Which Graph Represents The Equation 12 = 3x + 4y? Explain How You Know.

Which Graph Represents The Equation 12 = 3x + 4y? Explain How You Know.

Understanding how to identify the correct graph that corresponds to a given linear equation is a fundamental skill in algebra and coordinate geometry. When presented with the equation 12 = 3x + 4y, students and learners often wonder how to determine which graph visually represents this equation. This article will guide you through the process of analyzing the equation, converting it into a slope-intercept form, and examining the key features that help you recognize the right graph. By the end, you'll understand not only which graph matches the equation but also why it does, based on the equation's properties.

Understanding the Equation 12 = 3x + 4y

Before selecting the correct graph, it is crucial to understand the structure of the equation itself. The equation 12 = 3x + 4y is linear, meaning it graphs as a straight line on the coordinate plane. To analyze it effectively, we should rewrite it in a more familiar form, such as the slope-intercept form y = mx + b.

Converting the Equation to Slope-Intercept Form

The current form of the equation is:

12 = 3x + 4y

To express it as y = mx + b, follow these steps:

1. Isolate the term with y: Subtract 3x from both sides:

12 - 3x = 4y

2. Divide both sides by 4 to solve for y:

y = (12 - 3x) / 4

3. Rewrite the equation to separate the terms:

y = (12 / 4) - (3x / 4)

Simplify:

$y = 3 - (3/4)x$

Now, the equation is in slope-intercept form:

$y = -(3/4)x + 3$

Key features of this form:

- Slope (m): $-3/4$
- Y-intercept (b): 3

How To Recognize the Correct Graph

Having the slope and y-intercept allows us to identify the correct graph visually. Here are the main features to look for:

1. Y-Intercept

- The y-intercept is the point where the line crosses the y-axis.
- For this equation, the y-intercept is at (0, 3).

When examining graphs, find the line that crosses the y-axis at $y = 3$.

2. Slope

- The slope indicates the steepness and direction of the line.
- A slope of $-3/4$ means:
- The line decreases as x increases.
- For every increase of 4 units in x, y decreases by 3 units.
- The negative sign indicates a downward slope from left to right.

On the graph, check for a line that goes downward from left to right with a gentle slope.

3. Additional Points

- To confirm, identify additional points on the line.
- For example, plug in $x = 4$:

$y = -(3/4)(4) + 3 = -3 + 3 = 0$

- So, the point (4, 0) should lie on the line.

- Similarly, for $x = 0$:

$y = 3$ (the y-intercept point).

- For $x = -4$:

$y = -(3/4)(-4) + 3 = 3 + 3 = 6$

- The point (−4, 6) should also be on the line.

Use these points to verify the line's position on the graph.

Steps to Identify the Correct Graph

To determine which graph matches the equation, follow these steps:

Step 1: Find the Y-Intercept

- Locate the point where the line crosses the y-axis.
- Confirm whether it is at (0, 3).

Step 2: Check the Slope

- From the y-intercept, count the rise and run to see if it matches the slope of −3/4.
- For example, move 4 units to the right (positive x direction), and see if the line goes down 3 units.

Step 3: Verify Additional Points

- Calculate points like (4, 0) and (−4, 6).
- Ensure these points lie on the line in the graph.

Step 4: Confirm the Line's Direction and Steepness

- The line should slope downward from left to right.
- Its steepness should correspond to a slope of −3/4.

Common Mistakes and How to Avoid Them

While analyzing graphs, a few common pitfalls can lead to incorrect conclusions:

1. Confusing the Slope Sign

- Make sure to note whether the slope is positive or negative.
- A positive slope slopes upward, negative slopes slope downward.

2. Misreading the Y-Intercept

- Confirm the point where the line crosses the y-axis.
- Sometimes, multiple lines may cross close to y=3, so check carefully.

3. Overlooking Additional Points

- Use multiple points to verify the line's position.
- Relying solely on the y-intercept can be misleading if the graph is misdrawn.

Conclusion: Which Graph Represents 12 = 3x + 4y?

Based on the analysis above, the graph that correctly represents the equation 12 = 3x + 4y is the one that:

- Crosses the y-axis at (0, 3)
- Has a slope of -3/4, descending gently from left to right
- Passes through points like (4, 0) and (-4, 6)

By applying the process of converting the equation into slope-intercept form, identifying key features, and verifying multiple points, you can confidently select the correct graph. Remember, understanding the fundamental properties of linear equations—such as slope and intercepts—is essential for interpreting and visualizing algebraic relationships accurately.

Whether you're studying for exams, solving real-world problems, or just enhancing your math skills, mastering how to connect equations to their graphs is a valuable tool. Keep practicing with different equations, and soon you'll find it easier to recognize their graphical representations with confidence.

Frequently Asked Questions

How can I determine which graph represents the equation 12 = 3x + 4y?

You can rewrite the equation in slope-intercept form (y = mx + b) and compare it to the graphs. For 12 = 3x + 4y, solving for y gives y = (12 − 3x) / 4. Then, check which graph matches this line by verifying points or slope and intercept.

What is the slope and y-intercept of the line 12 = 3x + 4y?

Rearranging the equation to slope-intercept form yields y = − (3/4)x + 3. Therefore, the slope is -3/4 and the y-intercept is at (0, 3). The correct graph should reflect these features.

How can plotting points help identify the correct graph for 12 = 3x + 4y?

By choosing specific x-values, calculating corresponding y-values, and plotting these points, you can see which graph passes through these points. The graph that contains points such as (0, 3), (4, 0), and (-4, 6) matches the equation.

Why is it important to check the intercepts when identifying the graph of 12 = 3x + 4y?

Because the intercepts directly relate to the equation's structure, verifying the x-intercept (when y=0) and y-intercept (when x=0) helps confirm which graph accurately represents the line. For this equation, x-intercept is at (4, 0), and y-intercept at (0, 3).

Can the equation 12 = 3x + 4y be represented by a vertical or horizontal line?

No, because the equation is linear with both x and y variables, resulting in a sloped line. Vertical lines have equations of the form x = constant, and horizontal lines have y = constant; this equation does not fit either form.

Additional Resources

Graphical Representation of Linear Equations: Deciphering the Equation 12 = 3x + 4y

Understanding how equations translate into visual graphs is a foundational skill in algebra and analytical geometry. Among these, linear equations—those of the first degree—are particularly straightforward yet powerful in their applications. The equation 12 = 3x + 4y exemplifies a linear relationship between two variables, x and y. Determining which graph accurately depicts this equation involves a thorough comprehension of its structure, properties, and how they manifest visually.

In this comprehensive analysis, we will explore the process of identifying the correct graph for the equation 12 = 3x + 4y. We will dissect the equation's form, convert it into a more familiar format, examine its slope and intercepts, and finally, analyze how these characteristics translate into graph features. This article aims to serve as an authoritative guide, akin to a product review or expert feature, providing clarity and depth to the process.

Understanding the Equation: The Basics of Linear Algebra

Before delving into graph identification, it's essential to understand what the equation 12 = 3x + 4y represents mathematically.

Standard Form of a Linear Equation

The given equation is in what's called the standard form of a linear equation:

$$ Ax + By = C $$

where A, B, and C are constants, and x and y are variables.

In our case:

- $A = 3$
- $B = 4$
- $C = 12$

This form is particularly useful because it makes the coefficients of the variables explicit, facilitating the calculation of intercepts and slope.

Conversion to Slope-Intercept Form

To better visualize and understand the graph, it’s often easiest to rewrite the equation in slope-intercept form:

$$ y = mx + b $$

where:

- m is the slope of the line,
- b is the y-intercept.

Let's perform the algebraic manipulation:

$$
\begin{aligned}
12 &= 3x + 4y \\
4y &= 12 - 3x \\
y &= \frac{12 - 3x}{4} \\
y &= -\frac{3}{4}x + 3
\end{aligned}
$$

This form reveals two critical features:

- Slope (m): $-\frac{3}{4}$
- Y-intercept (b): 3

Decoding the Graph: Key Features and Characteristics

Having converted the equation, we now turn to interpreting these features to determine the correct graph.

1. Slope: The Inclination of the Line

The slope $-\frac{3}{4}$ indicates:

- The line descends from left to right, as the slope is negative.
- For every 4 units moved horizontally to the right, the line drops by 3 units vertically.
- The steepness of the line is moderate; it's neither too flat nor too steep.

This characteristic helps dismiss graphs with positive slopes or slopes with different magnitudes when verifying options.

2. Y-Intercept: The Point Where the Line Crosses the Y-Axis

The y-intercept at 3 means:

- The line crosses the y-axis at the point $(0, 3)$.
- When $x=0$, $y=3$.

This is a precise point that can be plotted directly, serving as a reference.

3. X-Intercept: The Point Where the Line Crosses the X-Axis

Calculating the x-intercept:

$$
0 = -\frac{3}{4}x + 3
$$

$$
-\frac{3}{4}x = -3
$$

$$
x = \frac{-3 \times 4}{-3} = 4
$$

Thus, the line crosses the x-axis at $(4, 0)$.

This point can also be plotted to verify the accuracy of the graph.

Step-by-Step Approach to Identifying the Correct Graph

To confidently determine which graph corresponds to the equation, follow these steps:

Step 1: Check the Slope

- Verify that the graph has a line with a slope of approximately $-\frac{3}{4}$.
- The line should decline from left to right.
- Confirm the inclination visually or by calculating two points' rise over run.

Step 2: Confirm the Y-Intercept

- Ensure the line crosses the y-axis at about $y=3$.
- On the graph, find the point where the line intersects the y-axis; it should be close to $(0, 3)$.

Step 3: Confirm the X-Intercept

- The line should cross the x-axis at about $x=4$.
- Locate the x-intercept visually or by checking the point where the line crosses the x-axis, expecting $(4, 0)$.

Step 4: Cross-Verification with Additional Points

- Pick another value of x (say, $x=8$) and compute y:

$$
y = -\frac{3}{4} \times 8 + 3 = -6 + 3 = -3
$$

- The point $(8, -3)$ should lie on the graph.

- Check if this point is on the line in the candidate graph.

Step 5: Examine Multiple Graphs

- Discard graphs with:

- Positive slopes
- Incorrect intercepts
- Lines that do not pass through the calculated points

- The correct graph will be the one that accurately reflects all these features simultaneously.

Visual Interpretation and Practical Tips

Understanding the visual features of the graph enhances your confidence in

identification:

- Line Direction: Confirm that the line descends from left to right for a negative slope.
- Intercept Accuracy: Use precise points—intercepts are often marked or can be inferred from scale—to validate the graph.
- Steepness and Angle: The ratio of vertical change to horizontal change (rise over run) should approximate $-\frac{3}{4}$.

Practical Tip: When in doubt, plot the key points:

- Y-intercept: $(0, 3)$
- X-intercept: $(4, 0)$
- Additional point: $(8, -3)$

Plot these points on the candidate graphs. The one that passes through all of them with a straight line is the correct graph.

Conclusion: The Definitive Graph for 12 = 3x + 4y

Based on the algebraic analysis, the graph representing the equation 12 = 3x + 4y must exhibit the following features:

- A straight line
- Crossing the y-axis at $(0, 3)$
- Crossing the x-axis at $(4, 0)$
- Having a slope of $-\frac{3}{4}$

When these features are carefully checked against available options, the correct graph will be the one that aligns perfectly with these characteristics.

This process underscores the importance of translating equations into their graphical counterparts through slope and intercept calculations. Such understanding allows for quick and accurate identification, essential skills in algebraic reasoning, and critical for progressing in mathematics.

In essence, the graph representing 12 = 3x + 4y is the one that passes through the points $(0, 3)$, $(4, 0)$, and $(8, -3)$, with a consistent slope of $-\frac{3}{4}$. Recognizing these features ensures you select the correct graph confidently—an invaluable skill in the realm of algebra and beyond.

Which Graph Represents The Equation 12 3x 4y Explain How You Know

Which Graph Represents The Equation 12 3x 4y Explain How You Know

Related Articles

- I Need To Come Up With 5 Questions About Eukaryotic And Prokaryotic Cells
- Social Psychological Principles Have The Ability To Influence _____.
- Describe The Microscopic Structures You Would Expect To See In The Lungs.

Back to Home