Solve Using The Unit Column Method:(x-3y+z=3 3x 6y + 5z = 2 |-x+2y 2z=-1

Solve Using The Unit Column Method: (x - 3y + z = 3, 3x + 6y + 5z = 2, -x + 2y + 2z = -1) is a powerful technique for solving systems of linear equations efficiently. The Unit Column Method, also known as the Column Vector Method, simplifies the process by representing the equations and their coefficients as columns in matrices, making it easier to manipulate and find solutions systematically. This article will guide you through understanding and applying the Unit Column Method to solve the given system step-by-step.

Understanding the System of Equations

Before diving into the solution, it's important to understand the structure of the system:

- x - 3y + z = 3
- 3x + 6y + 5z = 2
- -x + 2y + 2z = -1

This is a system of three equations with three unknowns: x, y, and z. The goal is to find the values of these variables that satisfy all three equations simultaneously.

Representing the System Using the Unit Column Method

The Unit Column Method involves creating matrices that represent the coefficients of the variables and the constants on the right-hand side of the equations.

Step 1: Write the Coefficient Matrix

The coefficient matrix, often called matrix A, contains the coefficients of x, y, and z:

\[
A = \begin{bmatrix}
1 & -3 & 1 \\
3 & 6 & 5 \\
-1 & 2 & 2 \\
\end{bmatrix}
\]

Step 2: Write the Constant Column Vector

The constants on the right-hand side form a column vector:

\[
\mathbf{b} = \begin{bmatrix}
3 \\
2 \\
-1 \\
\end{bmatrix}

\]

Step 3: Write the Variable Vector

Let $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. The system can now be written as:

$$
A \mathbf{x} = \mathbf{b}
$$

The task reduces to solving for $\mathbf{x}$ using matrix operations.

Applying the Unit Column Method to Find the Solution

The key idea in the Unit Column Method is to generate special matrices called column matrices or unit matrices that help in solving for each variable independently.

Step 4: Constructing the Column Matrices for the Variables

To find each variable, replace the corresponding column of matrix A with the constants vector $\mathbf{b}$. This process is similar to Cramer's Rule but is interpreted through the lens of the Unit Column Method.

For each variable:

- Replace the first column with $\mathbf{b}$ to solve for x.
- Replace the second column with $\mathbf{b}$ to solve for y.
- Replace the third column with $\mathbf{b}$ to solve for z.

Construct the matrices:

- For x:

$$
A_x = \begin{bmatrix}
3 & -3 & 1 \\
2 & 6 & 5 \\
-1 & 2 & 2 \\
\end{bmatrix}
$$

- For y:

$$
A_y = \begin{bmatrix}
1 & 3 & 1 \\
3 & 2 & 5 \\
-1 & -1 & 2 \\
\end{bmatrix}
$$

- For z:

$$
A_z = \begin{bmatrix}
1 & -3 & 3 \\
3 & 6 & 2 \\
-1 & 2 & -1 \\
\end{bmatrix}
$$

Step 5: Calculate the Determinants

The determinants of these matrices are crucial because they relate to the solutions:

$$
x = \frac{\det(A_x)}{\det(A)}, \quad y = \frac{\det(A_y)}{\det(A)}, \quad z = \frac{\det(A_z)}{\det(A)}
$$

First, compute $\det(A)$.

Calculating $\det(A)$:

$$
A = \begin{bmatrix}
1 & -3 & 1 \\
3 & 6 & 5 \\
-1 & 2 & 2 \\
\end{bmatrix}
$$

Using the rule of Sarrus or cofactor expansion:

$$
\det(A) = 1 \times (6 \times 2 - 5 \times 2) - (-3) \times (3 \times 2 - 5 \times -1) + 1 \times (3 \times 2 - 6 \times -1)
$$

Calculations step-by-step:

$$
1 \times (12 - 10) = 1 \times 2 = 2
$$
$$
-(-3) \times (6 + 5) = 3 \times 11 = 33
$$
$$
1 \times (6 + 6) = 1 \times 12 = 12
$$

Sum:

$$
\det(A) = 2 + 33 + 12 = 47
$$

Next, compute $\det(A_x)$:

$$
A_x = \begin{bmatrix}
$$

3 & -3 & 1 \\
2 & 6 & 5 \\
-1 & 2 & 2 \\
\end{bmatrix}
\]

Similarly,

\[
\det(A_x) = 3 \times (6 \times 2 - 5 \times 2) - (-3) \times (2 \times 2 - 5 \times -1) + 1 \times (2 \times 2 - 6 \times -1)
\]

Calculations:

\[
3 \times (12 - 10) = 3 \times 2 = 6
\]
\[
-(-3) \times (4 + 5) = 3 \times 9 = 27
\]
\[
1 \times (4 + 6) = 1 \times 10 = 10
\]

Sum:

\[
\det(A_x) = 6 + 27 + 10 = 43
\]

Compute \(\det(A_y)\):

\[
A_y = \begin{bmatrix}
1 & 3 & 1 \\
3 & 2 & 5 \\
-1 & -1 & 2 \\
\end{bmatrix}
\]

Determinant:

\[
1 \times (2 \times 2 - 5 \times -1) - 3 \times (3 \times 2 - 5 \times -1) + 1 \times (3 \times -1 - 2 \times -1)
\]

Calculations:

\[
1 \times (4 + 5) = 1 \times 9 = 9
\]
\[
-3 \times (6 + 5) = -3 \times 11 = -33
\]
\[
1 \times (-3 + 2) = 1 \times -1 = -1
\]

Sum:

$$
\det(A_y) = 9 - 33 - 1 = -25
$$

Compute $\det(A_z)$:

$$
A_z = \begin{bmatrix}
1 & -3 & 3 \\
3 & 6 & 2 \\
-1 & 2 & -1 \\
\end{bmatrix}
$$

Determinant:

$$
1 \times (6 \times -1 - 2 \times 2) - (-3) \times (3 \times -1 - 2 \times -1) + 3 \times (3 \times 2 - 6 \times -1)
$$

Calculations:

$$
1 \times (-6 - 4) = 1 \times -10 = -10
$$
$$
-(-3) \times (-3 + 2) = 3 \times -1 = -3
$$
$$
3 \times (6 + 6) = 3 \times 12 = 36
$$

Sum:

$$
\det(A_z) = -10 - 3 + 36 = 23
$$

Calculating the Variables

Now, apply Cramer's Rule via the Unit Column Method:

$$
x = \frac{\det(A_x)}{\det(A)} = \frac{43}{47}
$$

$$
y = \frac{\det(A_y)}{\det(A)} = \frac{-25}{47}
$$

$$
z = \frac{\det(A_z)}{\det(A)} = \frac{23}{47}
$$

Final Solution:

$$
\boxed{
x = \frac{43}{47}, \quad y = -\frac{25}{47}, \quad z = \frac{23}{47}
}
$$

Summary and Benefits

Frequently Asked Questions

What is the step-by-step approach to solve the system of equations using the unit column method?

First, write the system in matrix form, then perform row operations to convert the matrix into a form where each variable's column becomes a unit (identity) column, solving for each variable sequentially.

How do you set up the augmented matrix for the given system of equations?

Arrange the coefficients of x, y, and z along with the constants into a matrix:
[[1, -3, 1, 3], [3, -6, 5, 2], [-1, 2, 2, -1]].

What are common challenges faced when using the unit column method for such systems?

Challenges include correctly performing row operations without errors, maintaining the pivot positions, and ensuring the columns are converted into unit columns without losing solutions.

Can the unit column method be applied to any system of linear equations? Why or why not?

It is most effective for systems that are square (same number of equations and variables) and have a unique solution. For inconsistent or dependent systems, other methods may be more appropriate.

What is the final solution obtained for the system using the unit column method?

After applying the method, the solution is x = 2, y = -1, z = 0.

Additional Resources

Solve Using The Unit Column Method: A Comprehensive Guide to Solving Systems of Equations

When tackling systems of equations, various methods can be employed to find solutions efficiently. One such technique, the Unit Column Method, offers a structured and insightful approach by leveraging matrix representations and column operations. This method is particularly useful for understanding the underlying relationships between variables and for solving systems with multiple equations. In this guide, we'll explore how to solve using the unit column method through a step-by-step process, using the example system:

\[
\begin{cases}
x - 3y + z = 3 \\

3x + 6y + 5z = 2 \\
-x + 2y + 2z = -1
\end{cases}
\]

We'll break down the procedure thoroughly, ensuring clarity for learners and practitioners alike.

Understanding the Unit Column Method

The Unit Column Method is rooted in matrix algebra, where systems of linear equations are expressed in matrix form as \(A \mathbf{x} = \mathbf{b}\). The core idea involves manipulating the coefficient matrix \(A\) and the constant vector \(\mathbf{b}\) using column operations, especially focusing on unit columns—columns with a single '1' and zeros elsewhere—to systematically isolate variables.

Why Use the Unit Column Method?

- Structured Approach: It provides a clear framework that aligns with matrix operations.
- Insight into Variable Relationships: By working through columns, you can better understand how each variable contributes to the system.
- Facilitates Row Operations: It complements methods like Gaussian elimination, offering an alternative perspective.

Step-by-Step Solution to the System Using the Unit Column Method

Step 1: Write the System in Matrix Form

Express the system as:

$$
A = \begin{bmatrix}
1 & -3 & 1 \\
3 & 6 & 5 \\
-1 & 2 & 2
\end{bmatrix},
\quad
\mathbf{x} = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix},
\quad
\mathbf{b} = \begin{bmatrix}
3 \\
2 \\
-1
\end{bmatrix}
$$

The goal is to find $\mathbf{x}$ such that $A \mathbf{x} = \mathbf{b}$.

Step 2: Form the Column Matrices

Break down the coefficient matrix into its individual columns:

$$
A = \left[\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3 \right]
$$

Where:
- $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$
- $\mathbf{a}_2 = \begin{bmatrix} -3 \\ 6 \\ 2 \end{bmatrix}$
- $\mathbf{a}_3 = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$

The system becomes:

$$
x \mathbf{a}_1 + y \mathbf{a}_2 + z \mathbf{a}_3 = \mathbf{b}
$$

Step 3: Use the Unit Column Approach

The core idea is to manipulate the columns to isolate each variable—this involves creating unit columns (columns with a single '1' in a position and zeros elsewhere)—and then perform column operations to reduce the system.

Procedure:

- Create unit columns corresponding to each variable.
- Express the right-hand side as a combination of these columns.
- Perform column operations to simplify the system step by step.

Applying the Unit Column Method to the System

Step 4: Find the First Variable (x)

Goal: Isolate x by eliminating other variables from the first equation.

- First, observe the first equation:

$$
x - 3y + z = 3
$$

- To focus on x, consider the column $\mathbf{a}_1 = [1, 3, -1]^T$. It already has a leading 1 in the first position, which is ideal.

- Next, eliminate x from the second and third equations by subtracting appropriate multiples of the first column.

Perform column operations:

- Replace $\mathbf{a}_2$ with $\mathbf{a}_2 - 3 \mathbf{a}_1$:

$$
\mathbf{a}_2' = \mathbf{a}_2 - 3 \mathbf{a}_1 = \begin{bmatrix} -3 - 3(1) \\ 6 - 3(3) \\ 2 - 3(-1) \end{bmatrix} = \begin{bmatrix} -3 - 3 \\ 6 - 9 \\ 2 + 3 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \\ 5 \end{bmatrix}
$$

- Replace $\mathbf{a}_3$ with $\mathbf{a}_3 - 1 \times \mathbf{a}_1$:

$$
\mathbf{a}_3' = \mathbf{a}_3 - \mathbf{a}_1 = \begin{bmatrix} 1 - 1 \\ 5 - 3 \\ 2 - (-1) \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}
$$

Update the system with these new columns:

$$

$$
A' = \left[\mathbf{a}_1 \quad \mathbf{a}_2' \quad \mathbf{a}_3' \right]
$$

Corresponding constants:

$$
\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}
$$

Now, express the system as:

$$
x \mathbf{a}_1 + y \mathbf{a}_2' + z \mathbf{a}_3' = \mathbf{b}
$$

Step 5: Solve for x

Focus on the first equation:

$$
x \times 1 + y \times (-6) + z \times 0 = 3
$$

which simplifies to:

$$
x - 6 y = 3
$$

Express x in terms of y:

$$

$$
x = 3 + 6 y
$$

Step 6: Find y and z

Next, consider the second and third equations in the transformed system.

Second equation:

$$
\mathbf{a}_2' \text{ with constant } 2 \Rightarrow -6 y + 2 z = 2
$$

Third equation:

$$
\mathbf{a}_3' \text{ with constant } -1 \Rightarrow 2 y + 3 z = -1
$$

Now, we have two equations:

$$
\begin{cases}
-6 y + 2 z = 2 \\
2 y + 3 z = -1
\end{cases}
$$

Step 7: Solve for y and z

Equation 1:

$$
-6 y + 2 z = 2
$$

Divide through by 2:

$$
-3 y + z = 1
$$

Express z:

$$
z = 1 + 3 y
$$

Equation 2:

$$
2 y + 3 z = -1
$$

Substitute z:

$$
2 y + 3 (1 + 3 y) = -1
$$

Simplify:

\[

$$
2y + 3 + 9y = -1
$$

$$
(2y + 9y) + 3 = -1
$$

$$
11y + 3 = -1
$$

$$
11y = -4
$$

$$
y = -\frac{4}{11}
$$

Now, find z:

$$
z = 1 + 3 \times \left(-\frac{4}{11}\right) = 1 - \frac{12}{11} = \frac{11}{11} - \frac{12}{11} = -\frac{1}{11}
$$

Finally, find x:

$$
x = 3 + 6y = 3 + 6 \times \left(-\frac{4}{11}\right) = 3 - \frac{24}{11} = \frac{33}{11} - \frac{24}{11} = \frac{9}{11}
$$

Final Solution:

\[
\boxed{

x = \frac{9}{11}, \quad y = -\frac{4}{11}, \quad z = -
\frac{1}{11}
}
\]

Key Takeaways and Tips for Using the Unit Column Method

- Start with matrix representation: Express the system clearly in matrix form, breaking down into columns.
- Use column operations strategically to eliminate variables and simplify the system.
- Create unit columns when possible to isolate variables, making the system easier to solve.
- Apply

[Solve Using The Unit Column Method X 3y Z 3 3x 6y 5z 2 X 2y 2z 1]

Solve Using The Unit Column Method X 3y Z 3 3x 6y 5z 2 X 2y 2z 1

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