(1 Point) Find The Centroid (y) Of The Region Bounded By: Y=7x^2, Y=0, And X = 4

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Introduction to Centroid Calculation of a Bounded Region

Understanding how to find the centroid of a region is a fundamental concept in calculus and engineering. The centroid, often referred to as the geometric center, represents the average position of all the points in a shape or region. When dealing with a planar region bounded by specific curves and lines, calculating its centroid involves integrating over the region's area. In this article, we will explore how to find the y-coordinate of the centroid for the region bounded by the parabola $y=7x^2$, the x-axis $y=0$, and the vertical line $x=4$. This problem provides a practical illustration of applying integral calculus to determine the centroid's position.

Understanding the Problem and the Region

Given Curves and Boundaries

The problem specifies three boundaries:

- The parabola $y = 7x^2$
- The x-axis $y = 0$
- The vertical line $x = 4$

These boundaries enclose a region in the xy-plane. To visualize:

- The parabola opens upward.
- The region extends from $x=0$ (where the parabola intersects the x-axis) to $x=4$.
- The lower boundary is the x-axis $y=0$.
- The right boundary is the line $x=4$.

Region Description

The region is bounded:

- On the bottom by $y=0$,
- On the top by $y=7x^2$,
- On the right by $x=4$,
- On the left by the y-axis $x=0$ (since the parabola intersects $y=0$ at $x=0$).

This creates a finite, well-defined area suitable for centroid calculation.

Mathematical Foundations for Finding the Centroid

Definition of the Centroid Coordinates

For a planar region R, the centroid coordinates $(\bar{x}, \bar{y})$ are given by:

$$
\bar{x} = \frac{1}{A} \iint_R x \, dA
$$

$$
\bar{y} = \frac{1}{A} \iint_R y \, dA
$$

where A is the area of the region.

Calculating the Area A

The area of the region is:

$$
A = \int_{x=0}^{4} \left(y_{top} - y_{bottom} \right) dx = \int_{0}^{4} (7x^2 - 0) dx
$$

which simplifies to:

$$
A = \int_{0}^{4} 7x^2 dx
$$

Calculating the Moments for $\bar{y}$

The y-coordinate of the centroid involves the first moment about the x-axis:

$$

$$
\iint_R y \, dA = \int_{x=0}^{4} \left(\int_{y=0}^{7x^2} y \, dy \right) dx
$$

This double integral computes the total "first moment" of the area with respect to the x-axis, which helps in finding $\bar{y}$.

Step-by-Step Solution to Find $\bar{y}$

Step 1: Compute the Area A

$$
A = \int_0^4 7x^2 dx
$$

Calculating the integral:

$$
A = 7 \int_0^4 x^2 dx = 7 \left[\frac{x^3}{3} \right]_0^4 = 7 \times \frac{4^3}{3} = 7 \times \frac{64}{3} = \frac{448}{3}
$$

Thus, the area of the region is:

$$
A = \frac{448}{3}
$$

Step 2: Compute the First Moment About the x-axis M_y

This involves integrating y over the region:

$$
M_y = \int_0^4 \left(\int_0^{7x^2} y \, dy \right) dx
$$

The inner integral:

$$
\int_0^{7x^2} y \, dy = \left[\frac{y^2}{2} \right]_0^{7x^2} = \frac{(7x^2)^2}{2} = \frac{49x^4}{2}
$$

Now, integrate with respect to x:

$$
M_y = \int_0^4 \frac{49x^4}{2} dx = \frac{49}{2} \int_0^4 x^4 dx
$$

Calculating the integral:

$$
\int_0^4 x^4 dx = \left[\frac{x^5}{5} \right]_0^4 = \frac{4^5}{5} = \frac{1024}{5}
$$

Therefore,

$$
M_y = \frac{49}{2} \times \frac{1024}{5} = \frac{49 \times 1024}{10} = \frac{49 \times 1024}{10}
$$

Simplify numerator:

$$
49 \times 1024 = (50 - 1) \times 1024 = 50 \times 1024 - 1024 = 51200 - 1024 = 50176
$$

Thus,

$$
M_y = \frac{50176}{10} = 5017.6
$$

Step 3: Calculate $\bar{y}$

Using the formula:

$$
\bar{y} = \frac{1}{A} M_y
$$

Substituting the values:

$$
\bar{y} = \frac{1}{\frac{448}{3}} \times 5017.6 = \frac{3}{448} \times 5017.6
$$

Calculating:

$$
\bar{y} = \frac{3 \times 5017.6}{448} = \frac{15052.8}{448}
$$

Simplify:

$$
\bar{y} \approx 33.6
$$

Therefore, the y-coordinate of the centroid of the region is approximately 33.6 units.

Summary of the Calculation Process

To find the y-coordinate of the centroid:

1. Determine the region boundaries based on the given curves.
2. Calculate the area using the integral of the top function $y=7x^2$.
3. Compute the first moment M_y about the x-axis by integrating y over the region.
4. Divide the moment M_y by the area A to find $\bar{y}$.

Additional Insights and Applications

Why is the Centroid Important?

The centroid is essential in various fields such as structural engineering, physics, and design. It helps determine how a structure will balance, how forces are distributed, and where to position components for optimal stability.

Extensions of the Problem

- Finding the x-coordinate of the centroid, $\bar{x}$, involves similar steps but integrating x times the area element.
- Considering more complex regions, such as those bounded by multiple curves or involving rotation, expands the application of calculus techniques.
- Using centroid calculations in real-world scenarios, such as designing beams or analyzing the center of mass in physics.

Conclusion

The process of finding the centroid of a region bounded by curves involves understanding the geometry of the region, setting up appropriate integrals, and performing careful calculations. In this specific problem, we found that the y-coordinate of the centroid for the region bounded by $y=7x^2$, $y=0$, and $x=4$ is approximately 33.6 units. Mastering these techniques equips students and professionals with essential tools for analyzing and designing various structures and systems with precision and confidence.

References and Further Reading

- Calculus Textbooks: For a deeper understanding of double integrals and centroid calculations.
- Engineering Mechanics: To explore how centroid and center of mass concepts are applied in practical scenarios.
- Online Calculus Resources: Websites like Khan Academy and Paul's Online Math Notes offer tutorials and examples on centroid calculations.

By mastering the integral calculus techniques demonstrated here, you can confidently approach similar problems involving centroid determination in various fields of science and engineering.

Frequently Asked Questions

What is the region bounded by the curves in the problem?

The region is bounded by the parabola $y = 7x^2$, the x-axis $y = 0$, and the vertical line $x = 4$.

How do we set up the integral to find the y-coordinate of the centroid?

The y-coordinate of the centroid, $\bar{y}$, is given by $(1/A) \int y \, dA$ over the region, where A is the area of the region.

What is the formula for the area of the region?

The area A is calculated by integrating with respect to x: A = ∫ from 0 to 4 of y dx, which simplifies to $A = \int_0^4 7x^2 \, dx$.

How do we compute the centroid's y-coordinate for this region?

Calculate $\bar{y} = (1/A) \int_0^4 y^2 \, dx$, where $y = 7x^2$, so $\bar{y} = (1/A) \int_0^4 (7x^2) \, y \, dx$.

What is the value of the area A for the given region?

$A = \int_0^4 7x^2 \, dx = 7 \, (x^3 / 3)$ evaluated from 0 to 4 $= 7 \, (64 / 3) = 448/3$.

How do you compute the y-coordinate $\bar{y}$ of the centroid?

$Y = (1/A) \int_0^4 y \, y \, dx = (1/A) \int_0^4 (7x^2) (7x^2) \, dx = (1/A) \int_0^4 49x^4 \, dx$.

What is the numerical value of $\bar{y}$ after calculation?

Calculating the numerator: $\int_0^4 49x^4\,dx = 49\,(x^5 / 5)$ from 0 to 4 $= 49\,(1024 / 5) = 49\ 204.8 = 10{,}035.2$. Then $\bar{y} = (1 / (448/3))\ 10{,}035.2 = (3/448)\ 10{,}035.2 \approx 67.2$.

What is the final answer for the y-coordinate of the centroid?

The y-coordinate of the centroid $\bar{y}$ is approximately 67.2 units.

Additional Resources

Centroid Calculation of the Region Bounded by $y=7x^2$, $y=0$, and $x=4$

Introduction: Unlocking the Geometric Secret of a Parabolic Region

Understanding the centroid of a region is a fundamental concept in calculus and engineering, offering insights into the "center of mass" of a shape when uniform density is assumed. Imagine you're tasked with designing a structural component, or perhaps analyzing the balance point of a physical object. Knowing the centroid helps determine how forces distribute or how to position the shape for optimal stability.

In this detailed exploration, we analyze a specific bounded region defined by the parabola $y=7x^2$, the x-axis $y=0$, and the vertical line $x=4$. Our goal? To find the y-coordinate of its centroid, denoted as $\bar{y}$. This process not only involves calculus but also showcases the elegance of geometric reasoning.

Let's dive into this mathematical adventure, step by step, with clarity and precision.

Understanding the Region of Interest

Before calculating the centroid, it's essential to visualize and define the region precisely.

Boundaries of the Region

- Lower boundary: $y=0$ (the x-axis)
- Upper boundary: $y=7x^2$ (a parabola opening upwards)
- Vertical boundary: $x=4$

Note: Since $y=7x^2$ is a parabola, the region is bounded between $x=0$ and $x=4$, with the parabola forming the top boundary and the x-axis the bottom.

Visual Summary:
- The region extends from the origin along the x-axis up to $x=4$.
- The parabola arches above the x-axis within this range.
- The region is the area between $y=0$ and $y=7x^2$, from $x=0$ to $x=4$.

Note on the region's shape: It resembles a "parabolic cap" stretching from the origin to $x=4$.

Mathematical Foundations: Setting Up the Problem

To find the centroid's y-coordinate, $\bar{y}$, we employ the standard formula for the centroid of a planar region with uniform density:

$$
\bar{y} = \frac{1}{A} \int_{R} y \, dA
$$

Where:
- A is the area of the region.
- dA is a differential element of area.

Given the region's shape, it's most straightforward to work with vertical slices (dy) or horizontal slices (dx). Here, because the boundaries are expressed in terms of y as a function of x, integrating with respect to x is most natural.

Key formulas:

- Area A:
$$
A = \int_{x=0}^{x=4} \left(\text{top boundary} - \text{bottom boundary} \right) dx
$$
- Centroid $\bar{y}$:
$$
\bar{y} = \frac{1}{A} \int_{x=0}^{x=4} \left(\frac{1}{2} \left[y_\text{top}^2 - y_\text{bottom}^2 \right] \right) dx
$$
or, more simply, recognizing that for a region bounded between $y=0$ and $y=7x^2$, the y-coordinate centroid is obtained via:

$$
\bar{y} = \frac{1}{A} \int_{x=0}^{x=4} \frac{1}{2} \left(y_\text{top}^2 \right) dx
$$

But in this case, it’s cleaner to use the standard method:

\[
\boxed{
\bar{y} = \frac{1}{A} \int_{x=0}^{x=4} \left(\frac{1}{2} \left(
y_\text{top}^2 - y_\text{bottom}^2 \right) \right) dx
}
\]

Given \(y_\text{bottom} = 0 \), this simplifies.

Step-by-Step Calculation

1. Calculating the Area \(A \)

The area of the region is:

\[
A = \int_{0}^{4} \left(y=7x^2 - 0 \right) dx = \int_{0}^{4} 7x^2 dx
\]

Performing the integral:

\[
A = 7 \int_{0}^{4} x^2 dx = 7 \left[\frac{x^3}{3} \right]_{0}^{4} = 7 \times
\frac{4^3}{3} = 7 \times \frac{64}{3} = \frac{448}{3}
\]

Result:

\[
\boxed{
A = \frac{448}{3} \text{ square units}
}
\]

---

2. Calculating the Moment About the x-axis \(M_y \)

The y-coordinate of the centroid involves the first moment of the area:

\[
M_y = \int_{A} y \, dA
\]

Expressed as an integral:

\[
M_y = \int_{x=0}^{4} \left(y \times \text{area of a vertical slice} \right)
dx
\]

Since each vertical slice has height $y=7x^2$ and width dx, the differential element of the moment is:

$$
dM_y = y \times y \, dx = y^2 dx
$$

Thus,

$$
M_y = \int_{0}^{4} y^2 dx
$$

Substitute $y=7x^2$:

$$
M_y = \int_{0}^{4} (7x^2)^2 dx = \int_{0}^{4} 49x^4 dx
$$

Perform the integration:

$$
M_y = 49 \int_{0}^{4} x^4 dx = 49 \left[\frac{x^5}{5} \right]_0^4 = 49 \times \frac{4^5}{5}
$$

Calculate 4^5:

$$
4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024
$$

Therefore,

$$
M_y = 49 \times \frac{1024}{5} = \frac{49 \times 1024}{5}
$$

Compute numerator:

$$
49 \times 1024 = (50 - 1) \times 1024 = 50 \times 1024 - 1024 = 51200 - 1024 = 50176
$$

So,

$$
M_y = \frac{50176}{5}
$$

---

3. Finding $\bar{y}$

Recall:

$$
\bar{y} = \frac{M_y}{A}
$$

Plugging in the values:

$$
\bar{y} = \frac{\frac{50176}{5}}{\frac{448}{3}} = \frac{50176}{5} \times
\frac{3}{448}
$$

Simplify:

$$
\bar{y} = \frac{50176 \times 3}{5 \times 448}
$$

Calculate numerator:

$$
50176 \times 3 = 150528
$$

Calculate denominator:

$$
5 \times 448 = 2240
$$

Thus,

$$
\boxed{
\bar{y} = \frac{150528}{2240}
}
$$

Simplify the fraction:

Divide numerator and denominator by 64:

$$
150528 \div 64 = 2352
$$
$$
2240 \div 64 = 35
$$

So,

$$
\bar{y} = \frac{2352}{35}
$$

Express as a decimal:

$$
2352 \div 35 \approx 67.2
$$

Final Result and Interpretation

The y-coordinate of the centroid of the bounded region is:

$$
\boxed{
\bar{y} = \frac{2352}{35} \approx 67.2
}
$$

This value indicates that the region's center of mass (assuming uniform density) lies approximately 67.2 units above the x-axis, reflecting the influence of the parabolic shape which extends upward significantly as x increases.

Additional Insights and Practical Applications

Understanding the centroid isn't just a theoretical exercise; it has practical implications across various fields:

- Structural engineering: Design of beams or components where the centroid determines the neutral axis.
- Physics: Calculation of center of mass in planar objects.
- Manufacturing: For tasks like cutting or material distribution, knowing the centroid ensures balance and stability.
-

1 Point Find The Centroid Y Of The Region Bounded By Y 7x 2 Y 0 And X 4

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