

The Length Of The Latus Rectum Is 10 And The Parabola Opens To The Left.

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Understanding the properties of parabolas is fundamental in the study of conic sections, which have wide-ranging applications in mathematics, physics, engineering, and computer graphics. Among the various features of a parabola, the latus rectum holds particular significance because it provides insights into the parabola's width and shape relative to its focus. In this article, we delve into the detailed analysis of a specific parabola characterized by a latus rectum of length 10 and an orientation that opens to the left, exploring its geometric properties, equations, and applications.

Introduction to Parabolas and Their Geometric Properties

A parabola is a symmetrical, U-shaped curve that is defined as the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix. The parabola's key features include:

- Vertex: The point where the parabola changes direction.
- Focus: The fixed point inside the parabola from which distances are measured.
- Directrix: A fixed line outside the parabola, serving as a reference.
- Axis of symmetry: The line passing through the focus and vertex, dividing the parabola into mirror images.
- Latus rectum: A line segment perpendicular to the axis of symmetry that passes through the focus and is contained entirely within the parabola.

The parabola's shape and size are influenced heavily by the position of these features and the lengths associated with them, especially the latus rectum.

Understanding the Latus Rectum

Definition and Significance

The latus rectum of a parabola is a line segment perpendicular to the axis of

symmetry, passing through the focus, and whose endpoints lie on the parabola. Its length provides a measure of the parabola's "width" at the focus level.

- Mathematically: If the focus is at point (F) and the parabola opens in a specific direction, the latus rectum is the segment through (F) , perpendicular to the axis, with its endpoints on the parabola.

- Significance: The length of the latus rectum is directly related to the parabola's eccentricity and other geometric properties. It helps in deriving the standard equation of the parabola and understanding its size and shape.

Relation to the Parabola's Equation

In standard form, the parabola's equation is often expressed based on the orientation and the length of the latus rectum:

- For a parabola opening upwards or downwards:

$$(x - h)^2 = 4p(y - k)$$

- For a parabola opening to the left or right:

$$(y - k)^2 = 4p(x - h)$$

Where:

- (h, k) is the vertex.
- (p) is the distance from the vertex to the focus, and also from the vertex to the directrix.

The length of the latus rectum (L) is related to (p) by:

$$L = 4p$$

So, knowing the length of the latus rectum allows us to determine (p) :

$$p = \frac{L}{4}$$

Given Data: Latus Rectum Length = 10 and Parabola Opens to the Left

In this scenario, the length of the latus rectum is given as 10 units, and the parabola opens towards the left. This information helps us construct the standard equation and analyze its properties.

Determining the Parameters

Since:

$$L = 4p \rightarrow p = \frac{L}{4} = \frac{10}{4} = 2.5$$

The parabola opens to the left, which indicates that the focus is located to the left of the vertex along the x-axis.

Assuming the vertex at the origin $((0, 0))$ for simplicity, the focus will be at:

$$F = (-p, 0) = (-2.5, 0)$$

And the directrix will be a vertical line located to the right of the vertex:

$$x = p = 2.5$$

Standard Equation of the Parabola

Given the focus at $((-2.5, 0))$ and the parabola opening to the left, the standard form of the parabola's equation is:

$$(y - k)^2 = 4p(x - h)$$

Since the vertex is at $((h, k) = (0, 0))$, and the parabola opens leftward:

$$(y)^2 = -4p x$$

Substituting the value of $(p = 2.5)$:

$$y^2 = -4 \times 2.5 \times x$$
$$y^2 = -10x$$

This is the standard form of the parabola with the given properties.

Graphical Representation and Key Features

Plotting the Parabola

To visualize the parabola:

- Vertex: $((0, 0))$
- Focus: $((-2.5, 0))$
- Directrix: $(x = 2.5)$
- Latus rectum endpoints: For the focus at $((-2.5, 0))$, the endpoints are located vertically at a distance of $(\frac{L}{2} = 5)$ units above and below the focus:

```
\[
\text{Endpoints:} \quad (-2.5, 5) \quad \text{and} \quad (-2.5, -5)
\]
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These points lie on the parabola, confirming the length of the latus rectum.

Key Characteristics

- Symmetry: The parabola is symmetric about the x-axis.
- Opening direction: To the left, as indicated by the negative coefficient in the standard equation.
- Width at focus: The latus rectum length of 10 indicates the width of the parabola at the focus level.

Applications of Parabolas with a Latus Rectum of 10

Understanding the properties of such a parabola has several practical applications across various fields:

1. Satellite Dish Design

Satellite dishes often have a parabolic shape to focus signals on the receiver. Knowing the precise dimensions, such as the length of the latus rectum, helps in designing dishes with optimal focus and signal strength.

2. Reflector and Antenna Engineering

Parabolic reflectors are used in antennas and lighting fixtures. The width at the focus (related to the latus rectum) influences the beam width and intensity.

3. Physics and Projectile Motion

Certain types of projectile trajectories approximate parabolas. Understanding their geometric features helps in modeling and predicting projectile paths.

4. Optical Systems and Lenses

Parabolic mirrors are used in telescopes and headlights to direct light efficiently. Precise knowledge of the parabola's dimensions enhances the design of these optical devices.

Additional Calculations and Geometric Insights

Focus-Vertex Distance

As established, $(p = 2.5)$. The focus is located 2.5 units to the left of the vertex.

Directrix Equation

The directrix is the vertical line:

$$\begin{aligned} & \{ \\ & x = 2.5 \\ & \} \end{aligned}$$

Finding Points on the Parabola

To find points on the parabola, select (y) values and solve for (x) :

$$\{$$

$$y^2 = -10x \rightarrow x = -\frac{y^2}{10}$$

For example:

- When $(y = 0)$:

$$x = 0$$

The vertex at $((0, 0))$.

- When $(y = 5)$:

$$x = -\frac{25}{10} = -2.5$$

Corresponds to the focus point.

Similarly, at $(y = -5)$:

$$x = -2.5$$

which confirms the endpoints of the latus rectum.

Summary and Conclusion

The exploration of a parabola with a latus rectum of length 10 that opens to the left reveals key geometric and algebraic features:

- The standard equation is $(y^2 = -10x)$.
- The focus is at $((-2.5, 0))$, and the directrix is at $(x = 2.5)$.
- The latus rectum length directly relates to (p) , which is 2.5.
- The parabola's width at the focus level is 10 units, indicating a relatively wide opening.

Understanding these properties is crucial for practical applications in engineering, physics, and design. Precise knowledge of the parabola's dimensions enables optimized construction and utilization in various technological solutions.

Final Remarks

The study of parabolas, especially those with specific features like a

Frequently Asked Questions

What is the length of the latus rectum for a parabola that opens to the left with a given length of 10?

The length of the latus rectum is 10 units, as specified in the problem statement.

How do you find the focus of a parabola that opens to the left with a latus rectum length of 10?

For a parabola opening to the left, the focus is located at a distance of 5 units from the vertex along the axis of symmetry, which can be determined using the latus rectum length and the parabola's equation.

What is the general equation of a parabola opening to the left with a latus rectum of length 10?

The standard form is $(y - k)^2 = -4a(x - h)$, where $4a$ equals the length of the latus rectum, so here $4a = 10$, thus $a = 2.5$. The specific equation depends on the vertex coordinates (h, k) .

How does the length of the latus rectum relate to the parameter 'a' in the parabola's equation?

The length of the latus rectum is equal to $4a$; therefore, if the latus rectum is 10, then $a = 2.5$.

Where is the vertex located for a parabola opening to the left with a latus rectum of 10?

The vertex is at the point (h, k) , which can be chosen based on the specific parabola. In the standard form, it is at (h, k) , and the focus is located at $(h - a, k)$.

What is the significance of the latus rectum in

understanding the shape of the parabola?

The latus rectum indicates the width of the parabola at the focus, helping to determine how 'wide' or 'narrow' the parabola is at that point.

Can the latus rectum length be used to find the focus and directrix of the parabola?

Yes, knowing the latus rectum length allows you to find the focus (located at a distance of 'a' from the vertex) and the directrix (a line perpendicular to the axis of symmetry at a distance 'a' on the opposite side).

How does the direction of opening (to the left) affect the standard form of the parabola's equation?

For a parabola opening to the left, the standard form is $(y - k)^2 = -4a(x - h)$, with the negative sign indicating the leftward opening.

If the parabola opens to the left with a latus rectum of 10, what are the coordinates of the focus assuming the vertex is at (h, k)?

The focus would be at $(h - a, k)$, where $a = 2.5$. So, the focus coordinates are $(h - 2.5, k)$.

Additional Resources

The Length Of The Latus Rectum Is 10 And The Parabola Opens To The Left

Introduction

In the fascinating world of conic sections, parabolas stand out not only for their elegant curves but also for their wide-ranging applications—from satellite dishes and car headlights to the principles of projectile motion. Today, we delve into a specific parabola characterized by two defining features: it has a latus rectum (the focal width) measuring exactly 10 units, and it opens to the left. These conditions, while seemingly straightforward, reveal rich geometric properties and mathematical insights that are essential for students, engineers, and mathematicians alike. Understanding how the length of the latus rectum influences the parabola's shape and orientation provides a deeper appreciation of the underlying principles governing conic sections.

Understanding the Parabola: Basic Concepts

To appreciate the significance of the given conditions, it's crucial to revisit some fundamental concepts about parabolas.

What Is a Parabola?

A parabola is a symmetric, U-shaped curve that can be defined as the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix. Its standard equation depends on its orientation and position in the coordinate plane.

Key Elements of a Parabola

- Focus (F): A fixed point that helps define the parabola.
- Directrix: A fixed line, equidistant from the parabola's points relative to the focus.
- Vertex (V): The point where the parabola changes direction; closest point to the focus along the axis of symmetry.
- Axis of Symmetry: The line passing through the focus and the vertex, around which the parabola is symmetric.
- Latus Rectum: A line segment perpendicular to the axis of symmetry, passing through the focus, with endpoints on the parabola. Its length is a measure of the parabola's "width" at the focus.

The Significance of the Latus Rectum

The latus rectum is a crucial parameter because it directly relates to the parabola's shape and size.

Definition and Properties

- The latus rectum is a line segment passing through the focus, perpendicular to the axis of symmetry.
- Its endpoints lie on the parabola.
- The length of the latus rectum (denoted as "l") is a fixed value that influences the parabola's curvature.

The Role of the Latus Rectum in the Equation

For a parabola opening to the right or left with vertex at the origin, the standard form is:

$$[y^2 = 4ax]$$

- When $(a > 0)$, the parabola opens to the right.
- When $(a < 0)$, it opens to the left.

The length of the latus rectum in this form is:

$$[l = 4|a|]$$

Thus, knowing the latus rectum length allows us to determine the value of (a) , which in turn defines the parabola's shape and orientation.

Determining the Equation of the Parabola

Given the problem statement: the length of the latus rectum is 10, and the parabola opens to the left. Let's explore how to formulate its equation step by step.

Step 1: Establish the Focus and Vertex

Since the parabola opens to the left, the axis of symmetry is horizontal, pointing leftward. Its vertex can be assumed at the origin or another point, depending on the specific context.

Step 2: Use the Latus Rectum to Find (a)

Given:

$$[l = 10]$$

And for a parabola $(y^2 = 4ax)$, the latus rectum length:

$$[l = 4|a|]$$

So,

$$[4|a| = 10 \Rightarrow |a| = \frac{10}{4} = 2.5]$$

Because the parabola opens to the left, (a) is negative:

$$[a = -2.5]$$

Step 3: Write the Standard Equation

Thus, the parabola's equation in its vertex form:

$$[y^2 = 4a x \Rightarrow y^2 = -10 x]$$

This equation describes a parabola with vertex at the origin, opening to the left, and having a latus rectum of length 10.

Step 4: Locate the Focus and Directrix

- Focus: located at $(a, 0)$:

$$(-2.5, 0)$$

- Directrix: the line perpendicular to the axis, at a distance $|a|$ from the vertex on the opposite side:

$$x = 2.5$$

Geometric Features and Implications

Understanding the precise placement of the focus and directrix helps in visualizing the parabola and applying it in real-world scenarios.

Focus-Directrix Definition

- The focus lies at $(-2.5, 0)$.
- The directrix is the vertical line $x = 2.5$.
- Every point (x, y) on the parabola satisfies:

$$\text{Distance to focus} = \text{Distance to directrix}$$

which is consistent with the standard parabola definition.

Shape and Orientation

- The parabola opens to the left, with the vertex at the origin.
- Its width at the focus (latus rectum) measures 10 units, giving a sense of its steepness and curvature.

Applications and Practical Significance

The mathematical properties of this specific parabola are not just theoretical but have practical implications across various fields.

Engineering and Design

- Satellite Dish Design: Parabolas with precise latus rectum lengths ensure optimal reflection and focus of signals.

- Headlights and Reflectors: The shape directs light efficiently, with the parabola's opening and width tailored to the desired beam pattern.

Physics and Trajectory Analysis

- Projectile Motion: Parabolas represent the paths of projectiles. Knowing the latus rectum helps in calculating the maximum height and range in specific scenarios.

Architecture and Art

- Parabolic arches and structures leverage these geometric principles for strength and aesthetic appeal.

Calculations and Further Exploration

Beyond the equation, various calculations can be performed to analyze the parabola's properties.

Length of the Latus Rectum and Focal Width

- For the parabola $(y^2 = -10x)$, the focus is at $(-2.5, 0)$.

- The endpoints of the latus rectum lie at:

$$\left[\left(-2.5, \pm \frac{l}{2} \right) = \left(-2.5, \pm 5 \right) \right]$$

- These points confirm the width of 10 units at the focus.

Computing Specific Points on the Parabola

- For $(y = 0)$:

$$\left[0^2 = -10x \Rightarrow x=0 \right]$$

- For $(y= 5)$:

$$\left[25 = -10x \Rightarrow x= -2.5 \right]$$

which aligns with the focus location.

Visualizing the Parabola

Plotting the parabola with these points helps in understanding the shape and verifying the theoretical calculations.

Conclusion

The statement, "The length of the latus rectum is 10 and the parabola opens to the left," encapsulates a precise geometric and algebraic configuration that exemplifies the elegance of conic sections. By establishing the relationship between the latus rectum length and the parabola's equation, we gain insights into its shape, focus, and directrix. Such understanding not only enriches mathematical knowledge but also underpins numerous technological and scientific applications. Whether in designing satellite dishes, analyzing projectile trajectories, or constructing architectural marvels, the properties of parabolas continue to demonstrate their relevance and utility across disciplines.

Mastering these concepts enables engineers, architects, and scientists to harness the power of this simple yet profound curve, turning abstract mathematics into tangible innovations that shape our world.

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