Third-degree, With Zeros Of 3, 1, And 2, And A Y-intercept Of 7.p(x)=?

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Understanding how to determine the polynomial function given specific zeros and a y-intercept is a fundamental skill in algebra. In this article, we will explore how to find a cubic polynomial $p(x)$ with zeros at 3, 1, and 2, and a y-intercept of 7. We will break down each step involved, discuss the general form of such polynomials, and provide examples to solidify your understanding.

What is a Third-Degree Polynomial?

A third-degree polynomial, also known as a cubic polynomial, is a polynomial function where the highest exponent of the variable x is 3. The general form of a cubic polynomial is:

$$
p(x) = ax^3 + bx^2 + cx + d
$$

where $a \neq 0$, and b, c, d are real coefficients.

Cubic polynomials are widely used in various fields such as physics, engineering, and economics because of their versatility in modeling complex behaviors like inflection points and turning points.

Zeros of a Polynomial and Their Significance

Zeros (or roots) of a polynomial are the values of x where $p(x) = 0$. They are critical in understanding the shape and intercepts of the graph of the polynomial.

For a cubic polynomial, the Fundamental Theorem of Algebra states that it will have exactly three roots (considering complex roots), which could be real or complex.

In our case, the zeros are given as 3, 1, and 2, which are real roots.

Constructing the Polynomial from Zeros

Given the zeros, the polynomial can be expressed as a product of factors corresponding to each zero:

$$
p(x) = a(x - r_1)(x - r_2)(x - r_3)
$$

where r_1, r_2, r_3 are the zeros.

For zeros at 3, 1, and 2, the factors are:

$$
p(x) = a(x - 3)(x - 1)(x - 2)
$$

The constant a is a scaling factor that affects the overall shape and size of the graph.

Determining the Leading Coefficient a

To fully specify $p(x)$, we need to find the value of a. This is where the y-intercept comes into play.

The y-intercept occurs when $x = 0$. The value of the polynomial at $x = 0$, denoted as $p(0)$, is given as 7.

So,

$$
p(0) = a(0 - 3)(0 - 1)(0 - 2) = 7
$$

Calculating the product inside:

$$
(0 - 3) = -3 \\
(0 - 1) = -1 \\
(0 - 2) = -2
$$

Multiplying these:

$$
(-3) \times (-1) \times (-2) = (-3) \times 2 = -6
$$

Therefore:

$$
p(0) = a \times (-6) = 7
$$

Solving for a:

$$
a = \frac{7}{-6} = -\frac{7}{6}
$$

Thus, the polynomial function is:

$$
p(x) = -\frac{7}{6}(x - 3)(x - 1)(x - 2)
$$

Expanded Form of the Polynomial

While the factored form is useful, expanding the polynomial provides a complete algebraic expression.

Step 1: Expand two factors:

$$
(x - 3)(x - 1) = x^2 - x - 3x + 3 = x^2 - 4x + 3
$$

Step 2: Multiply the result by $(x - 2)$:

$$
(x^2 - 4x + 3)(x - 2)
$$

Using distribution:

$$
x^2 \times x = x^3 \\
x^2 \times (-2) = -2x^2 \\
-4x \times x = -4x^2 \\
-4x \times (-2) = 8x \\
3 \times x = 3x \\
3 \times (-2) = -6
$$

Now, sum all terms:

\[

x^3 + (-2x^2) + (-4x^2) + 8x + 3x - 6
\]

Combine like terms:

\[
x^3 + (-2x^2 - 4x^2) + (8x + 3x) - 6 = x^3 - 6x^2 + 11x - 6
\]

Step 3: Incorporate the coefficient \(a = -\frac{7}{6} \):

\[
p(x) = -\frac{7}{6}(x^3 - 6x^2 + 11x - 6)
\]

Distribute:

\[
p(x) = -\frac{7}{6}x^3 + \frac{7}{6} \times 6 x^2 - \frac{7}{6} \times 11 x + \frac{7}{6} \times 6
\]

Simplify each term:

\[
p(x) = -\frac{7}{6}x^3 + 7x^2 - \frac{77}{6}x + 7
\]

This is the expanded form of the polynomial:

\[
\boxed{
p(x) = -\frac{7}{6}x^3 + 7x^2 - \frac{77}{6}x + 7
}
\]

Graphing the Polynomial and Its Features

Understanding the features of \(p(x) \) helps in visualizing its graph.

Zeros (Roots):
The polynomial crosses the x-axis at \(x = 1, 2, 3 \).

Y-intercept:
At \(x=0 \), \(p(0) = 7 \).

End Behavior:
Since the leading coefficient is negative (\(-\frac{7}{6} \)), as \(x \to \infty \), \(p(x) \to -\infty \), and as \(x \to -\infty \), \(p(x) \to

∞.

Turning Points:
A cubic polynomial can have up to two turning points (local maxima and minima). The exact locations can be found by taking the derivative and analyzing critical points.

Applications and Importance of Understanding Such Polynomials

Knowing how to construct a polynomial from zeros and a y-intercept is essential in various mathematical and real-world contexts:

- Designing Curves: Engineers and designers use polynomial functions to create smooth curves with specific intersection points and behaviors.

- Data Modeling: Polynomial regression relies on these principles to fit data points with a curve that best represents the data.

- Physics and Economics: Modeling phenomena like projectile motion or supply-demand curves often involves cubic functions with known roots and intercepts.

- Mathematical Education: Learning to construct polynomials from roots and intercepts reinforces understanding of polynomial properties and the Fundamental Theorem of Algebra.

Summary and Key Takeaways

- A third-degree polynomial with zeros at 3, 1, and 2 can be expressed as $p(x) = a(x - 3)(x - 1)(x - 2)$.

- To find the coefficient a, use the given y-intercept $p(0) = 7$.

- Substituting $x = 0$ into the factored form allows solving for a.

- The resulting polynomial in expanded form is:

$$
p(x) = -\frac{7}{6}x^3 + 7x^2 - \frac{77}{6}x + 7
$$

- Understanding how zeros, intercepts, and coefficients relate helps in graphing and analyzing polynomial functions.

By mastering these concepts, students and professionals can confidently construct and analyze cubic functions tailored to specific criteria, enhancing their problem-solving toolkit in algebra and calculus.

Keywords: cubic polynomial, third-degree polynomial, polynomial zeros, y-intercept, algebra, polynomial expansion, graphing polynomial functions, algebraic modeling

Frequently Asked Questions

What is the general form of a third-degree polynomial with zeros at 3, 1, and 2?

The general form is p(x) = a(x - 3)(x - 1)(x - 2), where a is a non-zero constant.

How do you determine the value of 'a' in the polynomial p(x) if the y-intercept is 7?

Substitute x = 0 into the polynomial and set p(0) = 7, then solve for 'a'.

What is the polynomial p(x) with zeros at 3, 1, and 2 and a y-intercept of 7?

First write p(x) = a(x - 3)(x - 1)(x - 2). Then find 'a' by using p(0) = 7, resulting in p(x) = 7(x - 3)(x - 1)(x - 2).

How do you expand the polynomial p(x) = 7(x - 3)(x - 1)(x - 2)?

First expand (x - 3)(x - 1)(x - 2) step by step, then multiply the resulting polynomial by 7.

What is the expanded form of p(x) for the given zeros and y-intercept?

Expanding gives p(x) = 7x^3 - 49x^2 + 105x - 42.

Why is the coefficient 'a' equal to 7 in this polynomial?

Because the y-intercept occurs when x=0, so p(0) = a(-3)(-1)(-2) = 7; solving for 'a' gives a = 7.

Can the polynomial p(x) have different leading coefficients while maintaining the zeros and y-intercept?

Yes, but the specific polynomial with y-intercept 7 has a leading coefficient of 7; different 'a' values would change the y-intercept.

What is the significance of the zeros 3, 1, and 2 in the polynomial?

They are the roots of the polynomial, where p(x) equals zero.

How does knowing the zeros and y-intercept help in constructing the polynomial?

Zeros determine the factors, and the y-intercept allows solving for the leading coefficient to fully specify the polynomial.

What is the final form of the polynomial p(x) with zeros at 3, 1, 2, and y-intercept of 7?

The polynomial is p(x) = 7x^3 - 49x^2 + 105x - 42.

Additional Resources

Third-degree polynomials occupy a fundamental role in algebra and calculus, often serving as the foundation for understanding more complex functions. When analyzing such polynomials, key features such as roots (zeros), intercepts, and coefficients provide valuable insights into their behavior, shape, and intersections with axes. This article delves into a specific scenario: determining the explicit form of a third-degree polynomial \(p(x) \) that has zeros at 3, 1, and 2, combined with a y-intercept of 7. Through comprehensive exploration, we will uncover the precise structure of this polynomial, interpret its characteristics, and understand the implications of its features.

Understanding the Fundamentals of Cubic Polynomials

What Is a Third-Degree Polynomial?

A third-degree polynomial, also known as a cubic polynomial, is a polynomial equation of degree three. Its general form can be expressed as:

$$
p(x) = ax^3 + bx^2 + cx + d
$$

where $a \neq 0$, and a, b, c, d are real coefficients. The degree indicates the highest power of x present, which directly influences the polynomial's end behavior and the number of potential real roots.

Key characteristics of cubic polynomials:

- They can have up to three real roots.
- Their graphs tend to infinity in opposite directions (one end up, the other down), or vice versa.
- They may possess one real root and two complex conjugates, depending on the discriminant.

Roots and Zeros of the Polynomial

Defining Zeros and Their Significance

Zeros of a polynomial are the values of x where $p(x) = 0$. These points correspond to the x-intercepts of the graph. For a cubic polynomial, the zeros can be real or complex, but in our case, they are explicitly real and given as 3, 1, and 2.

Implications of zeros:

- The zeros dictate the polynomial's factorization.
- The multiplicity of zeros influences the graph's behavior at those points:
- Zero with odd multiplicity (e.g., 1) crosses the x-axis.
- Zero with even multiplicity (e.g., 2) touches the x-axis and turns around.

In our problem, the zeros are simple (multiplicity 1), simplifying the analysis.

Constructing the Polynomial from Zeros

Given the zeros at 3, 1, and 2, the polynomial can be expressed as:

\[
p(x) = a(x - 3)(x - 1)(x - 2)
\]

where \(a \) is a non-zero constant that scales the polynomial.

The process involves:

- Recognizing that each zero corresponds to a factor \((x - \text{zero}) \).
- Multiplying these factors to obtain the cubic expression.
- Adjusting the scale with \(a \) to satisfy additional conditions (such as the y-intercept).

Determining the Leading Coefficient and the Polynomial Equation

Using the Y-Intercept to Find \(a \)

The y-intercept of a polynomial occurs where \(x = 0 \), and \(p(0) \) gives the value of the polynomial at that point. The problem states that the y-intercept is 7, which means:

\[
p(0) = 7
\]

Substituting \(x = 0 \) into the general form:

\[
p(0) = a(0 - 3)(0 - 1)(0 - 2) = a(-3)(-1)(-2)
\]

Calculating the product:

\[
(-3)(-1) = 3 \\
3 \times (-2) = -6
\]

Thus:

\[

$$
p(0) = a \times (-6) = 7
$$

Solving for a:

$$
a = \frac{7}{-6} = -\frac{7}{6}
$$

Result:

$$
a = -\frac{7}{6}
$$

Explicit Form of the Polynomial

Substituting a back into the factorized form:

$$
p(x) = -\frac{7}{6} (x - 3)(x - 1)(x - 2)
$$

This expression fully characterizes the polynomial. To gain deeper insights into its behavior, we can expand it into standard polynomial form.

Expanding the Polynomial for Analysis

Step-by-Step Expansion

1. Multiply two factors first:

$$
(x - 3)(x - 1) = x^2 - x - 3x + 3 = x^2 - 4x + 3
$$

2. Multiply the result by the third factor:

$$
(x^2 - 4x + 3)(x - 2)
$$

Expanding:

$$
x^2 \times x = x^3 \\
x^2 \times (-2) = -2x^2 \\
-4x \times x = -4x^2 \\
-4x \times (-2) = 8x \\
3 \times x = 3x \\
3 \times (-2) = -6
$$

Combining like terms:

$$
x^3 + (-2x^2 - 4x^2) + (8x + 3x) - 6 = x^3 - 6x^2 + 11x - 6
$$

3. Apply the leading coefficient:

$$
p(x) = -\frac{7}{6} (x^3 - 6x^2 + 11x - 6)
$$

4. Distribute $-\frac{7}{6}$:

$$
p(x) = -\frac{7}{6} x^3 + 7 x^2 - \frac{77}{6} x + 7
$$

Final expanded form:

$$
p(x) = -\frac{7}{6} x^3 + 7 x^2 - \frac{77}{6} x + 7
$$

Analyzing the Polynomial's Behavior

End Behavior and Graphical Insight

Given the leading term:

$$
-\frac{7}{6} x^3
$$

Since the coefficient is negative, the polynomial descends toward $-\infty$ as $x \to +\infty$ and ascends toward $+\infty$ as $x \to -\infty$. This "twisted" end behavior is characteristic of cubic functions with a negative leading coefficient.

Implication:

- The graph will cross the x-axis at the zeros 1, 2, and 3.
- The polynomial touches or crosses the axes at these points, with smooth curves connecting these intercepts.

Critical Points and Inflection

- The polynomial, being cubic, can have up to two turning points (local maximum and minimum).
- The inflection point occurs where the second derivative is zero.
- These features influence the shape and the steepness of the graph near the zeros.

Calculating derivatives can provide numerical insights into the slope and concavity, but for the scope of this article, the primary focus remains on the explicit form and intercepts.

Summary and Significance

This analysis showcases how algebraic principles enable us to determine the explicit form of a polynomial based on key features. Starting from the given zeros at 3, 1, and 2, and a y-intercept of 7, we derived:

$$
p(x) = -\frac{7}{6} (x - 3)(x - 1)(x - 2)
$$

which, upon expansion, becomes:

$$
p(x) = -\frac{7}{6} x^3 + 7 x^2 - \frac{77}{6} x + 7
$$

This polynomial exemplifies the elegance of factorization and the power of algebraic manipulation in understanding polynomial behavior. The negative leading coefficient indicates the graph's end behavior, while the roots and intercepts reveal key points of intersection with axes.

Applications of such an analysis include:

- Graphing cubic functions accurately.
- Understanding the relationship between zeros and polynomial coefficients.
- Extending the approach to more complex polynomials with multiple roots or multiplicities.
- Using derivatives to analyze the polynomial's critical points, concavity, and inflection points.

Concluding Remarks

The problem of finding a third-degree polynomial with specified roots and intercepts is a classic exercise in algebra that underpins much of higher mathematics. It illustrates the interconnectedness of roots, factors, coefficients, and graph behavior. Mastery of these concepts enhances one's capacity to analyze and interpret polynomial functions, both in theoretical contexts and practical applications such as physics, engineering, and data modeling. Understanding how to reconstruct a polynomial from its roots and intercepts not only deepens mathematical insight but also equips practitioners with tools to manipulate and interpret complex functions across disciplines.

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