Solve The Given Initial-value Problem. X' = 8 1 5 6 X, X(0) = 4 6X(t) =

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Understanding the Initial-Value Problem (IVP)

In the realm of differential equations, an initial-value problem (IVP) is a differential equation coupled with specific initial conditions. Solving an IVP involves finding a function that satisfies both the differential equation and the initial conditions.

The problem presented appears to involve a system of differential equations characterized by matrices and initial conditions:

- Differential equation: X' = A X
- Initial condition: X(0) = X□

Where A is a matrix, and X(t) is a vector function.

Deciphering the Given Problem

The problem states:

"Solve The Given Initial-value Problem. X' = 8 1 5 6 X, X(0) = 4 6X(t) ="

This notation suggests that:

- The differential equation is a system of two equations involving matrix multiplication.
- The matrix A is:

$$
A = \begin{bmatrix}
8 & 1 \\
5 & 6
\end{bmatrix}
$$

- The initial condition vector X(0) is:

$$
X(0) = \begin{bmatrix}
4 \\
6
\end{bmatrix}
$$

- The goal is to find X(t), a function of time t, satisfying the IVP.

Mathematical Formulation of the System

Given the above, the initial-value problem can be written as:

$$
\frac{d}{dt}X(t) = A X(t)
$$

$$
X(0) = \begin{bmatrix} 4 \\ 6 \end{bmatrix}
$$

where

$$
A = \begin{bmatrix}
8 & 1 \\
5 & 6
\end{bmatrix}
$$

The solution involves matrix exponentials and eigenvalues/eigenvectors.

Methodology for Solving the System

To solve the system, we typically proceed with the following steps:

1. Find Eigenvalues of Matrix A

Eigenvalues are scalars λ satisfying:

\[

$$
\det(A - \lambda I) = 0
$$

where I is the identity matrix.

2. Find Eigenvectors corresponding to each Eigenvalue

Eigenvectors v satisfy:

$$
(A - \lambda I) v = 0
$$

3. Construct the General Solution

The general solution X(t) is a linear combination of eigenvectors scaled by exponentials:

$$
X(t) = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2
$$

where c_1 and c_2 are constants determined by initial conditions.

4. Apply Initial Conditions to Find Constants

Substitute t=0 into X(t) and solve for c_1 and c_2:

$$
X(0) = c_1 v_1 + c_2 v_2
$$

Calculating Eigenvalues of Matrix A

Let's compute the eigenvalues:

$$
A - \lambda I = \begin{bmatrix}
8 - \lambda & 1 \\
5 & 6 - \lambda
\end{bmatrix}
$$

The characteristic polynomial:

$$
\det(A - \lambda I) = (8 - \lambda)(6 - \lambda) - (5)(1) = 0
$$

Calculating:

$$
(8 - \lambda)(6 - \lambda) - 5 = 0
$$

$$
(8)(6) - 8\lambda - 6\lambda + \lambda^2 - 5 = 0

\]

\[
48 - 8\lambda - 6\lambda + \lambda^2 - 5 = 0
\]

\[
\lambda^2 - 14 \lambda + 43 = 0
\]

Solve the quadratic:

\[
\lambda^2 - 14 \lambda + 43 = 0
\]

Discriminant:

\[
D = (14)^2 - 4 \times 1 \times 43 = 196 - 172 = 24
\]

Eigenvalues:

\[
\lambda_{1,2} = \frac{14 \pm \sqrt{24}}{2} = \frac{14 \pm 2\sqrt{6}}{2} = 7 \pm \sqrt{6}
\]

Thus,

\[
\boxed{
\lambda_1 = 7 + \sqrt{6} \quad,\quad \lambda_2 = 7 - \sqrt{6}

}

\]

Finding Eigenvectors

Let's find eigenvectors for each eigenvalue.

Eigenvector for \(\lambda_1 = 7 + \sqrt{6}\)

Solve:

\[

(A - \lambda_1 I) v = 0

\]

\[

\begin{bmatrix}

8 - (7 + \sqrt{6}) & 1 \\

5 & 6 - (7 + \sqrt{6})

\end{bmatrix}

\begin{bmatrix}

v_{1} \\

v_{2}

\end{bmatrix}

= \begin{bmatrix}

0 \\

0

\end{bmatrix}

\]

Simplify:

\[

\begin{bmatrix}

1 - \sqrt{6} & 1 \\

5 & -1 - \sqrt{6}

\end{bmatrix}

\]

From the first row:

\[

(1 - \sqrt{6}) v_1 + v_2 = 0 \Rightarrow v_2 = - (1 - \sqrt{6}) v_1

\]

Choose \(v_1 = 1\):

\[

v^{(1)} = \begin{bmatrix}

1 \\

- (1 - \sqrt{6})

\end{bmatrix}

\]

Similarly, for \(\lambda_2 = 7 - \sqrt{6}\):

\[

A - \lambda_2 I = \begin{bmatrix}

8 - (7 - \sqrt{6}) & 1 \\

5 & 6 - (7 - \sqrt{6})

\end{bmatrix}

= \begin{bmatrix}

1 + \sqrt{6} & 1 \\

5 & -1 + \sqrt{6}

\end{bmatrix}

\]

From the first row:

\[

(1 + \sqrt{6}) v_1 + v_2 = 0 \Rightarrow v_2 = - (1 + \sqrt{6}) v_1

\]

Choose \(v_1=1\):

\[

v^{(2)} = \begin{bmatrix}

1 \\

- (1 + \sqrt{6})

\end{bmatrix}

\]

Constructing the General Solution

The general solution:

$$
X(t) = c_1 e^{\lambda_1 t} v^{(1)} + c_2 e^{\lambda_2 t} v^{(2)}
$$

or explicitly:

$$
X(t) = c_1 e^{(7 + \sqrt{6}) t} \begin{bmatrix} 1 \\ - (1 - \sqrt{6}) \end{bmatrix} + c_2 e^{(7 - \sqrt{6}) t} \begin{bmatrix} 1 \\ - (1 + \sqrt{6}) \end{bmatrix}
$$

which simplifies to:

$$
X(t) = c_1 e^{(7 + \sqrt{6}) t} \begin{bmatrix} 1 \\ - (1 - \sqrt{6}) \end{bmatrix} + c_2 e^{(7 - \sqrt{6}) t} \begin{bmatrix} 1 \\ - (1 + \sqrt{6}) \end{bmatrix}
$$

Applying Initial Conditions

At $t=0$, $X(0) = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$. Plugging in:

$$
\begin{bmatrix} 4 \\ 6 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ - (1 - \sqrt{6}) \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ - (1 + \sqrt{6}) \end{bmatrix}
$$

This leads to the system:

$$
4 = c_1 + c_2
$$

$$
6 = - c_1 (1 - \sqrt{6}) - c_2 (1 + \sqrt{6})
$$

Let's write the second equation explicitly:

$$
6 = - c_1 (1 - \sqrt{6}) - c_2 (1 + \sqrt{6})
$$

Express \(c_

Frequently Asked Questions

What is the initial-value problem given in the equation X' = 8 1 5 6 X with X(0) = 4 6?

The problem appears to be a differential equation X' = 8 1 5 6 X with the initial condition X(0) = 4 6, but the exact notation is unclear. Likely, it represents X' = 8X, with initial condition X(0) = 4.

How do you interpret the differential equation X' = 8X?

The differential equation X' = 8X is a first-order linear ordinary differential equation indicating that the rate of change of X is proportional to X itself, with proportionality constant 8.

What is the general solution to the differential equation $X' = 8X$?

The general solution is $X(t) = C e^{8t}$, where C is a constant determined by initial conditions.

How do you find the particular solution given the initial condition $X(0) = 4$?

Substitute t=0 into the general solution: $X(0) = C e^{0} = C$. Given $X(0) = 4$, so $C=4$. Therefore, the particular solution is $X(t) = 4 e^{8t}$.

What is the solution to the initial-value problem $X' = 8X$ with $X(0) = 4$?

The solution is $X(t) = 4 e^{8t}$.

Can this differential equation be solved using separation of variables?

Yes. Separating variables gives $dX / X = 8 dt$, which integrates to $\ln|X| = 8t + C$, leading to the exponential solution.

Are there any special considerations when solving this initial-value problem?

The main consideration is ensuring the initial condition is correctly applied to determine the constant C. Also, the exponential solution is valid for all real t.

What is the significance of the initial condition in solving the differential equation?

The initial condition $X(0) = 4$ allows us to find the specific solution by determining the constant C in the general solution.

Additional Resources

Solve The Given Initial-Value Problem: An In-Depth Analytical Approach

The process of solving initial-value problems (IVPs) is fundamental in understanding the dynamic behavior of systems described by differential equations. These problems not only provide solutions that describe how a system evolves over time but also offer insights into the underlying physics, engineering, or mathematical principles at play. In this article, we delve into a specific IVP that involves a first-order linear differential equation, examining each step meticulously to ensure clarity and comprehension.

Understanding the Initial-Value Problem

Before proceeding with the solution, it is crucial to interpret the problem statement accurately. The given problem appears to be:

- Differential equation: X' = 8 1 5 6 X
- Initial condition: X(0) = 4 6X(t) =

However, the notation seems somewhat ambiguous or possibly corrupted. Based on common forms and context, a reasonable interpretation could be:

Assumed Corrected Form:
X' = A X, where A is a coefficient matrix or scalar, and initial condition X(0) = X□.

Given the entries "8 1 5 6" and "4 6X(t) =", it seems that the problem involves a 2x2 matrix differential equation:

$$
\frac{d}{dt} \mathbf{X}(t) =
\begin{bmatrix}
8 & 1 \\
5 & 6
\end{bmatrix}
\mathbf{X}(t)
$$

with initial condition:

$$
\mathbf{X}(0) =
\begin{bmatrix}
4 \\
6
\end{bmatrix}
$$

This format aligns with typical systems of linear differential equations encountered in engineering and applied mathematics.

Mathematical Formulation of the Problem

System of Differential Equations

The problem reduces to solving the following system:

$$
\frac{d}{dt}
\begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix}
=
\begin{bmatrix}
8 & 1 \\
5 & 6
\end{bmatrix}
\begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix}
$$

with initial conditions:

$$
x(0) = 4, \quad y(0) = 6
$$

Significance of the System

Such systems are prevalent in modeling coupled phenomena — for instance, population dynamics, electrical circuits, or mechanical systems with interconnected components. The matrix encapsulates the rates of change and interactions between variables.

Analytical Solution Approach

The solution to a linear system of differential equations involves several key steps:

1. Eigenvalue and Eigenvector Analysis: Find eigenvalues and eigenvectors of the coefficient matrix.
2. Construct the General Solution: Use eigenvalues/eigenvectors to express the general solution.
3. Apply Initial Conditions: Determine constants in the solution to satisfy initial conditions.

Let's proceed step by step.

Step 1: Find Eigenvalues of the Coefficient Matrix

Given matrix:

$$
A =
\begin{bmatrix}
8 & 1 \\
5 & 6
\end{bmatrix}
$$

Eigenvalues λ satisfy:

$$
\det(A - \lambda I) = 0
$$

Calculating:

$$
\det
\begin{bmatrix}
8 - \lambda & 1 \\
5 & 6 - \lambda
\end{bmatrix}
= (8 - \lambda)(6 - \lambda) - 5 \times 1 = 0
$$

Expanding:

$$
(8 - \lambda)(6 - \lambda) - 5 = 0
$$

$$
(8 \times 6) - 8 \lambda - 6 \lambda + \lambda^2 - 5 = 0
$$

$$
48 - 14 \lambda + \lambda^2 - 5 = 0
$$

$$
\lambda^2 - 14 \lambda + 43 = 0
$$

This quadratic yields eigenvalues:

$$
\lambda_{1,2} = \frac{14 \pm \sqrt{(14)^2 - 4 \times 1 \times 43}}{2}
$$

Calculating discriminant:

$$
14^2 - 4 \times 43 = 196 - 172 = 24
$$

Thus:

$$
\lambda_{1,2} = \frac{14 \pm \sqrt{24}}{2} = \frac{14 \pm 2\sqrt{6}}{2} = 7 \pm \sqrt{6}
$$

Eigenvalues:

$$
\boxed{
\lambda_1 = 7 + \sqrt{6} \quad \text{and} \quad \lambda_2 = 7 - \sqrt{6}
}
$$

Step 2: Find Eigenvectors

For each eigenvalue, solve:

$$
(A - \lambda I)\mathbf{v} = 0
$$

Eigenvector corresponding to \(\lambda_1 = 7 + \sqrt{6}\):

\[
A - \lambda_1 I =
\begin{bmatrix}
8 - \lambda_1 & 1 \\
5 & 6 - \lambda_1
\end{bmatrix}
\]

Compute entries:

\[
8 - (7 + \sqrt{6}) = 1 - \sqrt{6}
\]
\[
6 - (7 + \sqrt{6}) = -1 - \sqrt{6}
\]

So,

\[
\begin{bmatrix}
1 - \sqrt{6} & 1 \\
5 & -1 - \sqrt{6}
\end{bmatrix}
\]

Set the first row:

\[

(1 - \sqrt{6}) v_1 + v_2 = 0 \Rightarrow v_2 = - (1 - \sqrt{6}) v_1

\]

Choosing \(v_1 = 1\), the eigenvector is:

\[

\mathbf{v}_1 =

\begin{bmatrix}

1 \\

- (1 - \sqrt{6})

\end{bmatrix}

\]

Similarly, for \(\lambda_2 = 7 - \sqrt{6}\):

\[

8 - (7 - \sqrt{6}) = 1 + \sqrt{6}

\]

\[

6 - (7 - \sqrt{6}) = -1 + \sqrt{6}

\]

Eigenvector:

\[

(1 + \sqrt{6}) v_1 + v_2 = 0 \Rightarrow v_2 = - (1 + \sqrt{6}) v_1

\]

Choosing \(v_1 = 1\):

\[

\mathbf{v}_2 =

\begin{bmatrix}

1 \\

- (1 + \sqrt{6})

\end{bmatrix}

\]

Step 3: Construct the General Solution

The general solution is a linear combination of eigenvectors scaled by exponentials of eigenvalues:

\[

\mathbf{X}(t) = c_1 e^{\lambda_1 t} \mathbf{v}_1 + c_2 e^{\lambda_2 t} \mathbf{v}_2

\]

Substituting eigenvalues and eigenvectors:

\[

\mathbf{X}(t) = c_1 e^{(7 + \sqrt{6}) t}

\begin{bmatrix}

1 \\

- (1 - \sqrt{6})

\end{bmatrix}

+ c_2 e^{(7 - \sqrt{6}) t}

\begin{bmatrix}

1 \\

- (1 + \sqrt{6})

\end{bmatrix}

\]

Step 4: Apply Initial Conditions

Given:

\[
\mathbf{X}(0) = \begin{bmatrix} 4 \\ 6 \end{bmatrix}
\]

At \(t=0\):

\[
\mathbf{X}(0) = c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2
\]

Expressed explicitly:

\[
\begin{bmatrix}
4 \\
6
\end{bmatrix}
= c_1
\begin{bmatrix}
1 \\
- (1 - \sqrt{6})
\end{bmatrix}

+ c_2

\begin{bmatrix}

1 \\

- (1 + \sqrt{6})

\end{bmatrix}

\]

This yields a system:

1. \(c_1 + c_2 = 4 \)
2. \(- c_1 (1 - \sqrt{6}) - c_2 (1 + \sqrt{6}) = 6 \)

Rewrite the second equation:

\[

- c_1 (1 - \sqrt{6}) - c_2 (1 + \sqrt{6}) = 6

\]

Expressed as:

\[

- c_1 + c_1 \sqrt{6} - c_2 - c_2 \sqrt{6} = 6

\]

Group terms:

\[

(- c_1 - c_2)

Solve The Given Initial Value Problem X 8 1 5 6 X X 0 4 6x T

Solve The Given Initial Value Problem X 8 1 5 6 X X 0 4 6x T

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