Let U = 4i-j, V = 5i + J. And W=i+6j.Find The Specified ScalarU*V+U*W

Let U = 4i - j, V = 5i + j, and W = i + 6j. Find the specified scalar U V + U W. This problem involves the calculation of dot products of vectors, which is a fundamental concept in vector algebra. Understanding how to compute these operations is essential for various applications in physics, engineering, and computer science. In this article, we will explore the step-by-step process to solve this problem, delve into the properties of dot products, and explain how to interpret the results effectively.

Understanding the Vectors and Operations

Before diving into calculations, it's important to understand the given vectors and the operations involved.

The Given Vectors

- U = 4i - j
- V = 5i + j
- W = i + 6j

In component form, these vectors can be written as:
- U = (4, -1)
- V = (5, 1)
- W = (1, 6)

The problem involves computing the scalar quantities U · V and U · W, and then summing these results.

Dot Product Fundamentals

The dot product (also known as the scalar product) of two vectors A = (a_1, a_2) and B = (b_1, b_2) in two-dimensional space is given by:

$$\mathbf{A} \cdot \mathbf{B} = a_1 b_1 + a_2 b_2$$

This operation produces a scalar value, which can be interpreted geometrically as:

- The product of the magnitudes of the two vectors and the cosine of the angle between them.
- A measure of how much one vector extends in the direction of another.

The properties of the dot product include:

- Commutativity: $A \cdot B = B \cdot A$
- Distributivity: $A \cdot (B + C) = A \cdot B + A \cdot C$
- Scalar multiplication: $(kA) \cdot B = k(A \cdot B)$

Understanding these properties helps simplify complex vector calculations.

Calculating U · V

Let's compute the dot product of vectors U and V.

Step-by-step Calculation

- $U = (4, -1)$
- $V = (5, 1)$

Applying the formula:

```
U · V = (4)(5) + (-1)(1) = 20 - 1 = 19
```

Result: $U \cdot V = 19$

This scalar value indicates the extent to which U and V point in the same general direction.

Calculating U · W

Similarly, compute the dot product of U and W.

Step-by-step Calculation

- $U = (4, -1)$
- $W = (1, 6)$

Applying the formula:

```
U · W = (4)(1) + (-1)(6) = 4 - 6 = -2
```

Result: $U \cdot W = -2$

The negative value suggests that U and W are oriented in somewhat opposite directions.

Combining the Results

The original problem asks for the sum of the two scalar products:

```
U · V + U · W = 19 + (-2) = 17
```

Final Answer: The value of $U \cdot V + U \cdot W$ is 17.

Applications and Significance

Understanding how to compute and interpret dot products is critical across various disciplines:

Physics

- Calculating work done by a force: $W = F \cdot d$
- Determining angles between vectors, which relate to directions and orientations

Engineering

- Analyzing forces in structures
- Computing projections of vectors

Computer Science and Data Analysis

- Similarity measures in machine learning algorithms
- Calculating cosine similarity between vectors

Summary of Steps for Similar Problems

To solve similar vector problems involving dot products:

1. Convert vectors to component form.
2. Apply the dot product formula: $a_1b_1 + a_2b_2$.
3. Compute each dot product separately.
4. Sum or combine the scalar results as required.

Ensuring accuracy in each step is critical for correct results, especially when dealing with more complex vectors in higher dimensions.

Conclusion

In this detailed walkthrough, we successfully calculated the scalar quantities $U \cdot V$ and $U \cdot W$, then combined them to find the total value. The problem demonstrates fundamental vector operations essential for various scientific and engineering applications. Mastery of dot products allows for deeper insights into vector relationships, projections, and angles, forming a foundational skill in vector algebra and analysis.

Whether you are working on physics problems, engineering designs, or data analysis, understanding how to manipulate and interpret vectors and their dot products is invaluable. Practice with different vectors and operations will strengthen your skills and enhance your ability to solve complex problems efficiently.

Frequently Asked Questions

How do you compute the dot product U·V given U = 4i - j and V = 5i + j?

$U \cdot V = (4)(5) + (-1)(1) = 20 - 1 = 19$.

What is the dot product U·W if U = 4i - j and W = i + 6j?

U·W = (4)(1) + (-1)(6) = 4 - 6 = -2.

How do you compute the dot product U·V + U·W with the given vectors?

First compute U·V = 19 and U·W = -2, then sum: 19 + (-2) = 17.

What is the value of the scalar U·V + U·W for the given vectors?

The value is 17.

Can you explain the steps to find the scalar U·V + U·W for vectors U, V, and W?

Yes. First, find U·V by multiplying corresponding components and summing: (4)(5) + (-1)(1) = 20 - 1 = 19. Next, find U·W: (4)(1) + (-1)(6) = 4 - 6 = -2. Finally, add the two results: 19 + (-2) = 17.

Additional Resources

Understanding Vector Operations: Find the Scalar U·V + U·W for Given Vectors U, V, and W

When working with vectors in mathematics and physics, understanding how to compute dot products (also known as scalar products) is fundamental. In this guide, we will explore the problem: "Let U = 4i - j, V = 5i + j, and W = i + 6j. Find the scalar U·V + U·W." This exercise not only emphasizes the calculation of dot products but also showcases how to manipulate and interpret vector expressions in a clear, step-by-step manner. By the end of this article, you'll have a thorough understanding of how to approach similar vector problems with confidence.

Introduction to Vectors and Dot Products

Before diving into calculations, it’s vital to review some core concepts:

- Vectors: Quantities that have both magnitude and direction, represented in component form such as U = ai + bj, where a and b are real numbers indicating the vector's components along the x and y axes respectively.

- Dot Product (Scalar Product): An operation that takes two vectors and returns a scalar (number). For vectors $A = a_1i + b_1j$ and $B = a_2i + b_2j$, the dot product is calculated as:

$$
A \cdot B = a_1a_2 + b_1b_2
$$

- Significance: The dot product measures how much one vector extends in the direction of another and is used in calculating angles, projections, and in physics for work calculations.

Step-by-Step Breakdown of the Problem

Given vectors:

- $U = 4i - j$
- $V = 5i + j$
- $W = i + 6j$

Our goal: Calculate the scalar $U \cdot V + U \cdot W$.

This involves two main steps:

1. Computing $U \cdot V$
2. Computing $U \cdot W$
3. Adding the results together

Step 1: Expressing the Vectors in Component Form

While the vectors are already given in component form, let's explicitly write them:

- $U = (4, -1)$
- $V = (5, 1)$
- $W = (1, 6)$

This simplifies the calculation process.

Step 2: Calculating $U \cdot V$

Recall:

$$
U \cdot V = (a_1)(a_2) + (b_1)(b_2)
$$

Plugging in the components:

$$
U \cdot V = (4)(5) + (-1)(1) = 20 - 1 = 19
$$

Result: $U \cdot V = 19$

Step 3: Calculating $U \cdot W$

Similarly,

$$
U \cdot W = (4)(1) + (-1)(6) = 4 - 6 = -2
$$

Result: $U \cdot W = -2$

Step 4: Summing the Results

Now, compute:

$$
U \cdot V + U \cdot W = 19 + (-2) = 17
$$

Final answer: 17

Additional Insights and Applications

Understanding how to perform these calculations is crucial beyond simple exercises. Let’s explore some broader applications:

1. Angle Between Vectors

The dot product relates to the angle θ between two vectors:

$$
U \cdot V = |U||V|\cos \theta
$$

Knowing the dot product helps determine the angle, which is vital in physics and engineering.

2. Projection of One Vector onto Another

The projection of U onto V:

$$
\text{proj}_V U = \left(\frac{U \cdot V}{|V|^2} \right) V
$$

This is useful in resolving forces, velocities, or other vector quantities into components.

3. Work Done in Physics

In physics, the work W done by a force $\vec{F}$ moving an object through displacement $\vec{d}$:

$$
W = \vec{F} \cdot \vec{d}
$$

Calculating dot products is essential in such contexts.

Common Pitfalls and Tips

- Misinterpreting signs: Pay close attention to negative signs in components.
- Component consistency: Ensure that vectors are expressed in the same basis and order.
- Unit consistency: When applying in real-world problems, verify all units are compatible.

Summary of the Calculation

Step	Operation	Result
1	$U \cdot V$	$20 - 1 = 19$
2	$U \cdot W$	$4 - 6 = -2$
3	Sum	$19 + (-2) = 17$

Final Thoughts

Calculating the scalar $U \cdot V + U \cdot W$ provides a clear illustration of vector dot products and their applications. Mastering these fundamental operations paves the way for tackling more complex vector algebra problems, whether in theoretical mathematics, physics, or engineering disciplines. Remember, practice with various vectors and scenarios will deepen your understanding and enhance your problem-solving skills.

By breaking down the problem into manageable steps and understanding the underlying principles, you can confidently approach similar vector computations in your academic or professional journey.

Let U 4i J V 5i J And W I 6j Find The Specified Scalaru V U W

Let U 4i J V 5i J And W I 6j Find The Specified Scalaru V U W

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