So The Determinant For Finding The Equation Of Plane Is DetX12y11z34=0

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Understanding how to find the equation of a plane in three-dimensional space is a fundamental concept in vector algebra and analytic geometry. The statement "DetX12y11z34=0" encapsulates a crucial method for deriving the plane's equation through the use of determinants. This approach leverages the properties of determinants to effectively handle the geometric relationships between points, vectors, and planes. In this comprehensive article, we delve into the significance of determinants in plane equations, interpret the specific determinant notation, and explore the step-by-step process to derive and verify the equation of a plane using determinants.

Fundamentals of Plane Equations in 3D Space

Understanding the Geometric Setup

In three-dimensional space, a plane is a flat, two-dimensional surface extending infinitely in all directions. To define a plane explicitly, typically, three non-collinear points or a point and a normal vector are required.

- Points in Space: Denoted as $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, and $P_3(x_3, y_3, z_3)$.

- Normal Vector: A vector perpendicular to the plane, often derived via cross product of two vectors lying on the plane.

- Plane Equation: Usually expressed in the form $Ax + By + Cz + D = 0$, where (A, B, C) is the normal vector.

Role of Determinants in Plane Geometry

Determinants provide a compact, algebraic way to verify the coplanarity of points and derive the plane's equation. Specifically:

- The determinant of a matrix formed by the coordinates of points can indicate whether points are coplanar.

- Using determinants simplifies calculations involving multiple points and vectors.

- The vanishing of certain determinants indicates the points lie on the same plane.

Interpreting the Determinant Notation: DetX12y11z34=0

Decoding the Notation

The notation "DetX12y11z34=0" suggests a determinant involving points or vectors labeled with specific coordinates:

- X12: Likely refers to point $P_1(x_1, y_1, z_1)$ or a vector associated with the point 12.

- Y11: Corresponds to point $P_2(x_2, y_2, z_2)$ or associated with the label 11.

- Z34: Corresponds to point $P_3(x_3, y_3, z_3)$ or associated with label 34.

Alternatively, these could denote specific coordinate values or placeholders in a matrix.

Given the context, it probably signifies a determinant constructed with the coordinates of three points:

$$
\begin{vmatrix}
x_1 & y_1 & z_1 \\
x_2 & y_2 & z_2 \\
x_3 & y_3 & z_3 \\
\end{vmatrix} = 0
$$

which indicates the points are coplanar.

Why the Determinant Equals Zero?

- If the determinant of the matrix formed by the coordinates of three points is zero, it implies the points are coplanar.

- The determinant essentially computes the volume of the parallelepiped formed by vectors from these points.

- Zero volume indicates the points lie on the same plane.

Deriving the Equation of a Plane Using Determinants

Step-by-Step Procedure

To find the equation of a plane passing through three points P_1, P_2, P_3, follow these steps:

1. Identify Coordinates:

- $P_1(x_1, y_1, z_1)$

- $P_2(x_2, y_2, z_2)$

- $P_3(x_3, y_3, z_3)$

2. Form Vectors on the Plane:

- $\vec{AB} = P_2 - P_1 = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

- $\vec{AC} = P_3 - P_1 = (x_3 - x_1, y_3 - y_1, z_3 - z_1)$

3. Calculate the Normal Vector:

- $\vec{n} = \vec{AB} \times \vec{AC}$

- The cross product yields the normal vector (A, B, C).

4. Form the Plane Equation:

- The general form is:

$$
A(x - x_1) + B(y - y_1) + C(z - z_1) = 0
$$

- Expand and simplify to:

$$
A x + B y + C z + D = 0
$$

where

$$
D = -(A x_1 + B y_1 + C z_1)
$$

5. Express the Equation:

- Final form:

$$
A x + B y + C z + D = 0
$$

Note: Alternatively, one can directly derive the coefficients using determinants.

Using Determinants to Find the Plane Equation

The equation of the plane passing through points P_1, P_2, P_3 can be expressed as a determinant:

$$
\begin{vmatrix}
x - x_1 & y - y_1 & z - z_1 \\
x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\
x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\
\end{vmatrix} = 0
$$

Expanding this determinant yields a linear equation in x, y, z.

Alternatively, a more direct way involves constructing a 4x4 determinant:

$$
\begin{vmatrix}
x & y & z & 1 \\
x_1 & y_1 & z_1 & 1 \\
x_2 & y_2 & z_2 & 1 \\
x_3 & y_3 & z_3 & 1 \\
\end{vmatrix} = 0
$$

which upon expansion provides the plane's equation in the form $Ax + By + Cz + D = 0$.

Application of the Determinant Method: An Example

Given Points:

Suppose we have three points:

- $P_1(1, 2, 3)$
- $P_2(4, 5, 6)$
- $P_3(7, 8, 9)$

Step 1: Form the Determinant

Construct the matrix:

$$
\begin{vmatrix}
x & y & z & 1 \\
1 & 2 & 3 & 1 \\
4 & 5 & 6 & 1 \\
7 & 8 & 9 & 1 \\
\end{vmatrix} = 0
$$

Step 2: Expand the Determinant

Expanding along the first row, we get:

$$
x \cdot \begin{vmatrix}
2 & 3 & 1 \\
5 & 6 & 1 \\
8 & 9 & 1 \\
\end{vmatrix}
- y \cdot \begin{vmatrix}
1 & 3 & 1 \\
4 & 6 & 1 \\
7 & 9 & 1 \\
\end{vmatrix}
+ z \cdot \begin{vmatrix}
1 & 2 & 1 \\
4 & 5 & 1 \\
7 & 8 & 1 \\
\end{vmatrix}
- 1 \cdot \begin{vmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9 \\
$$

\end{vmatrix} = 0
\]

Calculating each minor leads to the explicit plane equation.

Step 3: Final Plane Equation

Once minors are computed, the resulting linear equation in \(x, y, z \) represents the plane passing through the three points.

Significance and Limitations of the Determinant Method

Advantages

- Provides a systematic way to verify coplanarity.

- Easily derives the plane's equation from given points.

- Useful in computational applications and analytical geometry.

Limitations

- Requires precise calculation of determinants, which can be cumbersome with complex data.

- Not suitable for degenerate cases where points are collinear (determinant equals

Frequently Asked Questions

What is the significance of using a determinant to find the equation of a plane?

Using a determinant helps to derive the equation of a plane when given three points or a point and a normal vector, by ensuring the plane passes through the points and is perpendicular to the normal vector.

How is the determinant format set up for finding the plane equation from points?

The determinant is set up as a 3x3 matrix with the coordinates of points or vectors, such as |x - x0, y - y0, z - z0|, and the plane equation is derived by setting the determinant to zero.

In the expression Det|X, 12y, 11z, 34|=0, what do the variables X, y, and z represent?

X, y, and z represent the variables corresponding to the coordinates of a general point (x, y, z) in the plane, used to formulate the plane's equation via the determinant.

How can I interpret the determinant expression Det|X, 12y, 11z, 34|=0 in terms of plane geometry?

This determinant represents a condition that all points (x, y, z) satisfying the plane's equation must fulfill, ensuring the point lies on the plane defined by the given parameters.

What is the step-by-step process to derive the plane equation from the determinant expression?

First, expand the determinant to obtain a linear equation in x, y, and z, then simplify to express the plane equation in the standard form $Ax + By + Cz + D = 0$.

Can the determinant method be used for planes defined by points, vectors, or both?

Yes, the determinant method can be applied to find the plane equation when given three points, a point and a normal vector, or two vectors lying on the plane.

What does setting the determinant equal to zero imply about the points involved?

Setting the determinant to zero indicates that the points or vectors are coplanar, meaning they all lie on the same plane.

How does the coefficients in the determinant relate to the normal vector of the plane?

The coefficients obtained from the expanded determinant form correspond to the components of the plane's normal vector, which is perpendicular to the plane.

Are there any specific conditions or limitations when using determinants to find the plane equation?

Yes, the points used must be non-collinear (not all on a single line), and the determinant method assumes the points or vectors provided are sufficient to define a unique plane.

How does the determinant method compare to other methods for finding the plane equation?

The determinant method provides a systematic approach for deriving the plane equation from three points or two vectors, often simplifying calculations compared to algebraic methods, especially in coordinate geometry.

Additional Resources

So The Determinant For Finding The Equation Of Plane Is DetX12y11z34=0—this statement encapsulates a fundamental approach in vector and analytical geometry. It highlights how determinants serve as a powerful algebraic tool to derive the equation of a plane in three-dimensional space. Whether you're a student trying to grasp the core concepts, a teacher preparing instructional material, or a professional solving geometric problems, understanding how to use determinants to find the equation of a plane is essential. This guide aims to demystify this process, providing a comprehensive breakdown of the concepts, steps, and applications involved.

Understanding the Equation of a Plane

Before delving into determinants, it's crucial to understand what an equation of a plane represents in three-dimensional space.

What Is a Plane?

A plane is a flat, two-dimensional surface extending infinitely in 3D space. It can be uniquely defined by:

- A point through which it passes, or
- A point and a normal vector (a vector perpendicular to the plane).

Standard Form of the Plane Equation

The general form of a plane's equation is:

$$Ax + By + Cz + D = 0$$

where:

- (A, B, C) is the normal vector to the plane.
- D is a scalar constant.

Given three non-collinear points, or a point and a normal vector, we can determine the values of $A, B, C,$ and D.

The Role of Determinants in Finding the Equation of a Plane

Using determinants is a systematic way to derive the equation of a plane that passes through three points. The expression:

$$\det \begin{bmatrix}
x - x_0 & y - y_0 & z - z_0 \\
x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\
x_2 - x_0 & y_2 - y_0 & z_2 - z_0
\end{bmatrix} = 0$$

ensures that the point (x, y, z) lies on the same plane passing through points (x_0, y_0, z_0), (x_1, y_1, z_1), and (x_2, y_2, z_2).

Why determinants?

- The determinant of a matrix composed of vectors signifies the volume of the parallelepiped they form.
- When the volume is zero (determinant equals zero), the vectors are coplanar, meaning all points lie on the same plane.

Step-by-Step Guide: Deriving the Plane Equation Using a Determinant

Let's explore how to derive the plane's equation with a concrete example, connecting to the expression:

det X12 y11 z34 = 0

Step 1: Identify the Points

Suppose you are given three points:

- $P_0 = (x_0, y_0, z_0)$
- $P_1 = (x_1, y_1, z_1)$
- $P_2 = (x_2, y_2, z_2)$

These points are known or chosen based on the problem context.

Step 2: Form Vectors

Construct vectors relative to one point, say P_0:

- $\vec{A} = P_1 - P_0 = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$
- $\vec{B} = P_2 - P_0 = (x_2 - x_0, y_2 - y_0, z_2 - z_0)$

Step 3: Set Up the Determinant

The determinant setup involves the general point (x, y, z):

$$
\det
\begin{bmatrix}
x - x_0 & y - y_0 & z - z_0 \\
x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\
x_2 - x_0 & y_2 - y_0 & z_2 - z_0
\end{bmatrix} = 0
$$

This determinant equals zero when (x, y, z) lies on the same plane as the three known points.

Step 4: Expand the Determinant

Expanding the determinant, you get the scalar equation:

$$
(x - x_0) \left((y_1 - y_0)(z_2 - z_0) - (z_1 - z_0)(y_2 - y_0) \right) - (y - y_0) \left((x_1 - x_0)(z_2 - z_0) - (z_1 - z_0)(x_2 - x_0) \right) + (z - z_0) \left((x_1 - x_0)(y_2 - y_0) - (y_1 - y_0)(x_2 - x_0) \right) = 0
$$

This is the general equation of the plane.

Applying the Concept: Deciphering the Expression `detX12y11z34=0`

The notation `detX12y11z34=0` appears to be a condensed or shorthand way of representing a determinant involving specific points or coefficients.

Interpreting the Notation

- `X12` suggests a component involving the x-coordinate, perhaps x_1 or x_2.
- `y11` could imply y_1 or perhaps a coefficient associated with the y-coordinate.
- `z34` likely references points or coefficients involving z_3 and z_4.

In many contexts, such shorthand may denote a determinant built from coordinates:

\[
\det

$$
\begin{bmatrix}
x & y & z \\
x_1 & y_1 & z_1 \\
x_2 & y_2 & z_2
\end{bmatrix} = 0
$$

which is the standard form for the plane passing through points (x_1, y_1, z_1) and (x_2, y_2, z_2), using the general point (x, y, z).

Clarifying the Expression

Suppose the expression is a placeholder or a specific notation used in a problem to denote the determinant involving points or coefficients, which equal zero when the point lies on the plane.

Practical Example: Deriving the Equation of a Plane

Let's work through a practical example step-by-step.

Given Points:

- $P_0 = (1, 2, 3)$
- $P_1 = (4, 0, 2)$
- $P_2 = (0, 5, 1)$

Step 1: Create Vectors

$$
\vec{A} = (4 - 1, 0 - 2, 2 - 3) = (3, -2, -1)
$$

$$
\vec{B} = (0 - 1, 5 - 2, 1 - 3) = (-1, 3, -2)
$$

Step 2: Compute the Cross Product (Normal Vector)

The normal vector $\vec{N}$ to the plane is:

$$
\vec{N} = \vec{A} \times \vec{B} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
3 & -2 & -1 \\
-1 & 3 & -2
\end{vmatrix}
$$

Calculating:

$$
\mathbf{i}((-2)(-2) - (-1)(3)) = \mathbf{i}(4 - (-3)) = \mathbf{i}(7)
$$

$$
- \mathbf{j}(3(-2) - (-1)(-1)) = - \mathbf{j}(-6 - 1) = - \mathbf{j}(-7) = 7 \mathbf{j}
$$

$$
\mathbf{k}(3 \times 3 - (-2)(-1)) = \mathbf{k}(9 - 2) = 7 \mathbf{k}
$$

Thus,

$$
\vec{n} = (7, 7, 7)
$$

which simplifies to $(1, 1, 1)$ after dividing by 7.

Step 3: Write the Plane Equation

Using point $P_0 = (1, 2, 3)$ and normal vector $(1, 1, 1)$:

$$
1(x - 1) + 1(y - 2) + 1(z - 3) = 0
$$

Simplify:

$$
x - 1 + y - 2 + z - 3 = 0
$$

$$
x + y + z - 6 = 0
$$

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