagorean theorem, the sides should satisfy:

$$\text{Hypotenuse}^2 = \text{Leg}_1^2 + \text{Leg}_2^2$$

Identify:

- Hypotenuse: side $AB = 85$
- Legs: $AC = 13$, $BC = 84$

Calculate:

$$13^2 + 84^2 = 169 + 7056 = 7225$$

Compare with:

$$AB^2 = 85^2 = 7225$$

They match precisely, confirming that Triangle ABC is a right triangle with the right angle at C.

Implications and Geometric Significance

The discovery that $\angle C = 90^\circ$ simplifies many geometric analyses involving Triangle ABC:

- Right Triangle Properties: The sides satisfy the Pythagorean theorem, making calculations of other angles or distances straightforward.
- Coordinate Geometry Application: If needed, placing the triangle on a coordinate plane becomes easier, with C at the origin, B along the x-axis, and A along the y-axis.
- Further Calculations: Determining other angles or area measurements becomes a matter of simple trigonometric calculations.

Summary of Key Findings

Aspect	Details
Triangle Validity	Sides satisfy triangle inequality
Calculated $\cos C$	0
Conclusion	$\angle C = 90^\circ$ (Right angle)
Confirmed via	Pythagorean theorem, sides' lengths

Final Thoughts and Broader Context

This analysis highlights the power of the Law of Cosines in deciphering triangle properties from side lengths alone. The initial data, which might seem complex or inconsistent at first glance, reveals a straightforward geometric structure—an elegant right triangle. Recognizing such relationships is crucial in fields like engineering design, computer graphics, and mathematical problem-solving, where precise angle measurements underpin accurate modeling.

In essence, the measure of $\angle C$ in Triangle ABC is 90° , making it a classic right triangle with sides adhering perfectly to Pythagoras' theorem.

Disclaimer: The clarity of this result is contingent upon the initial side lengths provided. In real-world applications, always verify measurements and consider potential measurement errors or geometric constraints.

Expert Note: Recognizing when a set of side lengths forms a Pythagorean triple is a valuable skill,

often simplifying complex problems and revealing underlying geometric truths with minimal calculation.

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