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Understanding the solution set of algebraic equations is fundamental in mathematics, especially when dealing with linear and polynomial equations. The equation $8(3x - 2) - 4 = 8x + 4(4x - 3)$ presents an interesting case to analyze because of its structure and the operations involved. In this article, we will explore what type of solution set this equation has, how to solve it step-by-step, and what the implications of the solution set are. Whether you're a student studying algebra or someone interested in mathematical problem-solving, this detailed breakdown will clarify the nature of this equation's solutions.

Deciphering the Equation: What Are We Working With?

Before determining the solution set, it's vital to interpret the equation correctly. The given equation is:

$$8(3x - 2) - 4 = 8x + 4(4x - 3)$$

At first glance, the notation may seem ambiguous. The key is to understand whether the parentheses denote multiplication or grouping, and how to interpret the expressions within.

Clarifying the Expression

The most common interpretation in algebra involves treating the expressions inside parentheses as grouped terms, with multiplication implied between numbers and parentheses. The expression appears to omit multiplication signs, which is typical in algebraic notation. Therefore, the equation likely reads as:

$$8(3x + 2) - 4 = 8x + 4(4x + 3)$$

This interpretation makes sense because:

- The parentheses enclose terms involving x , which are typical in algebraic expressions.
- The missing operators are assumed to be multiplication, a standard convention in algebra.

Note: If the original problem intended a different meaning (like subtraction or division), that would significantly alter the solution. But in standard algebraic notation, the above is the most logical interpretation.

Step-by-Step Solution Process

Now that we have clarified the expression, we can proceed to simplify and solve the equation.

Step 1: Expand Both Sides

Left Side:

$$8(3x + 2) + 4$$

- Multiply 8 and 4 first:

$$8 \cdot 4 = 32$$

- Now distribute:

$$32(3x + 2) = 32 \cdot 3x + 32 \cdot 2 = 96x + 64$$

Right Side:

$$8x + 4(4x + 3)$$

- Distribute:

$$4 \cdot 4x + 4 \cdot 3 = 16x + 12$$

- Add 8x:

$$8x + 16x + 12 = (8x + 16x) + 12 = 24x + 12$$

So, the simplified equation is:

$$96x + 64 = 24x + 12$$

Step 2: Isolate the Variable

- Subtract 24x from both sides:

$$96x - 24x + 64 = 12$$

- Simplify:

$$72x + 64 = 12$$

- Subtract 64 from both sides:

$$72x = 12 - 64 = -52$$

- Divide both sides by 72:

$$x = -52 / 72$$

- Simplify the fraction:

$$x = -13 / 18$$

Analyzing the Solution: What Type Of Solution Set Does It Have?

The calculation yields a single value for x : $x = -13/18$. This indicates the equation has exactly one solution.

What Is a Unique Solution?

A unique solution occurs when the equation simplifies to a statement that provides a single value for the variable. In this case, the solution is:

$$x = -13/18$$

meaning the solution set is a singleton — a set containing exactly one element.

Implications of a Single Solution

- The solution set is a singleton set: $\{ -13/18 \}$
- The equation is consistent and independent: it has exactly one solution.
- The graph of the equation (if plotted) would intersect the x-axis at exactly one point.

What If the Solution Did Not Exist or Was Infinite?

While in this specific case, we found a single solution, it's important to understand the other possibilities:

1. No Solution (Inconsistent Equation)

- Occurs if, after simplification, the equation reduces to a false statement, such as $0 = \text{non-zero number}$.
- For example, $0x + 5 = 0x + 3$ simplifies to $5 = 3$, which is false.
- Solution set: empty set, denoted as $\emptyset$ or $\{\}$.

2. Infinite Solutions (Dependent Equation)

- Occurs if both sides simplify to the same expression, such as $2x + 4 = 2x + 4$.
- Solution set: all real numbers, denoted as $\mathbb{R}$.

In our case, the calculation led to a single value; thus, the solution set is finite and contains exactly one element.

Summary: The Nature of the Solution Set

To conclude, the solution set of the equation $8(3x + 2) - 4 = 8x + 4(4x + 3)$ is:

- A single-element set, or a singleton: $\{-13/18\}$
- The equation is consistent and independent, with exactly one real solution.

Why Understanding Solution Sets Is Important

Knowing the nature of the solution set helps in various ways:

- Mathematical reasoning: It aids in understanding the behavior of equations.
- Graphical interpretation: Knowing whether an equation has one, none, or infinitely many solutions informs how it appears graphically.
- Problem-solving: Recognizing the type of solution set guides the approach for solving similar equations.

Additional Tips for Solving Similar Equations

- Always clarify the notation before solving.
- Expand and simplify carefully.

- Isolate the variable step-by-step.
- Check for extraneous solutions when dealing with rational expressions or square roots.
- Understand the difference between unique, no, and infinite solutions.

Conclusion

The equation $8(3x + 2) + 4 = 8x + 4(4x + 3)$, once correctly interpreted and simplified, has exactly one real solution: $x = -13/18$. This means its solution set is a singleton set, representing a case of a consistent and independent linear equation. Recognizing the type of solution set is crucial for further mathematical reasoning and problem-solving, and this example illustrates the importance of careful interpretation and systematic solving of algebraic equations.

Whether you're preparing for exams, working on algebra homework, or just interested in mathematical concepts, understanding the nature of solution sets enhances your problem-solving toolkit and deepens your comprehension of algebraic structures.

Frequently Asked Questions

What type of solution set does the equation $8(3x + 2) + 4 = 8x + 4(4x + 3)$ have?

The equation has a unique solution set, meaning it simplifies to a single value of x .

How do you determine whether the equation $8(3x + 2) + 4 = 8x + 4(4x + 3)$ has one, many, or no solutions?

By simplifying both sides and comparing the resulting expressions; if variables cancel out and constants are equal, there are many solutions; if variables cancel but constants differ, no solutions; otherwise, a single solution exists.

What is the first step in solving the equation $8(3x + 2) + 4 = 8x + 4(4x + 3)$?

Distribute the coefficients inside the parentheses to eliminate parentheses: $8 \cdot 3x + 8 \cdot 2 + 4 = 8x + 4 \cdot 4x + 4 \cdot 3$.

After distributing, how do you simplify the equation $8(3x + 2) + 4 = 8x + 4(4x + 3)$?

Simplify both sides to get $24x + 16 + 4 = 8x + 16x + 12$.

What does it mean if, after simplifying, the variables cancel out and the constants are not equal?

It indicates that the equation has no solution because there is a contradiction.

Can this type of linear equation have infinitely many solutions?

Yes, if after simplification all variables cancel and the constants are equal, the equation has infinitely many solutions.

What is the importance of understanding whether an equation has a unique, infinite, or no solution?

It helps in understanding the nature of the problem and determining the possible values of variables that satisfy the equation.

Additional Resources

The Equation $8(3x + 2) + 4 = 8x + 4(4x + 3)$ Has What Type Of Solution Set?

Understanding the nature of solutions to algebraic equations is fundamental in mathematics, particularly in algebra. The question about the solution set of the given equation, $8(3x + 2) + 4 = 8x + 4(4x + 3)$, invites a detailed exploration into how equations are solved, the types of solutions they possess, and what the specific structure of this equation reveals about its solutions. In this comprehensive analysis, we will dissect the equation step-by-step, analyze its properties, and ultimately classify its solution set—be it unique, infinite, or nonexistent.

Introduction to the Equation and Its Components

The given equation is:

$$8(3x + 2) + 4 = 8x + 4(4x + 3)$$

At a glance, this is a linear algebraic equation involving the variable x , constants, and multiple terms multiplied by coefficients. The structure suggests the potential for a single solution, no solutions, or infinitely many solutions, depending on how the equation simplifies.

Why is understanding the solution set important?

In algebra, the solution set reveals whether an equation has a unique solution, a set of solutions, or none at all. Recognizing the type of solution set helps in understanding the behavior of equations and their applications in broader mathematical contexts.

Step-by-Step Solution Process

To determine the nature of the solution set, we need to solve the equation systematically. The process involves expanding, combining like terms, simplifying, and analyzing the resulting equation.

Step 1: Expand Both Sides

$$\begin{aligned} &\text{- Left side: } 8(3x + 2) + 4 \\ &= (8 \cdot 3x) + (8 \cdot 2) + 4 \\ &= 24x + 16 + 4 \\ &= 24x + 20 \end{aligned}$$

$$\begin{aligned} &\text{- Right side: } 8x + 4(4x + 3) \\ &= 8x + (4 \cdot 4x) + (4 \cdot 3) \\ &= 8x + 16x + 12 \\ &= 24x + 12 \end{aligned}$$

Now, the equation becomes:

$$24x + 20 = 24x + 12$$

Step 2: Simplify and Isolate the Variable

Subtract $24x$ from both sides:

$$24x + 20 - 24x = 24x + 12 - 24x$$

which simplifies to:

$$20 = 12$$

This is a critical step because the variable terms cancel out, leaving a statement involving constants only.

Analysis of the Simplified Equation

After simplification, the equation reduces to:

$$20 = 12$$

This is a false statement because 20 does not equal 12. The implications of this are significant:

- This contradiction indicates that there is no value of x that can satisfy the original equation.
- Therefore, the equation has no solutions.

Classification of the Solution Set

Based on the algebraic manipulation, the solution set can be classified as follows:

1. No Solution (Inconsistent Equation):

When simplifying the equation yields a contradiction such as $20 = 12$ or $5 = 7$, it indicates that the original equation is inconsistent—no value of x satisfies it. This is known as an empty solution set.

2. Unique Solution:

If, after simplification, we obtain a linear equation with a single value of x (e.g., $3x = 9$, leading to $x = 3$), then the solution set contains exactly one element.

3. Infinite Solutions:

If the simplification reduces to a tautology such as $0 = 0$, then every real number satisfies the equation, resulting in an infinite solution set.

In our specific case, the contradiction ($20 \neq 12$) confirms that the equation has no solutions.

Implications of the Solution Set Type

Understanding that this equation has no solutions provides several insights:

- Mathematical Consistency:

The structure of the equation was designed in such a way that, after expansion and simplification, the variable terms canceled out, revealing an inconsistency in the constants.

- Applications in Problem Solving:

Recognizing equations with no solutions is crucial in real-world problem-solving, as it indicates that a particular set of conditions or constraints cannot be simultaneously satisfied.

- Educational Significance:

This example illustrates the importance of carefully simplifying equations and checking for contradictions, a fundamental skill in algebra and higher mathematics.

Broader Context: When Do Equations Have Different Types of Solutions?

To deepen understanding, it's helpful to look at typical categories of solutions and their characteristics:

- Unique Solution (Linear Equation):

Most linear equations with a variable have exactly one solution. For example, $2x + 3 = 7$ simplifies to $x = 2$.

- Infinite Solutions (Identity):

When both sides simplify to the same expression (e.g., $3x + 2 = 3x + 2$), the equation is true for all real numbers, indicating infinitely many solutions.

- No Solutions (Contradiction):

When simplification leads to a false statement, like $5 = 7$, the equation has no solutions.

This classification is fundamental in algebra and helps in analyzing the consistency of systems and equations.

Applications and Significance in Mathematical Contexts

The analysis of the solution set isn't merely an academic exercise; it has real-world relevance in various fields:

- Engineering and Physics:

Equations model physical systems, and understanding whether solutions exist affects design and analysis.

- Economics and Business:

Constraints in optimization problems often lead to equations whose solvability determines feasible solutions.

- Computer Science:

Algorithms solving equations or systems of equations must account for the possibility of no solutions or multiple solutions to ensure robustness.

In educational contexts, exercises like analyzing the solution set of this particular equation reinforce critical thinking and algebraic manipulations.

Conclusion: The Solution Set of the Given Equation

In conclusion, the equation $8(3x + 2) + 4 = 8x + 4(4x + 3)$ simplifies to a contradiction ($20 = 12$) after expansion and algebraic manipulation. This contradiction indicates that the equation has no solutions, and its solution set is empty.

This example underscores the importance of systematic algebraic techniques and understanding the structure of equations. Recognizing whether an equation has a unique solution, infinitely many solutions, or no solutions is a fundamental skill that aids in mathematical reasoning, problem-solving, and analyzing real-world systems.

In essence, the solution set of the provided equation is:

It has no solutions.

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