ThWrite Anexplicit Formula Foransthenterm Of The Sequence 40,50,60, ...

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Understanding sequences and their explicit formulas is fundamental in mathematics, especially in algebra and discrete mathematics. The sequence 40, 50, 60, ... is a classic example of an arithmetic progression, where each term increases by a constant difference. In this article, we will explore how to derive an explicit formula for the nth term of this sequence. We will break down the process step by step, providing clear explanations and practical examples to ensure a comprehensive understanding.

Understanding the Sequence: 40, 50, 60, ...

Before deriving the explicit formula, it’s essential to analyze the sequence's properties:

1. Recognizing the Pattern

- The sequence starts at 40.
- Each subsequent term increases by 10.
- The sequence progresses as: 40, 50, 60, 70, 80, ...

2. Identifying the Type of Sequence

- This sequence is an arithmetic sequence because the difference between consecutive terms is constant.
- The common difference (d) here is 10.

3. Significance of an Explicit Formula

- An explicit formula allows you to find any term directly without calculating all previous terms.
- It is especially useful for large values of n or when analyzing the sequence's behavior.

Deriving the Explicit Formula for the Sequence

The general form of an arithmetic sequence's explicit formula is:

\[a_n = a_1 + (n - 1)d \]

Where:
- \(a_n \) is the nth term.

- a_1 is the first term.
- d is the common difference.
- n is the position of the term in the sequence.

Applying the Formula to Our Sequence

Given:
- $a_1 = 40$
- $d = 10$

Substitute these into the general form:

$$a_n = 40 + (n - 1) \times 10$$

Simplify:

$$a_n = 40 + 10n - 10$$

$$a_n = (10n) + (40 - 10)$$

$$a_n = 10n + 30$$

Thus, the explicit formula for the sequence is:

$$\boxed{a_n = 10n + 30}$$

Verifying the Explicit Formula

To ensure the correctness of the derived formula, let's verify with some initial terms:

- For $n=1$:

$$a_1 = 10(1) + 30 = 10 + 30 = 40$$

- For $n=2$:

$$a_2 = 10(2) + 30 = 20 + 30 = 50$$

- For $n=3$:

$$a_3 = 10(3) + 30 = 30 + 30 = 60$$

All of these match the initial sequence, confirming that the formula is correct.

Applications of the Explicit Formula in Real-Life Contexts

Understanding and using explicit formulas like $a_n = 10n + 30$ have numerous practical applications:

1. Financial Planning and Budgeting

- Calculating cumulative savings or investments where contributions increase by a fixed amount each period.

2. Engineering and Physics

- Modeling phenomena where quantities increase linearly over time, such as distance traveled at constant speed.

3. Computer Science

- Determining the value of sequences in algorithms, especially in iterative processes or data structures.

Key Points to Remember

- The sequence is arithmetic with a first term $a_1 = 40$.
- The common difference $d = 10$.
- The explicit formula derived is $a_n = 10n + 30$.
- This formula allows quick computation of any term in the sequence.

Additional Tips for Working with Arithmetic Sequences

- Always identify the first term and common difference before deriving the explicit formula.
- Use the explicit formula for large term calculations to save time.
- Verify your formula with initial sequence terms to ensure accuracy.
- Be aware of the difference between explicit (direct) and recursive formulas; the explicit formula provides a direct way to find terms.

Conclusion

Deriving the explicit formula for the sequence 40, 50, 60, ... is straightforward once you recognize it as an arithmetic sequence. By identifying the first term and common difference, applying the general formula, and simplifying, you obtain:

$$a_n = 10n + 30$$

This formula empowers you to calculate any term directly, facilitating problem-solving across various mathematical and real-world applications. Mastering the process of deriving explicit formulas enhances your ability to analyze and work with sequences efficiently. Whether you're studying algebra, tackling engineering problems, or managing financial plans, understanding how to find explicit formulas is an essential skill in mathematics.

Keywords for SEO Optimization:
- explicit formula for arithmetic sequence
- sequence 40,50,60, ...
- nth term of sequence
- arithmetic progression formula
- how to find explicit formula
- sequence formula derivation
- mathematical sequences
- algebra sequences
- sequence analysis
- sequence applications

Frequently Asked Questions

What is the explicit formula for the nth term of the sequence 40, 50, 60, ...?

The explicit formula is an = 40 + (n - 1) × 10.

How do I derive the explicit formula for this arithmetic sequence?

Since the sequence increases by 10 each time, the formula is an = a_1 + (n - 1) × d, where a_1 = 40 and d = 10, resulting in an = 40 + (n - 1) × 10.

What is the common difference in the sequence 40, 50, 60, ...?

The common difference is 10.

Can you explain how to find the nth term of an arithmetic sequence?

Yes, the nth term of an arithmetic sequence is given by an = a_1 + (n - 1) × d, where a_1 is the first term and d is the common difference.

What is the value of the 10th term in the sequence 40, 50, 60, ...?

Using the formula an = 40 + (n - 1) × 10, for n=10: an = 40 + (10 - 1) × 10 = 40 + 90 = 130.

Is the sequence 40, 50, 60, ... an arithmetic sequence?

Yes, because each term increases by a constant difference of 10.

How can I verify if the explicit formula matches the sequence?

You can substitute values of n into the formula and check if the results match the sequence terms. For example, for n=1, an=40+0=40; for n=2, an=50; for n=3, an=60, etc.

What is the general form of an explicit formula for an arithmetic sequence?

The general form is an = a_1 + (n - 1) × d, where a_1 is the first term and d is the common difference.

Additional Resources

Explicit Formula for the nth Term of the Sequence 40, 50, 60, ...

Introduction

Sequences are fundamental objects in mathematics, serving as the backbone for understanding patterns, functions, and various analytical constructs. Among the many sequences studied, arithmetic sequences hold a special place due to their simplicity and broad applicability. This article aims to provide a comprehensive exploration into deriving an explicit formula for the nth term of the sequence 40, 50, 60, ..., a classic example of an arithmetic progression.

Understanding how to formulate the nth term explicitly is crucial for numerous mathematical and real-world applications, from computational algorithms to modeling natural phenomena. This review will delve into the derivation process, theoretical underpinnings, and potential extensions, providing a detailed and rigorous analysis suitable for academic review or scholarly publication.

Background on Arithmetic Sequences

Definition and Properties

An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is constant. This constant difference is known as the common difference (denoted by \(d\)).

Mathematically, an arithmetic sequence $\{a_n\}$ can be expressed as:

$$
a_n = a_1 + (n - 1)d
$$

where:
- a_1 is the first term,
- d is the common difference,
- n is the position of the term in the sequence.

This formula provides an explicit or closed-form expression for the nth term, enabling direct computation without recursively calculating preceding terms.

Deriving the Explicit Formula for the Sequence 40, 50, 60, ...

Step 1: Identify the First Term and Common Difference

Given the sequence:

$$
40, 50, 60, \ldots
$$

- The first term (a_1) is 40.
- The subsequent terms increase by 10 each time, so the common difference (d) is 10.

Step 2: Apply the General Arithmetic Sequence Formula

Using the standard formula:

$$
a_n = a_1 + (n - 1)d
$$

we substitute the known values:

$$
a_n = 40 + (n - 1) \times 10
$$

Simplify:

$$
a_n = 40 + 10n - 10
$$
$$
a_n = 10n + 30
$$

Thus, the explicit formula for the sequence is:

$$
\boxed{
a_n = 10n + 30
}
$$

This formula allows for immediate calculation of any term in the sequence given its position n.

In-Depth Analysis of the Derivation

Verification of the Formula

To ensure correctness, verify the formula with known terms:

- For $n=1$:

$$
a_1 = 10(1) + 30 = 40 \quad \checkmark
$$

- For $n=2$:

$$
a_2 = 10(2) + 30 = 50 \quad \checkmark
$$

- For $n=3$:

$$
a_3 = 10(3) + 30 = 60 \quad \checkmark
$$

The formula accurately reproduces the initial sequence terms.

Theoretical Justification

The derivation hinges on the fundamental properties of arithmetic progressions. Since the sequence progresses linearly with a common difference of 10, the explicit formula must be linear in n, with a slope corresponding to the difference and an intercept determined by the initial term.

Generalization and Variations

Extending to Other Arithmetic Sequences

The methodology used here is universally applicable:

$$
a_n = a_1 + (n - 1)d
$$

where a_1 and d are specific to each sequence.

Handling Sequences with Different Starting Points or Differences

Suppose the sequence is $a, a + d, a + 2d, \ldots$, then the explicit formula becomes:

$$
a_n = a + (n - 1)d
$$

This universality underscores the importance of understanding the parameters a_1 and d.

Applications and Implications

Computational Efficiency

Having an explicit formula like $a_n = 10n + 30$ facilitates rapid computation of large terms without recursive iteration, which is particularly useful in programming and algorithmic contexts.

Modeling Real-World Phenomena

Arithmetic sequences model scenarios with constant rate changes, such as:

- Uniform salary increases
- Constant rate of depreciation
- Equidistant spacing in geometric arrangements

Identifying the explicit formula enables precise predictions and analysis.

Potential Extensions and Related Topics

Arithmetic Sequences with Variable Differences

While this discussion focused on constant differences, real-world data may exhibit non-uniform gaps, leading to more complex models like quadratic or exponential sequences.

Generating Series and Summations

The sum of the first n terms (S_n) of an arithmetic sequence can be derived as:

$$
S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2} [2a_1 + (n - 1)d]
$$

which is useful for aggregate calculations.

Conclusion

The explicit formula for the sequence 40, 50, 60, ... is:

$$
a_n = 10n + 30
$$

This formula provides a direct, efficient way to compute any term in the sequence, illustrating the power of the arithmetic progression framework. Its derivation is rooted in fundamental mathematical principles, emphasizing the importance of understanding sequence parameters and their roles in formulating closed-form expressions. Such explicit formulas are indispensable tools across mathematics, science, and engineering, enabling precise modeling, computation, and analysis.

This investigation underscores the elegance and utility of recognizing patterns and translating them into algebraic formulas, a core skill that underpins much of mathematical reasoning and application.

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