Perform The Indicated Operation G(x)=x^3+5x And F(x)=2x-2 Find (g O F)(x)

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Understanding how to compose functions is a fundamental aspect of algebra and calculus. In this article, we will explore the process of performing the indicated operation where $G(x) = x^3 + 5x$ and $F(x) = 2x - 2$, specifically focusing on finding the composition (g o f)(x). This task involves substituting one function into another, a key skill in mathematical problem-solving. By analyzing each step in detail, we aim to provide clarity and insight into function composition, making it accessible to students, educators, and mathematics enthusiasts alike.

Introduction to Function Composition

Function composition is a mathematical operation where two functions are combined to produce a new function. The notation (g o f)(x) represents the composition of functions g and f, meaning g is applied after f. Formally, this is expressed as:

$$(g \circ f)(x) = g(f(x))$$

This process involves two steps:
1. Calculate f(x).
2. Substitute the result into g to find g(f(x)).

Function composition is essential in various fields such as calculus, physics, engineering, and computer science, as it models complex operations built from simpler functions.

Given Functions: G(x) and F(x)

Let's examine the specific functions provided:

- $G(x) = x^3 + 5x$

- F(x) = 2x - 2

Our goal is to find (g o f)(x) = g(f(x)). To do this, we need to:
- Find f(x).
- Plug this into g(x) to obtain g(f(x)).

Step-by-Step Solution for (g o f)(x)

Step 1: Calculate f(x)

Given F(x) = 2x - 2, this is straightforward:
- For any input x, f(x) = 2x - 2.

Step 2: Substitute f(x) into G(x)

Recall G(x) = x3 + 5x. To find (g o f)(x), substitute f(x) into g:
- (g o f)(x) = G(f(x)) = (f(x))3 + 5(f(x)).

Now, replace f(x) with its expression:
- (g o f)(x) = (2x - 2)3 + 5(2x - 2).

Step 3: Simplify the expression

Let's compute each part separately:

a. Cube of (2x - 2):

(2x - 2)3 = (a - b)3 where a = 2x and b = 2.

Using the binomial expansion:

(2x - 2)3 = (2x)3 - 3(2x)2(2) + 3(2x)(2)2 - 23

Calculating each term:

- (2x)3 = 8x3
- 3(2x)2(2) = 3 4x2 2 = 3 4x2 2 = 24x2
- 3(2x)(2)2 = 3 2x 4 = 3 8x = 24x
- 23 = 8

Putting it all together:

(2x - 2)3 = 8x3 - 24x2 + 24x - 8

b. Simplify 5(2x - 2):

5(2x - 2) = 10x - 10

Step 4: Combine the results

Now, sum the two parts:

(g o f)(x) = (8x3 - 24x2 + 24x - 8) + (10x - 10)

Combine like terms:

- For x3: 8x3
- For x2: -24x2
- For x: 24x + 10x = 34x
- Constants: -8 - 10 = -18

Final expression:

$(g \text{ o } f)(x) = 8x^3 - 24x^2 + 34x - 18$

Key Points to Remember When Working with Function Composition

To effectively perform function compositions like (g o f)(x), keep in mind the following:

1. **Identify the functions clearly:** Know what each function does and its formula.

2. **Compute the inner function first:** Evaluate $f(x)$ before plugging into g.

3. **Substitute carefully:** Replace the variable in g with the entire expression for $f(x)$.

4. **Simplify step-by-step:** Expand and combine like terms for clarity and accuracy.

5. **Verify your work:** Double-check algebraic manipulations to avoid errors.

Applications of Function Composition in Mathematics and Real Life

Understanding how to perform and interpret function composition is crucial across various disciplines:

Mathematics and Calculus

- Used in chain rule for derivatives.
- Helps in modeling nested functions and complex equations.

Physics and Engineering

- Describes systems where outputs of one process feed into another, such as in control systems.

Computer Science

- Programming functions that call other functions.
- Building complex algorithms from simpler functions.

Economics and Social Sciences

- Modeling layered behaviors, such as consumer responses based on multiple factors.

Conclusion

Performing the indicated operation $G(x) = x^3 + 5x$ and $F(x) = 2x - 2$ to find (g o f)(x) involves understanding the concept of function composition, executing substitution carefully, and simplifying the resulting expression. The process exemplifies a fundamental skill in algebra, essential for higher-level mathematics and various applied sciences. The final result, (g o f)(x) = 8x3 - 24x2 + 34x - 18, showcases how composition combines the behaviors of both functions into a single, more complex function.

By mastering these techniques, students and professionals can analyze and solve more advanced problems involving nested functions, enabling deeper insights into mathematical modeling, problem-solving, and real-world applications.

Keywords for SEO Optimization:
- Function composition
- How to find (g o f)(x)
- $G(x) = x^3 + 5x$
- F(x) = 2x - 2
- Algebraic functions
- Mathematical problem-solving
- Composition of functions steps
- Polynomial functions
- Calculus basics
- Applying functions in real life

Frequently Asked Questions

How do you find the composition (g ∘ f)(x) for functions g(x) = x^3 + 5x and f(x) = 2x - 2?

To find (g ∘ f)(x), substitute f(x) into g(x), replacing every x in g(x) with f(x). So, (g ∘ f)(x) = g(f(x)) = (f(x))^3 + 5f(x).

What is the step-by-step process to compute (g ∘ f)(x) given g(x) = x^3 + 5x and f(x) = 2x - 2?

First, find f(x) = 2x - 2. Then, substitute f(x) into g(x): g(f(x)) = (2x - 2)^3 + 5(2x - 2). Expand and simplify the expression to get the composition.

What is the final simplified form of (g ∘ f)(x) when g(x) = x^3 + 5x and f(x) = 2x - 2?

The simplified form is (g ∘ f)(x) = 8x^3 - 12x^2 + 4x + 2.

Why is function composition important in mathematics, especially in the context of g(x) = x^3 + 5x and f(x) = 2x - 2?

Function composition allows us to analyze how functions work together, revealing more complex relationships. In this case, it shows how applying f first and then g transforms the input x through both functions.

Can you verify the composition (g ∘ f)(x) for the given functions by plugging in a specific value, say x=1?

Yes. For x=1: f(1) = 2(1) - 2 = 0. Then, g(0) = 0^3 + 5(0) = 0. So, (g ∘ f)(1) = 0, confirming the composition at x=1.

Additional Resources

Perform The Indicated Operation G(x)=x^3+5x And F(x)=2x-2 Find (g o f)(x)

Understanding how to evaluate composite functions such as (g o f)(x) is a fundamental skill in algebra and calculus. The phrase "Perform The Indicated Operation G(x)=x^3+5x And F(x)=2x-2 Find (g o f)(x)" signifies that we are asked to find the composition of two functions, G and F, specifically the composition where G is applied after F. This type of problem not only tests your understanding of function notation but also your ability to manipulate algebraic expressions efficiently. The process involves substituting one function into another, which is a core concept in higher mathematics and essential for solving more complex problems involving functions.

Understanding the Given Functions

Function G(x) = x^3 + 5x

The function G(x) is a polynomial of degree three, combining a cubic term and a linear term. This function takes an input x, cubes it, then adds five times x. Polynomial functions like G are fundamental in algebra because of their simplicity and the wide range of behaviors they can exhibit depending on their degree and coefficients.

Features of G(x):

- Degree: 3 (cubic)
- Type: Polynomial
- Behavior: As x approaches positive or negative infinity, the cubic term dominates, causing the function to grow rapidly in magnitude.
- Symmetry: G(x) is an odd function, which can be verified by checking if G(-x) = -G(x).
- Zeros: The roots of G(x) are solutions to $x^3 + 5x = 0$, which can be factored further.

Function F(x) = 2x - 2

F(x) is a linear function, representing a straight line with a slope of 2 and a y-intercept at -2. Linear functions are straightforward to evaluate and manipulate, making them an excellent starting point for understanding function compositions.

Features of F(x):

- Degree: 1 (linear)
- Slope: 2
- Y-intercept: -2
- Behavior: As x increases or decreases, F(x) increases or decreases linearly.
- Inverse Function: Since F(x) is linear and non-constant, it has an inverse given by $F^{-1}(x) = (x + 2)/2$.

What is a Function Composition?

Function composition, denoted as (g o f)(x), involves applying one function to the result of another. Specifically, (g o f)(x) means g(f(x)). In this problem, because G and F are given, our goal is to find G(F(x)).

This process involves:

- First calculating F(x).
- Then substituting that result into G(x).

Function composition is crucial in various areas of mathematics because it models processes where the output of one step becomes the input of another, such as in function chains, transformations, and in calculus when dealing with derivatives and integrals.

Step-by-Step Solution for (g o f)(x)

Step 1: Find F(x)

Given:

$$ F(x) = 2x - 2 $$

This is straightforward; we know the value of F(x) for any x.

Step 2: Substitute F(x) into G(x)

Since (g o f)(x) = G(F(x)), replace the variable x in G(x) with F(x):

$$ G(F(x)) = (F(x))^3 + 5 \times F(x) $$

Now, substitute F(x) = 2x - 2 into this expression:

$$ G(F(x)) = (2x - 2)^3 + 5 \times (2x - 2) $$

Step 3: Expand and Simplify

Let's expand each term:

1. Expand $(2x - 2)^3$:

Using the binomial expansion:

$$
(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3
$$

Set $a = 2x$, $b = 2$:

$$
(2x)^3 - 3 \times (2x)^2 \times 2 + 3 \times 2x \times 2^2 - 2^3
$$

Calculate each term:

- $(2x)^3 = 8x^3$
- $3 \times (2x)^2 \times 2 = 3 \times 4x^2 \times 2 = 3 \times 4x^2 \times 2 = 24x^2$
- $3 \times 2x \times 4 = 3 \times 2x \times 4 = 24x$
- $2^3 = 8$

Putting it all together:

$$
(2x - 2)^3 = 8x^3 - 24x^2 + 24x - 8
$$

2. Calculate $5 \times (2x - 2)$:

$$
5 \times 2x - 5 \times 2 = 10x - 10
$$

3. Combine all parts:

$$
G(F(x)) = (8x^3 - 24x^2 + 24x - 8) + (10x - 10)
$$

Simplify:

$$
G(F(x)) = 8x^3 - 24x^2 + 24x - 8 + 10x - 10
$$

Combine like terms:

$$
G(F(x)) = 8x^3 - 24x^2 + (24x + 10x) - (8 + 10)
$$

$$
G(F(x)) = 8x^3 - 24x^2 + 34x - 18
$$

Final Answer:

$$
\boxed{
(g \circ f)(x) = 8x^3 - 24x^2 + 34x - 18
}
$$

Features and Observations

- The composition results in a cubic polynomial, indicative of the combined transformations of the original functions.
- The degree of the composite function matches the highest degree among the functions, which is 3.
- The coefficients reflect the interplay of the linear and cubic components.

Applications and Significance

Understanding the composition of functions such as G and F extends beyond algebra into various fields:

- Calculus: Chain rule applications rely heavily on understanding composite functions.
- Engineering: Signal processing often involves function compositions to model system responses.
- Computer Science: Functional programming uses composition to build complex operations from simpler functions.
- Physics: Transformation sequences are modeled through composite functions.

Pros and Cons of Function Composition in Problem Solving

Pros:

- Simplifies complex transformations into manageable steps.
- Enhances understanding of how functions interact.
- Essential for advanced mathematical analysis and modeling.

Cons:

- Can become algebraically intensive, especially with higher-degree polynomials.
- Mistakes in substitution or expansion can lead to errors.
- Requires familiarity with binomial expansion and algebraic manipulation.

Conclusion

The task of performing the indicated operation G(x) = x^3 + 5x and F(x) = 2x - 2 to find (g o f)(x) exemplifies fundamental concepts in algebra related to function composition. Through systematic substitution and expansion, we derived the composite function as \(8x^3 - 24x^2 + 34x - 18\). Mastery of such operations is crucial for progressing into calculus, differential equations, and applied mathematics. The process underscores the importance of clear algebraic techniques and highlights the elegance of combining simple functions to produce complex behaviors. Whether in theoretical mathematics or practical applications, understanding and calculating composite functions remain a vital skill for students and professionals alike.

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