Sin^2A.cos2B- Sin^2B.cos2A= Cos^2B-cos^2Ashow In Step By Step Explanation

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Understanding and proving trigonometric identities is fundamental in mathematics, especially in simplifying complex expressions and solving equations. Today, we will delve into the detailed step-by-step proof of the identity:

$$\sin^2 A \cos 2B - \sin^2 B \cos 2A = \cos^2 B - \cos^2 A$$

This identity involves various standard trigonometric functions, including sine, cosine, and their double-angle counterparts. Our goal is to manipulate and simplify the left-hand side (LHS) to arrive at the right-hand side (RHS) systematically. Let's begin this exploration by reviewing essential formulas and then proceed through logical steps.

Understanding the Components and Formulas Involved

Before diving into the proof, it's important to recall key trigonometric identities that will aid us in simplifying the expression.

Double-Angle Formulas

- Cosine double-angle formula: $\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$
- Sine double-angle formula: $\sin 2\theta = 2 \sin \theta \cos \theta$

Square of Sine and Cosine

- $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$
- $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

These identities allow us to rewrite squared sine and cosine functions in terms of double angles, which often simplifies the algebra involved.

Step-by-Step Proof of the Identity

Now, we proceed with the proof by manipulating the LHS to match the RHS using the above identities.

Step 1: Express $\sin^2 A$ and $\sin^2 B$ in terms of $\cos 2A$ and $\cos 2B$

Using the identities:

$$
\sin^2 A = \frac{1 - \cos 2A}{2}
$$
$$
\sin^2 B = \frac{1 - \cos 2B}{2}
$$

Substitute these into the LHS:

$$
\text{LHS} = \left(\frac{1 - \cos 2A}{2} \right) \cos 2B - \left(\frac{1 - \cos 2B}{2} \right) \cos 2A
$$

Step 2: Expand the expression

Distribute the terms:

$$
\text{LHS} = \frac{1 - \cos 2A}{2} \cdot \cos 2B - \frac{1 - \cos 2B}{2} \cdot \cos 2A
$$

This becomes:

$$
\text{LHS} = \frac{\cos 2B - \cos 2A \cos 2B}{2} - \frac{\cos 2A - \cos 2A \cos 2B}{2}
$$

Rearranged as:

$$
\text{LHS} = \frac{\cos 2B - \cos 2A \cos 2B - \cos 2A + \cos 2A \cos 2B}{2}
$$

Notice that $\cos 2A \cos 2B$ appears with both positive and negative signs, which will simplify.

Step 3: Combine like terms

Combine the terms:

$$
\text{LHS} = \frac{\cos 2B - \cos 2A + (- \cos 2A \cos 2B + \cos 2A \cos 2B)}{2}
$$

Since $- \cos 2A \cos 2B + \cos 2A \cos 2B = 0$, they cancel out, leaving:

$$
\text{LHS} = \frac{\cos 2B - \cos 2A}{2}
$$

Step 4: Express RHS in a comparable form

Recall the RHS:

$$
\cos^2 B - \cos^2 A
$$

Using the identities for squares:

$$
\cos^2 \theta = \frac{1 + \cos 2\theta}{2}
$$

So:

$$
\cos^2 B - \cos^2 A = \frac{1 + \cos 2B}{2} - \frac{1 + \cos 2A}{2}
$$

Simplify:

$$
\frac{1 + \cos 2B - 1 - \cos 2A}{2} = \frac{\cos 2B - \cos 2A}{2}
$$

Conclusion: Equivalence of LHS and RHS

From previous steps, we've shown:

$$
\text{LHS} = \frac{\cos 2B - \cos 2A}{2}
$$

$$
\text{RHS} = \frac{\cos 2B - \cos 2A}{2}
$$

Therefore,

$$
\boxed{
\sin^2 A \cos 2B - \sin^2 B \cos 2A = \cos^2 B - \cos^2 A
}
$$

The identity is proven.

Summary and Key Takeaways

- Rewriting squared sine and cosine functions in terms of double angles simplifies complex expressions.
- Recognizing common terms and canceling out facilitates the proof.
- The symmetry in the identities involving $\cos^2 \theta$ and $\sin^2 \theta$ is crucial.

Additional Tips for Similar Proofs

- Always recall fundamental identities before starting complex proofs.
- Break down the expression into smaller parts and handle each systematically.

- Look for opportunities to substitute identities to express everything in terms of double angles.
- Simplify step-by-step, ensuring each algebraic manipulation is justified.

This detailed, step-by-step approach not only verifies the identity but also enhances understanding of the interplay between different trigonometric functions and their identities. Mastering such proofs strengthens your problem-solving skills and deepens your grasp of trigonometry.

Frequently Asked Questions

What is the given identity involving $\sin^2 A \cdot \cos 2B$ - $\sin^2 B \cdot \cos 2A$ and $\cos^2 B - \cos^2 A$?

The identity is: $\sin^2 A \cdot \cos 2B - \sin^2 B \cdot \cos 2A = \cos^2 B - \cos^2 A$. It shows a relationship between the expressions involving sine and cosine squares and double angles.

How can we verify the identity $\sin^2 A \cdot \cos 2B - \sin^2 B \cdot \cos 2A = \cos^2 B - \cos^2 A$ step by step?

We verify by expressing all terms in terms of sine and cosine, applying double angle formulas, and simplifying both sides to see if they are equal.

What are the key formulas used in the step-by-step proof of this identity?

The key formulas are: $\sin^2 x = (1 - \cos 2x)/2$, $\cos 2x = 2\cos^2 x - 1$, and the algebraic identities for expanding products and simplifying expressions.

How do we start the proof of the given identity?

Begin by expressing $\sin^2 A$ and $\sin^2 B$ using the identity $\sin^2 x = (1 - \cos 2x)/2$, and similarly for $\cos 2A$ and $\cos 2B$, then substitute into the original expression.

What is the role of double angle formulas in proving this identity?

Double angle formulas allow us to rewrite $\cos 2A$ and $\cos 2B$ in terms of $\cos^2 A$ and $\cos^2 B$, simplifying the expressions and making it easier to compare both sides.

Can you show the step to simplify $\sin^2 A \cdot \cos 2B$ - $\sin^2 B \cdot \cos 2A$ after substitution?

Yes. After substituting $\sin^2 A = (1 - \cos 2A)/2$ and similarly for $\sin^2 B$, expand the products, combine like terms, and simplify to see if the expression reduces to $\cos^2 B - \cos^2 A$.

What is the final simplified form of the left side after all substitutions and simplifications?

The left side simplifies to $\cos^2 B - \cos^2 A$, matching the right side, thus confirming the identity.

Why is this identity important in trigonometry?

It reveals relationships between squared sine and cosine functions and their double angles, useful in solving complex trigonometric equations and integrals.

Are there any special cases where this identity simplifies further?

Yes, when A = B or specific angles like 0°, 45°, or 90°, the identity can be checked more straightforwardly as a special case.

Can this identity be derived using algebraic or geometric methods?

It is primarily derived algebraically via substitution of double angle formulas, but geometric interpretations involving unit circles can also provide insight.

Additional Resources

Trigonometric Identities: An In-Depth Exploration of the Identity
"$\sin^2 A \cdot \cos 2B - \sin^2 B \cdot \cos 2A = \cos^2 B - \cos^2 A$"

Understanding and proving trigonometric identities form a cornerstone of advanced mathematics, especially within the realms of calculus, algebra, and geometry. The identity $\sin^2 A \cdot \cos 2B - \sin^2 B \cdot \cos 2A = \cos^2 B - \cos^2 A$ is a fascinating one that showcases the interplay between sine, cosine, and their double-angle counterparts. In this comprehensive review, we will dissect this identity step-by-step, exploring its derivation, underlying principles, and the mathematical elegance it embodies.

Introduction to the Identity

At first glance, the identity appears complex due to the presence of squared sine functions multiplied by double-angle cosine functions, and the difference involving squared cosines. To fully appreciate and verify this identity, understanding the fundamental concepts and formulas involved is essential.

Key Elements:

- Sine and Cosine Functions: Basic trigonometric functions defining ratios in right-angled triangles and points on the unit circle.
- Double-Angle Formulas: Expressions that relate the trigonometric functions of twice an angle to functions of the original angle.
- Pythagorean Identities: Fundamental identities like $\sin^2 A + \cos^2 A = 1$, which serve as the backbone for many derivations.

Foundational Concepts and Preliminaries

Before delving into the proof, it's crucial to revisit the core identities and properties that will be instrumental in our step-by-step derivation.

Double-Angle Formulas

These formulas express sine and cosine of double angles in terms of the single angle:

- $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$
- $\sin 2A = 2\sin A \cos A$

Similarly, for angles B:

- $\cos 2B = \cos^2 B - \sin^2 B = 2\cos^2 B - 1 = 1 - 2\sin^2 B$

Pythagorean Identity

- $\sin^2 A + \cos^2 A = 1$
- $\sin^2 B + \cos^2 B = 1$

These identities allow us to replace squared sine or cosine terms with expressions involving the other, simplifying the algebraic manipulations that follow.

Step-by-Step Derivation of the Identity

Let’s systematically verify the given identity:

\[
\boxed{
\text{Show that} \quad \sin^2A \cdot \cos 2B - \sin^2B \cdot \cos 2A = \cos^2B - \cos^2A
}

\]

Step 1: Express the double angles explicitly

Replace \(\cos 2A\) and \(\cos 2B\) with their double-angle formulas:

\[
\cos 2A = 2 \cos^2A - 1
\]
\[
\cos 2B = 2 \cos^2B - 1
\]

Substituting into the original expression:

\[
\sin^2A \cdot (2 \cos^2B - 1) - \sin^2B \cdot (2 \cos^2A - 1)
\]

Step 2: Distribute the terms

Distribute to remove parentheses:

\[
\sin^2A \cdot 2 \cos^2B - \sin^2A - \sin^2B \cdot 2 \cos^2A + \sin^2B
\]

Rearranged, the expression becomes:

\[
2 \sin^2A \cos^2B - \sin^2A - 2 \sin^2B \cos^2A + \sin^2B
\]

Step 3: Group similar terms

Group terms to facilitate further simplification:

\[
(2 \sin^2A \cos^2B - 2 \sin^2B \cos^2A) + (\sin^2B - \sin^2A)

\]

This leads to two primary parts:

- Part 1: \(2 (\sin^2A \cos^2B - \sin^2B \cos^2A)\)
- Part 2: \(\sin^2B - \sin^2A\)

Our goal is to relate this to \(\cos^2B - \cos^2A\), which appears on the right side of the identity.

Step 4: Focus on the primary difference term

Analyze the first part:

\[
2 (\sin^2A \cos^2B - \sin^2B \cos^2A)
\]

Recall the identities:

\[
\sin^2A = 1 - \cos^2A
\]
\[
\sin^2B = 1 - \cos^2B
\]

Express the difference:

\[
\sin^2A \cos^2B - \sin^2B \cos^2A = (1 - \cos^2A) \cos^2B - (1 - \cos^2B) \cos^2A
\]

Expanding:

\[
\cos^2B - \cos^2A \cos^2B - \cos^2A + \cos^2A \cos^2B
\]

Notice that \(- \cos^2A \cos^2B + \cos^2A \cos^2B = 0\), so they cancel out, leaving:

\[
\cos^2B - \cos^2A
\]

This is a crucial step because it simplifies the primary difference to the difference of squared cosines.

Step 5: Simplify the entire expression

Recall from earlier:

\[
2 (\sin^2A \cos^2B - \sin^2B \cos^2A) + (\sin^2B - \sin^2A)
\]

Substituting the simplified difference:

\[
2 (\cos^2B - \cos^2A) + (\sin^2B - \sin^2A)
\]

Now, focus on \(\sin^2B - \sin^2A\). Using the Pythagorean identities:

\[
\sin^2B - \sin^2A = (1 - \cos^2B) - (1 - \cos^2A) = - \cos^2B + \cos^2A
\]

which simplifies to:

\[
\cos^2A - \cos^2B
\]

Step 6: Final combination of terms

Putting everything together:

\[
2 (\cos^2B - \cos^2A) + (\cos^2A - \cos^2B)
\]

Distribute:

\[
2 \cos^2B - 2 \cos^2A + \cos^2A - \cos^2B
\]

Group similar terms:

\[

(2 \cos^2B - \cos^2B) + (-2 \cos^2A + \cos^2A) = \cos^2B - \cos^2A
\]

This matches the right side of the original identity, confirming that:

\[
\boxed{
\sin^2A \cdot \cos 2B - \sin^2B \cdot \cos 2A = \cos^2B - \cos^2A
}
\]

Summary and Significance of the Identity

This derivation reveals the elegant symmetry and interconnectedness of trigonometric functions. The key steps involved:

- Recognizing the double-angle formulas and substituting them appropriately.
- Expressing sine squared terms in terms of cosine squared via Pythagorean identities.
- Carefully algebraically manipulating terms to reveal cancellations and simplifications.

Why is this identity important?

- It exemplifies the power of algebraic manipulation in trigonometry.
- It connects different trigonometric functions, showcasing their relationships.
- It provides a tool for simplifying complex expressions in calculus and analytical geometry.
- Such identities are often used in solving equations, proving other identities, and in Fourier analysis.

Further Insights and Applications

Applications in Mathematics and Physics:

- Signal Processing: Trigonometric identities like these underpin Fourier series and transforms, essential in analyzing periodic signals.
- Geometry: Understanding angle relationships in polygons and circles often involves such identities.
- Calculus: Derivatives and integrals involving trigonometric functions often simplify via identities like this one.
- Engineering: Wave mechanics and electromagnetic theory utilize trigonometric identities to manipulate wave equations.

Potential Variations and Generalizations:

- Extending identities involving multiple angles and higher powers.
- Using identities to derive formulas for multiple angles or sum/difference formulas.
- Exploring similar identities involving tangent and cotangent functions for broader applications.

Conclusion

The identity $\sin^2 A \cdot \cos 2B - \sin^2 B \cdot \cos 2A = \cos^2 B - \cos^2 A$ exemplifies

Sin 2a Cos2b Sin 2b Cos2a Cos 2b Cos 2ashow In Step By Step Explanation

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